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GEOMETRIYADAN MASALALAR TO‘PLAMI

Akademik litsey va kasb-hunar kollejlari uchun o‘quv qo‘llanma

2-NASHRI

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Mazkur qo'llanma 2002-yilda "Ustoz" Respublika jamg'armasining "Yilning eng yaxshi darsligi va o'quv adabiyoti muallifi" respublika tanlovida 1-o'rinni egallagan.

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Mazkur qo'llanma "Geometriya" fanidan akademik litseylar uchun mavjud o'quv dasturi asosida yozilgan bo'lib, unda har bir bo'lim bo'yicha yechilishi zarur bo'lgan masalalar test topshiriqlari shaklida berilgan.

Qo'llanma akademik litsey va kasb-hunar kollejlari talabalari uchun mo'ljallangan, shuningdek, undan, oliy o'quv yurtlariga kirish uchun test sinovlariga mustaqil tayyorlanayotganlar ham foydalanishlari mumkin.

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SO'ZBOSHI

Mazkur qo'llanma "Geometriya" fani bo'yicha akademik litseylar uchun mavjud o'quv dasturida ko'rsatilgan barcha bo'limlarga doir masalalarni o'z ichiga oladi.

Qo'llanma ikki qismdan iborat bo'lib, o'n beshta paragrafdan tashkil topgan. Har bir paragrafning boshida mavzuga oid asosiy tushuncha, tasdiq va formulalar keltirilgan. So'ngra har bir paragraf mavzusi bo'yicha qator masalalar keltirilib, ularning yechilishlari bayon qilingan. Har bir paragrafning oxirida mustaqil yechish uchun masalalar ham berilgan.

Qo'llanmaning yozilishidan asosiy maqsad, Ta'lim to'g'risidagi Qonun va Kadrlar tayyorlash milliy dasturini amalga oshirish tadbirlaridan biri sifatida matematikadan adabiyotlar majmuasi (komplekti) yaratishdan iborat bo'lib, u akademik litseylar va kasb-hunar kollejlari talabalari uchun mo'ljallangan. Shuningdek, qo'llanma matematikani mustaqil o'rganib, oliy o'quv yurtlariga kirish test sinovlariga tayyorlanayotganlarga ham yordam beradi va, shu bilan birga, "Geometriya" fanidagi barcha tushunchalar, asosiy formulalar va tasdiqlarning masalalarni yechishda qo'llanilishini chuqurroq o'rganish imkoniyatini yaratadi.

Qo'llanma haqidagi fikr va mulohazalaringizni mualliflar mamnuniyat bilan qabul qiladilar.

Mualliflar

1-qism
PLANIMETRIYA

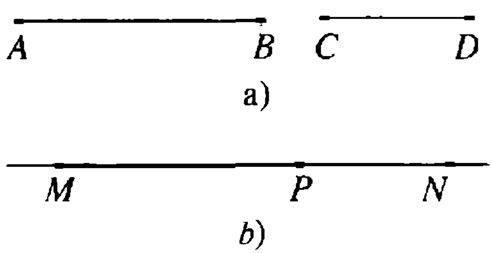
1-§. ASOSIY TUSHUNCHALAR

1.1 Kesma va burchaklar

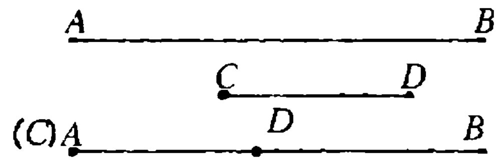
Eng sodda tushunchalar orqali ta'riflash mumkin bo'lmagan tushunchalar *boshlang'ich tushunchalar* deyiladi. Geometriyada shunday boshlang'ich tushunchalar jumlasiga *nuqta, to'g'ri chiziq, tekislik* kiradi. Boshlang'ich tushunchalarning xossalari aksiomalar yordamida kiritiladi. To'g'ri chiziqning ikki tomondan chegaralangan qismi *kesma* deyiladi. Bir tomondan chegaralangan to'g'ri chiziq *nur* (yarim to'g'ri chiziq) deb ataladi.

Chetki nuqtalari ustma-ust tushadigan kesmalar *teng* kesmalar deyiladi.

Berilgan ikkita *AB* va *CD* (1.1-chizma) kesmaning yig'indisini topish uchun to'g'ri chiziqni va unda biror *M* nuqtani (1.1-chizma) olamiz, so'ngra sirkul yordamida bu to'g'ri chiziqning M nuqtasidan avvalo *AB* kesmaga



1.1-chizma.



1.2-chizma.

teng MP kesma ajratamiz va uning oxiridan shu yo'nalish bo'yicha CD kesmaga teng PN kesma ajratamiz. Hosil qilingan MN kesma AB va CD kesmalarning yig'indisi deyiladi:

$$MN = AB + CD.$$

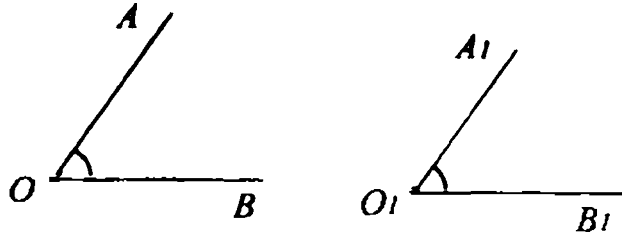
Faraz qilaylik, $|AB| > |CD|$ bo'lsin. CD kesmaning C uchini A nuqtadan qo'yib, CD kesmani AB kesmaning ichki qismida yasaymiz. U holda DB kesma AB va CD kesmalarning *ayirmasi* deb ataladigan kesmani beradi (1.2-chizma).

Umumiy uchga ega bo'lgan ikkita nurdan tashkil topgan geometrik shakl *burchak* deb ataladi. Nurlar burchakning *tomonlari,* ularning umumiy nuqtasi burchakning *uchi* deb ataladi va $\angle AOB$ yoki $\angle O$ kabi belgilanadi.

Tekislikda olingan burchakning tomonlari tekislikni ikki qismga bo'ladi. Har bir burchak uchun bu qismlarning biri uning ichki qismi, ikkinchisi tashqi qismi bo'ladi.

Agar burchakning tomonlari bir to'g'ri chiziqning to'ldiruvchi yarim to'g'ri chiziqlaridan iborat bo'lsa, u *yoyiq burchak* deyiladi.

Burchakning kattaligi transportir yordamida topiladi. Agar burchaklarning kattaliklari bir xil bo'lsa, ular teng burchaklar deyiladi. Boshqacha aytganda, agar $\angle A_1O_1B_1$ ni o'z-o'ziga parallel siljitib, O_1 nuqtani O nuqtaga, O_1B_1 nurni OB nurga ustma-ust tushirganda O_1A_1 tomon OA



1.3-chizma.

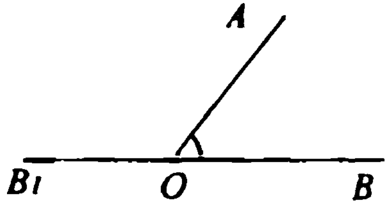
tomon bilan ustma-ust tushsa, $\angle AOB = \angle A_1O_1B_1$bo‘ladi (1.3-chizma).

Bitta tomoni umumiy bo‘lib, qolgan tomonlari bir to‘g‘ri chiziqni to‘ldiruvchi burchaklar *qo‘shni burchaklardir*. Masalan, $\angle AOB_1$ va $\angle AOB$ qo‘shni burchaklardir. $\angle BOB_1$ esa *yoyiq burchakdir* (1.4-chizma). Shuning uchun qo‘shni burchaklarning yig‘indisi

$$\angle AOB + \angle AOB_1 = 180^\circ \qquad (1.1)$$

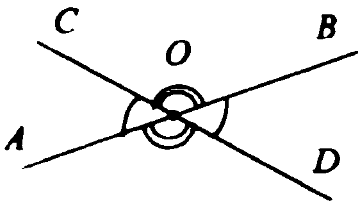
tenglikni qanoatlantiradi.

Qo‘shni burchaklar o‘zaro teng bo‘lsa, ularning har biri to‘g‘ri burchakdan iborat bo‘lib, kattaliklari 90° ga teng. Ikkita *AB* va *CD* to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklar *vertikal burchaklar* deb ataladi (1.5-chizmada $\angle AOC$ va $\angle BOD$; $\angle AOD$ va $\angle BOC$— vertikal burchaklar). Vertikal burchaklar o‘zaro teng bo‘ladi: $\angle AOC = \angle BOD$, $\angle AOD = \angle BOC$.

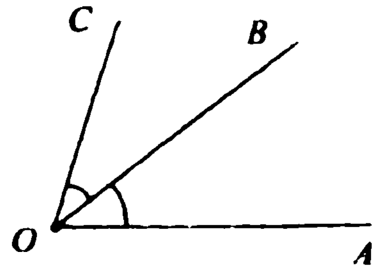


1.4-chizma.

Ikkita $\angle AOB$ va $\angle BOC$ burchakni qo‘shish (ayirish) uchun ularning uchlarini va bittadan tomonini ustma-ust tushiramiz. So‘ngra ularni qo‘shish uchun ikkinchi bur-



1.5-chizma.

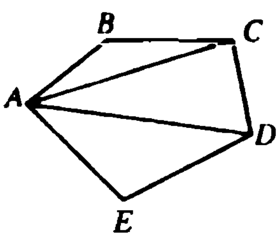


1.6-chizma.

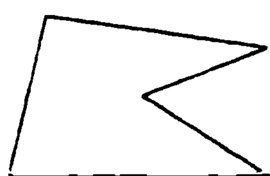
chakning ikkinchi tomonini birinchi burchakning tashqarisidan, ayirish uchun esa ichkarisidan yo'naltiramiz (1.6-chizma).

1.2. Ko'pburchaklar

Ko'pburchak tekislikda sodda yopiq siniq chiziqdan tashkil topgan shakldir (1.7- chizmada *ABCDE* beshburchak tasvirlangan). Siniq chiziqning bo'g'inlari ko'pburchakning *tomonlari* (*AB, BC, CD, DE, EA*), siniq chiziqning uchlari esa ko'pburchakning *uchlaridir* (*A, B, C, D, E*). Tomonlarining soniga qarab ko'pburchaklar *uchburchak, to'rtburchak, beshburchak* va hokazo deb nomlanadi.



qavariq beshburchak



qavariq bo'lmagan beshburchak

1.7- chizma.

Ko'pburchakning *perimetri* uning hamma tomonlari uzunliklarining yig'indisidan iborat.

Agar ko'pburchakning ixtiyoriy ikki nuqtasini tutashtiruvchi kesma shu ko'pburchakka tegishli bo'lsa, bu ko'pburchak *qavariq* bo'ladi. Aks holda ko'pburchak qavariq bo'lmaydi.

Ko'pburchakning ikkita qo'shni tomoni hosil qilgan burchaklar uning *ichki burchaklari,* ko'pburchakning ichki burchaklariga qo'shni bo'lgan burchaklar ko'pburchakning *tashqi burchaklari* deyiladi.

Ko'pburchak ichki burchaklarining yig'indisi quyidagi formula yordamida topiladi:

$$\Sigma=2d\,(n-2),\ (n \text{ — tomonlar soni, } d=90^\circ). \qquad (1.2)$$

Ko'pburchakning ikkita qo'shni bo'lmagan uchlarini tutashtiruvchi kesma ko'pburchakning *diagonali* deyiladi (1.7-chizmada *AC, AD*).

Ko'pburchakning muhim xossalari quyidagilardir.

1. Ixtiyoriy ko'pburchak tashqi burchaklarining yig'indisi 360° ga teng.

2. Muntazam ko'pburchakning hamma ichki burchaklari teng.

1.3. Parallel to'g'ri chiziqlar

Bir tekislikda yotib, kesishmaydigan *a* va *b* to'g'ri chiziqlar parallel to'g'ri chiziqlar deyiladi va ular $a \parallel b$ kabi belgilanadi.

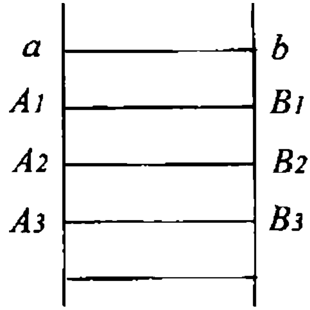
Parallel to'g'ri chiziqlarning xossalari:

4. Agar *a* va *b* to'g'ri chiziqlar parallel bo'lsa, ular orasidagi masofa o'zgarmas miqdordir.

5. Bitta to'g'ri chiziqqa parallel bo'lgan hamma to'g'ri chiziqlar o'zaro paralleldir.

6. Bir tekislikda yotib, bitta to'g'ri chiziqqa perpendikulyar bo'lgan hamma to'g'ri chiziqlar o'zaro paralleldir.

7. Ikkita parallel a va b to'g'ri chiziqlarga perpendikulyar bo'lgan to'g'ri chiziqlarning bu parallel to'g'ri chiziqlar orasidagi qismlari o'zaro tengdir: $A_1B_1 = A_2B_2 = A_3B_3$ (1.8-chizma).



1.8-chizma.

8. Burchak tomonlarini bir necha parallel to'g'ri chiziqlar kesib o'tsa, burchakning tomonlari o'zaro proporsional bo'lgan kesmalarga ajraladi (Fales teoremasi):

$$\frac{OA}{OA_1} = \frac{AA_1}{BB_1} = \ldots = \frac{A_{n-1}A_n}{B_{n-1}B_n} \quad \text{(1.9-chizma)}$$

Tekislikda ikkita a va b to'g'ri chiziqni uchinchi c to'g'ri chiziq kesib o'tgan bo'lsin, u holda hosil bo'lgan burchaklar quyidagicha nomlanadi (1.10-a chizma):

$\angle 3$ va $\angle 5$, $\angle 4$ va $\angle 6$ — *ichki almashinuvchi burchaklar;*

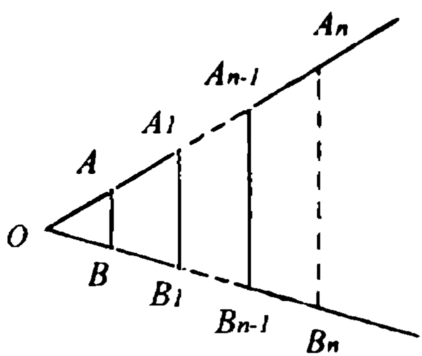
$\angle 1$ va $\angle 8$, $\angle 2$ va $\angle 7$ — *tashqi almashinuvchi burchaklar;*

$\angle 1$ va $\angle 5$, $\angle 2$ va $\angle 6$, $\angle 3$ va $\angle 7$, $\angle 4$ va $\angle 8$ — *mos burchaklar;*

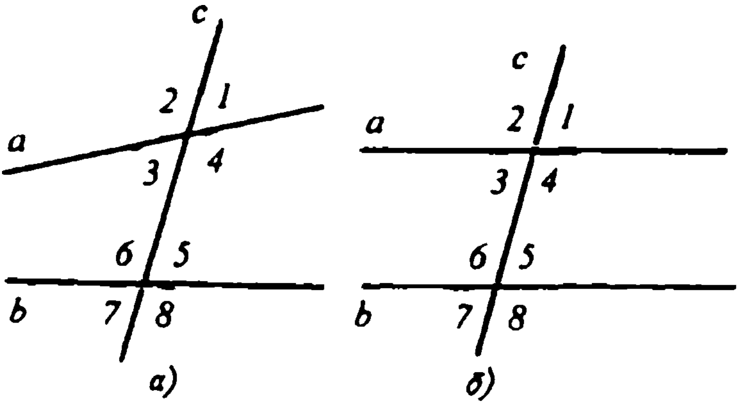
$\angle 3$ va $\angle 6$, $\angle 4$ va $\angle 5$ — *ichki bir tomonli burchaklar;*

$\angle 2$ va $\angle 7$, $\angle 1$ va $\angle 8$ — *tashqi bir tomonli burchaklar.*

9. Agar parallel a va b to'g'ri chiziqlar c to'g'ri chiziq bilan kesishgan bo'lsa (1.10-b chizma), u holda:



1.9- chizma.



1.10-chizma

1) ichki almashinuvchi burchaklar teng: $\angle 3=\angle 5$, $\angle 4=\angle 6$;

2) tashqi almashinuvchi burchaklar teng: $\angle 1=\angle 7$, $\angle 2=\angle 8$;

3) mos burchaklar teng: $\angle 1=\angle 5$, $\angle 2=\angle 6$, $\angle 3=\angle 7$, $\angle 4=\angle 8$;

4) ichki bir tomonli burchaklarning yig'indisi 180° ga teng: $\angle 3+\angle 6=180°$, $\angle 4+\angle 5=180°$;

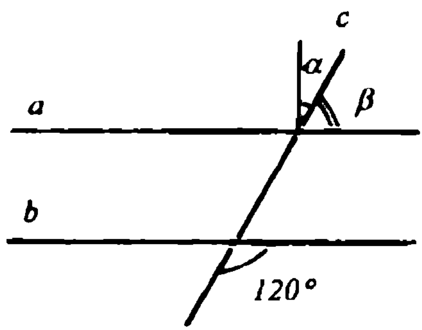
5) tashqi bir tomonli burchaklarning yig'indisi 180°ga teng, $\angle 2+\angle 7=180°$; $\angle 1+\angle 8=180°$.

10. Ikkita *a* va *b* to'g'ri chiziq uchinchi *c* to'g'ri chiziq bilan kesishganda: 1) ichki almashinuvchi burchaklar teng, 2) mos burchaklar teng, 3) bir tomonli ichki (tashqi) burchaklarning yig'indisi 180°ga teng bo'lsa, *a* va *b* to'g'ri chiziqlar paralleldir ($a \parallel b$).

1.4. Mavzuga oid ba'zi masalalarning yechimlari

1. Berilgan. $a \parallel b$, $\gamma=120°$.

α topilsin (1.4.1- chizma).



1.4.1-chizma.

Yechilishi. Tashqi bir tomonli burchaklar yig'indisi 180° ga teng: $\beta+120°=180°$ va $\beta=180°-120°=60°$. Shartga ko'ra $\alpha+\beta=90°$, u holda $\alpha=90°-\beta$, $\alpha=90°-60°=30°$.

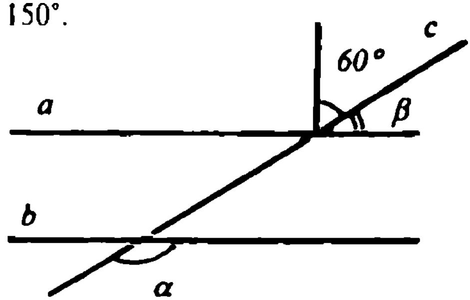
Javobi: 30°.

2. Berilgan. $a \parallel b$, $\gamma=60°$.

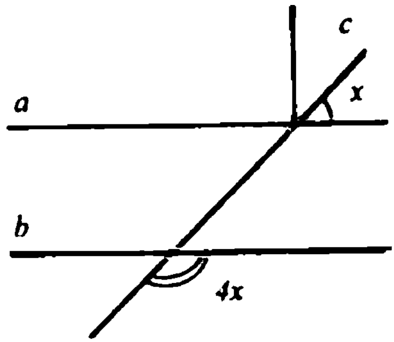
α topilsin (1.4.2-chizma).

Yechilishi. a va b bir tomonli tashqi burchaklardir. Shuning uchun $\alpha+\beta=180°$. Ikkinchi tomondan, $\beta+60°=90°$ tenglama: $\beta=90°-60°=30°$ va $\alpha=180°-30°=$ $=150°$ ekanligini beradi.

Javobi: 150°.



1.4.2- chizma.



1.4.3- chizma.

3. Berilgan. $a \parallel b$.

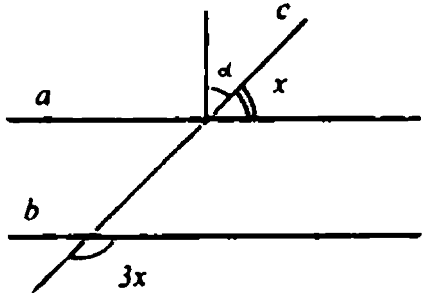
x topilsin (1.4.3-chizma).

Yechilishi. $a \parallel b$ bo'lgani uchun, 9- xossaga muvofiq tashqi bir tomonli burchaklarning yig'indisi 180°ga teng. Demak, $x+4x=180°$, $5x=180°$, $x=36°$.

Javobi: $x=36°$.

4. Berilgan. $a \parallel b$.

α topilsin (1.4.4- chizma).



1.4.4- chizma.

Yechilishi. $a \parallel b$ bo'lgani uchun, 9- xossaga muvofiq, $3x$ va x bir tomonli tashqi burchaklar bo'ladi va $3x+x=180°$, $4x=180°$, $x=45°$; $\alpha+x=90°$, u holda $\alpha=90°-$ $-45°=45°$.

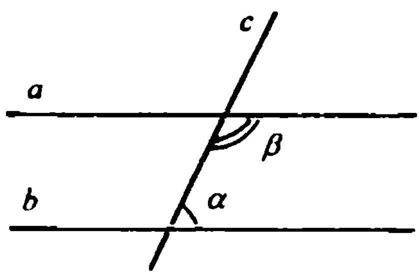
Javobi: $\alpha=45°$.

5. Berilgan. $a \parallel b$; $\beta=\alpha+34°$.

α, β topilsin (1.4.5- chizma).

Yechilishi. $a \parallel b$ bo'lganligi uchun 9- xossaga muvofiq, ichki bir tomonli α, β burchaklar uchun $\alpha+\beta=180°$ bo'ladi yoki $\alpha+\alpha+34°=180°$, $2\alpha=146°$, $\alpha=73°$ va $\beta=73°+34°=107°$.

Javobi: $\alpha=73°$, $\beta=107°$.

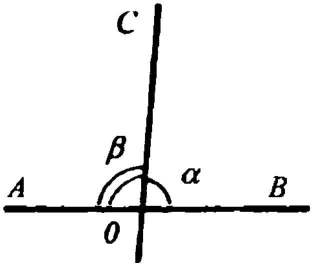


1.4.5- chizma.

6. Berilgan. $\angle AOC$, $\angle COB$ qo'shni burchaklar, $\angle AOC=\angle COB+20°$.

$\angle AOC$, $\angle COB$ topilsin (1.4.6-chizma).

Yechilishi. (1.1) formulaga asosan, qo'shni burchaklarning yig'indisi 180°ga teng, ular uchun $\angle COB=\alpha$, $\angle AOC=\beta$ belgilashlar kiritamiz. Ikkita α, β noma'lumga nisbatan



1.4.6- chizma.

$$\begin{cases} \alpha + \beta = 180^\circ, \\ \beta - \alpha = 20^\circ \end{cases}$$

sistemani hosil qildik. Sistemadagi tenglamalarni qo'shamiz: $\beta + \beta = 200^\circ$, $\beta = 100^\circ$ va $\alpha = 100^\circ - 20^\circ = 80^\circ$.

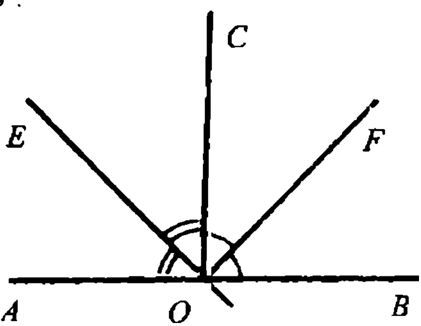
Javobi: 80° va 100°.

7. B e r i l g a n. $\angle AOC$, $\angle COB$ qo'shni burchaklar, OE, OF — ularning bissektrisalari.

$\angle EOF$ t o p i l s i n (1.4.7- chizma).

Y e c h i l i s h i. (1.1) formulaga asosan, qo'shni burchaklarning yig'indisi 180° ga teng, ya'ni $\angle AOE + \angle EOC + \angle COF + \angle FOB = 180^\circ$, bu yerdan, $2(\angle EOC + \angle COF) = 180^\circ$ va $\angle EOF = \angle EOC + \angle COF = 90^\circ$.

Javobi: 90°.

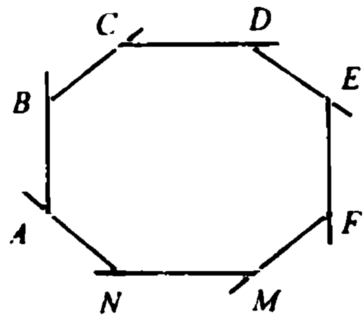


1.4.7- chizma.

8. B e r i l g a n. Ko'pburchak.
$S_{ich} = S_{tash} + 720^\circ$.

n t o p i l s i n (1.4.8- chizma).

Yechilishi. Ko'pburchakning tomonlari soni n bo'lsin. (1.2) formulaga muvofiq, ko'pburchak ichki burchaklari yig'indisi $180°(n-2)$ga teng, 1.2- banddagi 1- xossaga muvofiq bir yo'nalishda olingan tashqi burchaklari yig'indisi 360°ga teng. Shartga ko'ra $180°(n-2)=360°+720°$, $180°(n-2)=1080°$, $n-2=6$, $n=8$.



1.4.8-chizma.

Javobi: $n=8$.

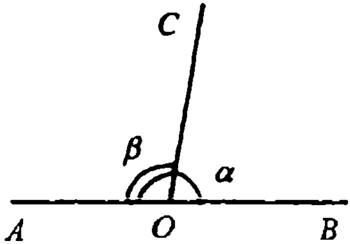
9. Berilgan. $\angle AOC$, $\angle COB$ qo'shni burchaklar, $\angle AOC : \angle COB = 11:7$.

$\angle AOC$, $\angle COB$ topilsin (1.4.9-chizma).

Yechilishi. $\angle AOC=\alpha$, $\angle COB=\beta$ bo'lsin. α va β ga nisbatan $\begin{cases}\alpha+\beta=180°,\\ \beta:\alpha=11:7\end{cases}$ tenglamalar sistemasini yozamiz.

Bu yerdan $\begin{cases}\beta=\frac{11}{7}\alpha,\\ \frac{11}{7}\alpha+\alpha=180°\end{cases} \Rightarrow \begin{cases}\alpha=110°,\\ \beta=70°.\end{cases}$

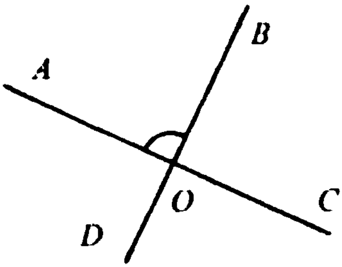
Javobi: 70° va 110°.



1.4.9- chizma.

10. Berilgan. $AC \cap BD = O$,
$\angle AOD + \angle AOB + \angle BOC = 255°$.

$\angle AOB$ topilsin (1.4.10-chizma).



1.4.10-chizma.

Yechilishi. AC va BD lar O nuqtada kesishganda hosil bo'lgan to'rtta burchakning yig'indisi 360°ga teng. Shartga ko'ra, uchtasining yig'indisi 255°ga teng bo'lsa, to'rtinchisi $\angle COD = 360° - 255° = 105°$ga teng. Vertikal burchaklar tengligidan $\angle AOB = \angle DOC = 105°$.

Javobi: 105°.

1.5. Mustaqil yechish uchun masalalar

1. AC to'g'ri chiziqda A va C nuqtalar orasida B nuqta yotadi. Agar $BC = 7{,}4$ sm bo'lib, AB kesmaning uzunligi AC kesmaning uzunligidan 3 marta kichik bo'lsa, AC topilsin.

A) 11,2; B) 10,6; C) 10,8; D) 11,1; E) 12,1 sm.

2. D, E, C nuqtalar bir to'g'ri chiziqda yotadi. $DE = 16$ sm, $DC = 9$ sm va D nuqta E va C nuqtalar orasida bo'lsa, CE kesmaning uzunligi topilsin.

A) 22; B) 24; C) 23; D) 26; E) 25 sm.

3. D va E nuqtalar orasida C nuqta joylashgan. Agar $DE = 16$ sm, $DC = 9$ sm bo'lsa, CE kesmaning uzunligi topilsin.

A) 7; B) 8; C) 6; D) 5; E) 9 sm.

4. $\angle ACD=80^\circ$, $\angle DCE=42^\circ$ hamda CE nur CA va CD nurlar orasidan o'tadi. $\angle ACE$ topilsin.

A) 40°; B) 39°; C) 38°; D) 42°; E) 43°.

5. $\angle AOC=48^\circ$, $\angle COD=27^\circ$ hamda OC nur OA va OD nurlar orasidan o'tsa, $\angle AOD$ topilsin.

A) 60°; B) 75°; C) 70°; D) 45°; E) 80°.

6. Qo'shni burchaklardan biri 37°ga teng bo'lsa, ikkinchisi topilsin.

A) 152°; B) 154°; C) 143°; D) 148°; E) 151°.

7. Qo'shni burchaklardan biri ikkinchisidan 8 marta katta. Katta burchakning kattaligi topilsin.

A) 160°; B) 150°; C) 130°; D) 140°; E) 145°.

8. Qo'shni burchaklarning kattaliklari 4:5 kabi nisbatda. Qo'shni burchaklardan kichigi topilsin.

A) 70°; B) 64°; C) 85°; D) 75°; E) 80°.

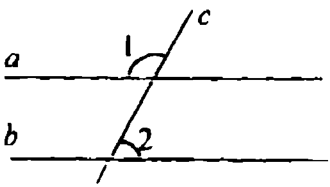
9. α burchak β burchakdan 2 marta katta, β burchak esa α burchakdan 50° kichik. Bu burchaklar qo'shni bo'lishi mumkinmi?

A) —; B) —; C) —; D) Mumkin emas; E) Mumkin.

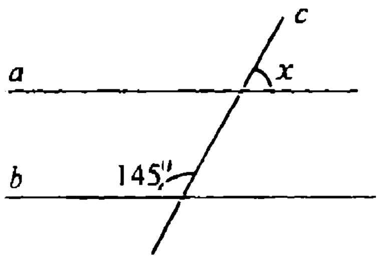
10. Yoyiq (aa_1) burchakning uchidan bitta yarimtekislikka b va c nurlar o'tkazilgan va $\angle(ac)=30^\circ$, $\angle(a_1b)=40^\circ$. d nur esa $\angle(bc)$ burchakning bissektrisasi bo'lsa, $\angle(dc)$ burchak topilsin.

A) 45°; B) 55°; C) 50°; D) 60°; E) 40°.

11. a to'g'ri chiziqqa nisbatan har xil yarimtekisliklarda A va C nuqtalar olingan. Ular a to'g'ri chiziqning



1.5.1- chizma.



1.5.2- chizma.

biror *O* nuqtasi bilan tutashtirilgan. Hosil qilingan to'rtta burchakdan biri 35°ga, ikkinchisi 115°ga teng. *O* nuqta *AC* to'g'ri chiziqda bo'lishi mumkinmi?

A) *C* nuqta *O* va *A* orasida; B) *O* nuqta *A* va *C* orasida; C) *A* nuqta *O* va *C* orasida; D) Ha; E) Yo'q.

12. Qo'shni burchaklardan biri ikkinchisidan 50° kichik. Katta burchak topilsin.

A) 105°; B) 90°; C) 110°; D) 115°; E) 120°.

13. $\angle(ab)=90°$, $\angle(ak)=30°$, $\angle(bk)=120°$ bo'lsa, *a* nur *b* va *k* nurlar orasidan o'tishi mumkinmi?

A) Mumkin; B) Mumkin emas; C)—; D)—; E)—.

14. *a* va *b* parallel to'g'ri chiziqlar *c* to'g'ri chiziq bilan kesishgan. $\angle 2=68°$ bo'lsa, $\angle 1$ topilsin (1.5.1-chizma).

A) 140°; B) 130°; C) 112°; D) 120°; E) 115°.

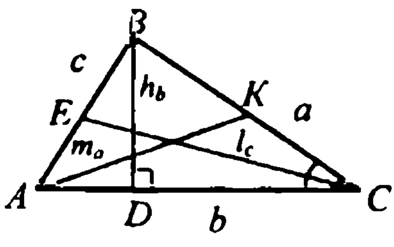
15. *a* va *b* to'g'ri chiziqlar parallel va *c* to'g'ri chiziq bilan kesishgan. *x* burchak topilsin (1.5.2-chizma).

A) 30°; B) 35°; C) 40°; D) 45°; E) 32°.

2-§. UCHBURCHAK VA UNING ELEMENTLARI

2.1 Asosiy tushunchalar va xossalar

Uchburchakning ixtiyoriy tomonini uning *asosi* deb olish mumkin. Asos qarshisida yotgan burchakning uchi uchburchakning *uchidir*.



2.1-chizma

Mediana uchburchakning uchi bilan unga qarshi tomonning o'rtasini tutashtiruvchi kesmadir. ABC uchburchakning tomonlarini $BC=a$, $AC=b$, $AB=c$ deb, uning A uchidan o'tkazilgan medianasini $AK=m_a$ deb belgilaymiz (2.1-chizma). Uchburchak medianasining uzunligi uning tomonlari uzunliklari orqali quyidagi formula bo'yicha topiladi:

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2};$$

$$m_b = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}; \qquad (2.1)$$

$$m_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}.$$

Uchburchakning *balandligi* uning uchidan qarshi tomonga o'tkazilgan BD perpendikulyardir (2.1- chizma).

Uchburchakning tomonlari a, b, c bo'lsin. U holda qarshi tomonga tushirilgan balandliklarning uzunliklari:

$$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)},$$

$$h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}, \qquad (2.2)$$

$$h_c = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)},$$

bunda $p = \frac{a+b+c}{2}$ — yarimperimetr.

Uchburchakning uchidan chiqib, shu uchdagi burchakni teng ikkiga bo'luvchi kesma uning bissektrisasidir (CE).

Tomonlari, a, b, c bo'lgan uchburchakda tomonlarga o'tkazilgan bissektrisalarning uzunliklari ushbu formulalar bo'yicha topiladi:

$$l_a = \frac{2}{b+c}\sqrt{bcp(p-a)} \text{ va } l_a = \frac{1}{b+c}\sqrt{bc((b+c)^2 - a^2)};$$

$$l_b = \frac{2}{a+c}\sqrt{acp(p-b)} \text{ va } l_b = \frac{1}{a+c}\sqrt{ac((a+c)^2 - b^2)};$$

$$l_c = \frac{2}{a+b}\sqrt{abp(p-c)} \text{ va } l_c = \frac{1}{a+b}\sqrt{ab((a+b)^2 - c^2)}.$$

Uchburchakning medianalari uning og'irlik markazi deb atalgan bitta nuqtada kesishadi. Balandliklar esa uchburchakning *ortomarkazi* deb atalgan nuqtada kesishadi. Shuningdek, bissektrisalar ham bitta nuqtada kesishadi.

Burchaklariga qarab uchburchaklar uch xil bo'ladi:

a) o'tkir burchakli (hamma burchaklari o'tkir);

b) o'tmas burchakli (bitta burchagi o'tmas);

d) to'g'ri burchakli (bitta burchagi to'g'ri).

Tomonlariga nisbatan uchburchaklar:

a) teng tomonli (hamma tomonlar uzunliklari o'zaro teng);

b) teng yonli (ikkita tomoni o'zaro teng);

d) ixtiyoriy uchburchak bo'ladi.

Teng tomonli uchburchak *muntazam uchburchak* ham deyiladi.

Teng yonli uchburchakda ikkita teng tomon uning *yon tomonlari*, uchinchi tomoni esa *asos* deyiladi.

Uchburchakning bitta tomonini tashqi sohaga davom ettirsak, ichki burchakka qo'shni bo'lgan burchak uchburchakning *tashqi burchagi* deb aytiladi.

To'g'ri burchakli uchburchakda to'g'ri burchak tashkil qilgan tomonlar *katetlar*, uchinchi tomon esa *gipotenuzadir*. Agar ikkita ΔABC va $\Delta A_1B_1C_1$ ning mos tomonlari proporsional, mos burchaklari esa teng bo'lsa, ular o'xshash deyiladi, ya'ni o'xshash ΔABC va $\Delta A_1B_1C_1$da

$$\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} = \frac{BC}{B_1C_1} \qquad (2.4)$$

va mos burchaklari teng, ya'ni

$$\angle A = \angle A_1, \ \angle B = \angle B_1, \ \angle C = \angle C_1. \qquad (2.5)$$

Uchburchaklar o'xshashligi belgisi ~ dir.

Uchburchak ikkita tomonining o'rtalarini tutashtiruvchi kesma uchburchakning *o'rta chizig'i* deyiladi.

Uchburchakning xossalarini keltiramiz.

1. Uchburchaklarning tenglik alomatlari:

a) agar bir uchburchakning ikkita tomoni va ular orasidagi burchagi ikkinchi uchburchakning mos ikkita tomoniga va ular orasidagi burchagiga teng bo'lsa, bu uchburchaklar tengdir.

b) agar bir uchburchakning bitta tomoni va unga yopishgan ikkita burchagi ikkinchi uchburchakning bitta tomoniga va unga yopishgan ikkita burchagiga teng bo'lsa, bu uchburchaklar tengdir.

d) agar bir uchburchakning uchta tomoni ikkinchi uchburchakning uchta tomoniga mos ravishda teng bo'lsa, uchburchaklar tengdir.

2. T e n g y o n l i uchburchakda: a) asosga tushirilgan balandlik uchburchakning ham medianasi, ham bissektrisasi bo'ladi; b) asosidagi burchaklar o'zaro teng.

3. Uchburchak ichki burchaklari yig'indisi 180° ga teng.

4. Uchburchakning tashqi burchagi uning shu burchakka qo'shni bo'lmagan ikkita ichki burchagining yig'indisiga teng.

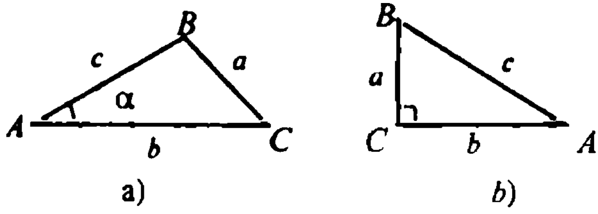
5. Har qanday uchburchakda medianalar bitta nuqtada kesishadi va kesishish nuqtasida uchburchak uchidan hisoblaganda 2:1 nisbatda bo'linadi.

6. K o s i n u s l a r t e o r e m a s i. Uchburchakda istalgan tomon uzunligining kvadrati qolgan tomonlar uzunliklari kvadratlarining yig'indisidan shu tomonlar uzunliklari va ular orasidagi burchak kosinusining ikkilangan ko'paytmasini ayirish natijasiga teng (2.2-*a* chizma):

$$a^2=b^2+c^2-2bc\cos\alpha,$$
$$b^2=a^2+c^2-2ac\cos\beta, \qquad (2.6)$$
$$c^2=a^2+b^2-2ab\cos\gamma.$$

7. P i f a g o r t e o r e m a s i. To'g'ri burchakli uchburchakda gipotenuza uzunligining kvadrati katetlar uzunliklari kvadratlarining yig'indisiga teng (2.2-*b* chizma):

$$c^2=a^2+b^2. \qquad (2.7)$$



2.2-chizma.

8. Sinuslar teoremasi. Uchburchakda tomonlar uzunliklari ular qarshisidagi mos burchaklarning sinuslariga proporsional (2.3-chizma):

$$a : \sin\alpha = b : \sin\beta = c : \sin\gamma$$

Eslatma. Ushbu nisbat uchburchakka tashqi chizilgan aylananing diametriga teng:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R. \qquad (2.9)$$

9. Uchburchaklarning o'xshashlik alomatlari:

a) bir uchburchakning ikkita burchagi ikkinchi uchburchakning ikkita burchagiga mos ravishda teng bo'lsa, ular o'xshash bo'ladi;

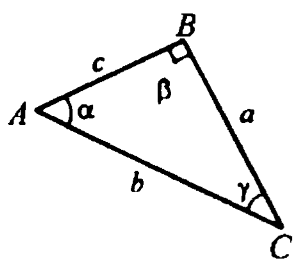
b) bir uchburchakning ikkita tomoni uzunliklari ikkinchi uchburchakning ikkita tomoni uzunliklariga mos ravishda proporsional, ular orasidagi burchaklar esa teng bo'lsa, bu uchburchaklar o'xshash bo'ladi;

d) bir uchburchakning tomonlari uzunliklari ikkinchi uchburchakning tomonlari uzunliklariga, mos ravishda, proporsional, ya'ni $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1} = k$ shart bajarilsa, bu ushburchaklar o'xshash bo'ladi, bunda k — o'xshashlik yoki proporsionallik koeffitsienti deyiladi.

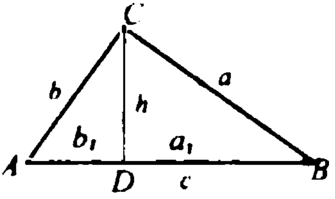
e) o'xshash uchburchaklar perimetrlari nisbati ularning o'xshash tomonlari nisbati kabi bo'ladi: $\frac{P}{P_1} = \frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$;

f) ularning yuzlari nisbati o'xshash tomonlar kvadratlari nisbati kabi bo'ladi: $S : S_1 = a^2 : a_1^2 = b^2 : b_1^2 = c^2 : c_1^2$.

10. Har qanday uchburchakka ichki aylana chizish mumkin. Uning markazi uchburchak bissektrisalarining kesishish nuqtasida bo'ladi.



2.3-chizma



2.4 -chizma.

11. Har qanday uchburchakka tashqi aylana chizish mumkin. Uning markazi uchburchak tomonlarining o'rta nuqtalaridan tomonlarga o'tkazilgan perpendikulyarlarning kesishish nuqtasida bo'ladi.

12. To'g'ri burchakli uchburchakda:

a) gipotenuzaga o'tkazilgan balandlik gipotenuzada hosil qilingan kesmalarning o'rta proporsional miqdoridir:

$$h^2=a_1 \cdot b_1; \qquad (2.10)$$

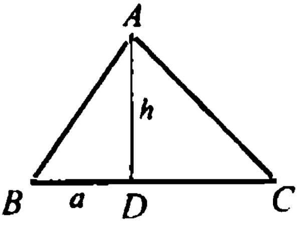
b) har bir katet gipotenuza va gipotenuzadagi proyeksiyasining o'rta proporsional miqdori bo'ladi (2.4-chizma):

$$a^2=c \cdot a_1 \text{ va } b^2=c \cdot b_1. \qquad (2.11)$$

To'g'ri burchakli uchburchakka ichki chizilgan aylana radiusi $r = \frac{a+b-c}{2}$ formuladan, tashqi chizilgan aylana radiusi esa $R = \frac{c}{a}$ formuladan topiladi, bunda a, b — katetlar uzunliklari, c — gipotenuza uzunligi.

13. Uchburchakning o'rta chizig'i asosga parallel va uning yarmiga teng.

14. Uchburchakning yuzini hisoblash formulalari:



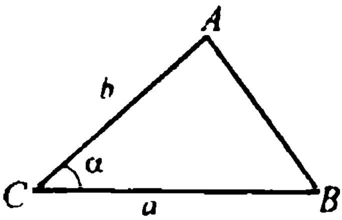
2.5- chizma.

$$S = \frac{ah}{2} \text{ (2.5- chizma)}; \qquad (2.12)$$

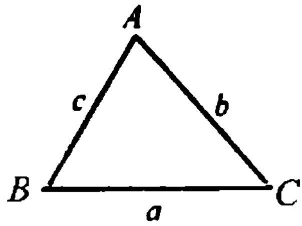
$$S = \frac{ab\sin\alpha}{2} \text{ (2.6- chizma)}; \qquad (2.13)$$

Geron formulasi:

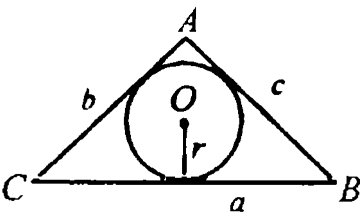
$$S = \sqrt{p(p-a)(p-b)(p-c)}, \qquad (2.14)$$



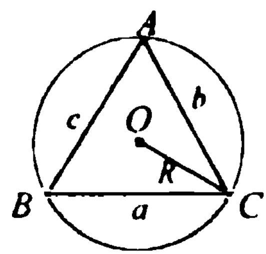
2.6- chizma



2.7- chizma



2.8- chizma.



2.9- chizma.

bunda $p = \frac{a+b+c}{2}$ (2.7- chizma)

$$S=pr, \qquad (2.15)$$

bunda r — ichki chizilgan aylananing radiusi (2.8-chizma);

$$S = \frac{abc}{4R}, \qquad (2.16)$$

bunda R — tashqi chizilgan aylananing radiusi. (2.9-chizma).

2.2. Mavzuga doir masalalar

1. a ning qanday qiymatlarida uzunliklari mos ravishda $1+a$, $1-a$ va 1,5 bo'lgan kesmalardan uchburchak yasash mumkin?

A) (0; 1,5]; B) (−0,75; 0,75); C) (−1; 1); D) (0; 1,5); E) (−3; −1).

2. Uchburchakning ikkita tomoni 0,5 va 7,9 ga teng. Uchinchi tomonining uzunligi butun son ekanligini bilgan holda shu tomoni topilsin.

A) 7; B) 9; C) 8; D) 10; E) 5.

3. Perimetri 30 sm ga teng bo'lgan uchburchak bissektrisasi bilan ikkita uchburchakka ajralgan. Bu uchburchaklarning perimetrlari 16 sm va 24 sm bo'lsa, bissektrisaning uzunligi topilsin.

A) 1; B) 3; C) 7; D) 4; E) 5 sm.

4. Teng yonli uchburchakning uchidagi burchagi 94°ga teng. Asosdagi burchaklarning bissektrisalari o'tkazilgan. Bu bissektrisalar orasidagi o'tkir burchak topilsin.

A) 37°; B) 43°; C) 48°; D) 47°; E) topish mumkin emas.

5. Uchburchakda burchaklar kattaliklari 1:2:3 nisbatda, kichik tomoni $2\sqrt{3}$ sm ga teng bo'lsa, uchburchakning perimetri topilsin.

A) $8+3\sqrt{3}$; B) $3(2+\sqrt{3})$; C) $11\sqrt{3}$; D) $9+4\sqrt{3}$; E) $6+6\sqrt{3}$ sm.

6. To'g'ri burchakli uchburchakda to'g'ri burchakning bissektrisasi gipotenuzani 1:2 nisbatda bo'ladi. To'g'ri burchak uchidan o'tkazilgan balandlik gipotenuzani qanday nisbatda bo'ladi?

A) 1:4; B) 1:5; C) 1:9; D) 1:25; E) 2:1.

7. Teng yonli uchburchakning uchidagi burchagi asosidagi burchakdan 30° katta. Uchburchakning burchaklari topilsin.

A) 30°, 30°, 60°; B) 60°, 60°, 90°; C) 40°, 50°, 80°; D) 50°, 50°, 80°; E) 50°, 80°, 80°.

8. Teng yonli uchburchakning perimetri 42 sm ga teng. Agar uchburchak asosining uzunligi yon tomonining uzunligidan 6 sm katta bo'lsa, uning tomonlari topilsin.

A) 10, 10, 16; B) 12, 12, 18; C) 13, 13, 19;
D) 14, 14, 20; E) 11 sm, 11 sm, 17 sm.

9. Uchburchakning tashqi burchagi 120°ga teng, unga qo'shni bo'lmagan burchaklar o'lchovlari nisbati 5:7 kabi bo'lsa, uchburchakning ichki burchaklari topilsin.

A) 60°, 40°, 80°; B) 50°, 80°, 50°; C) 50°, 60°, 70°;
D) 30°, 60°, 90°; E) 25°, 35°, 90°.

10. Teng yonli uchburchakda asosga tushirilgan balandlik 15 sm ga teng. Agar uning yon tomoni asosidan ikki marta katta bo'lsa, yon tomon uzunligi topilsin.

A) $2\sqrt{15}$; B) $4\sqrt{15}$; C) $\sqrt{15}$; D) 15; E) 30 sm.

11. To'g'ri burchakli uchburchakda gipotenuza 15 sm, katetlar esa 3:4 nisbatda bo'lsa, uning katta kateti topilsin.

A) 16; B) 12; C) 14; D) 9; E) 10 sm.

12. To'g'ri burchakli uchburchakda gipotenuza 20 sm, katetlar yig'indisi esa 28 sm ga teng. Uchburchakning yuzi hisoblansin.

A) 96; B) 100; C) 80; D) 120; E) 88 sm.

13. Teng tomonli uchburchakda balandlik 6 sm ga teng. Uchburchakka ichki chizilgan aylananing radiusi topilsin.

A) 6; B) 5; C) 4; D) 3; E) 2 sm.

14. To'g'ri burchakli uchburchakda katetlar 6 sm va 8 sm ga teng. To'g'ri burchak uchidan gipotenuzaga ba-

landlik o'tkazilgan. Hosil qilingan uchburchaklarning yuzlari hisoblansin.

A) 10,8 va 13,5; B) 12,4 va 15,3; C) 8,64 va 15,36; D) 9,12 va 16,48; E) 8,4 va 16.6 sm^2.

15. Tomonlari 13 sm, 14 sm, 15 sm bo'lgan uchburchakda eng kichik balandlik topilsin.

A) 11; B) 12; C) 12,2; D) 11,5; E) 11,2 sm.

16. Teng yonli uchburchakda yon tomon 5 sm ga, asosidagi burchakning kosinusi 0,6 ga teng. Uchburchakka ichki chizilgan aylananing radiusi topilsin.

A) 1; B) 1,5; C) 2; D) 2,5; E) 3 sm.

17. Uchburchakning a, b, c tomonlari $a^2=b^2+c^2+\sqrt{3}\,bc$ shartni qanoatlantirsa, a tomon qarshisidagi burchak topilsin.

A) 135°; B) 140°; C) 125°; D) 150°; E) 120°.

18. Ikkita o'xshash uchburchakning yuzlari 8 va 32 sm^2 ga, perimetrlarining yig'indisi 48 sm ga teng bo'lsa, kichik uchburchakning perimetri topilsin.

A) 12; B) 16; C) 20; D) 9,6; E) topish mumkin emas.

19. To'g'ri burchakli uchburchakda katet 7 sm ga, uning gipotenuzadagi proyeksiyasi esa 1,96 sm ga teng. Ikkinchi katetning uzunligi topilsin.

A) 12; B) 16; C) 24; D) 15; E) 26.

20. Teng yonli uchburchakning balandligi h, uchidagi burchagi β ga teng bo'lsa, uning asosi topilsin.

A) $h\sin\beta$; B) $h\cos\beta$; C) $h\sin\frac{\beta}{2}$; D) $h\cos\frac{\beta}{2}$; E) $2h\,\mathrm{tg}\,\frac{\beta}{2}$.

21. To'g'ri burchakli ABC uchburchakda tomonlar uzunliklari o'suvchi geometrik progressiyani tashkil qiladi. Uning kichik o'tkir burchagi topilsin.

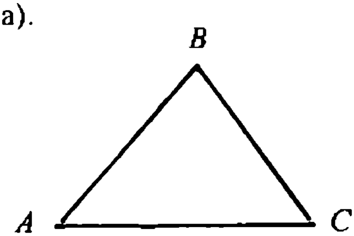
A) 35°; B) 40°; C) $\arcsin\frac{\sqrt{3}+1}{3}$; D) $\arccos\frac{\sqrt{3}+1}{3}$; E) $\arcsin\frac{\sqrt{5}-1}{2}$.

22. ABC uchburchakning AD medianasi AB va AC tomonlar bilan mos ravishda 60° va β burchaklar tashkil qiladi. $AB=\sqrt{3}$ sm, $AC=3$ sm ga teng bo'lsa, $\sin\beta$ topilsin.

A) $\frac{1}{3}$; B) $\frac{1}{2}$; C) $\frac{2}{3}$; D) $\frac{1}{4}$; E) $\frac{3}{4}$.

2.3. Mavzuga doir masalalarning yechimlari

1. Berilgan. $AB=1+a$, $AC=1-a$, $BC=1,5$. a topilsin (2.3.1- chizma).



2.3.1- chizma.

Yechilishi. Uchburchak tengsizligiga ko'ra, uchta kesma yordamida yasalgan uchburchakning ikkita tomoni uzunliklari yig'indisi uchinchi tomoni uzunligidan katta bo'lishi kerak. Shunga asosan,

$$\begin{cases} 1+a+1-a>1,5; \\ 1+a+1,5>1-a; \\ 1-a+1,5>1+a; \\ 1-a>0,\ 1+a>0 \end{cases}$$

tengsizliklar sistemasini yozamiz va uni a ga nisbatan yechamiz:

$$\begin{cases} a < 1, \\ a > -1; \\ 0a + 2 > 1{,}5; \\ 2a > -1{,}5; \\ 2a < 1{,}5, \end{cases} \quad \begin{cases} a < 1, \\ a > -1, \\ a \in R, \\ a > -0{,}75, \\ a < 0{,}75, \end{cases} \quad -0{,}75 < a < 0{,}75.$$

Javobi: B).

2. Berilgan. ΔABC, $AB=0{,}5$, $AC=7{,}9$, $BC \in N$. BC topilsin (2.3.2- chizma).

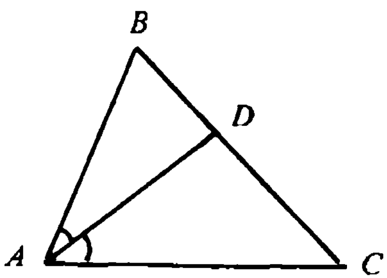
2.3.2- chizma

Yechilishi. Uchburchak tengsizligiga ko‘ra ushbu sistemani yozamiz:

$$\begin{cases} BC + 0{,}5 > 7{,}9; \\ BC + 7{,}9 > 0{,}5; \\ 7{,}9 + 0{,}5 > BC \end{cases} \quad \begin{cases} BC > 7{,}4; \\ BC > -7{,}4; \\ BC > 8{,}4 \end{cases} \quad 7{,}4 < BC < 8{,}4.$$

Shartga ko‘ra BC kesmaning uzunligi butun sondan iborat. Shuning uchun $BC=8$.

Javobi: C).



2.3.3-chizma

3. Berilgan. ΔABC, $P_{ABC}=30$ sm, AD bissektrisa, $P_{ABD}=16$ sm, $P_{ADC}=$ $=24$ sm.

AD bissektrisa topilsin (2.3.3-chizma).

Yechilishi. Uchburchakning perimetri ta’rifidan foydalanib,

$$\begin{cases} AB + BC + AC = 30, \\ AB + BD + AD = 16, \\ AC + DC + AD = 24 \end{cases}$$

tenglamalar sistemasini yozamiz. So‘ngra, oxirgi ikkita tenglamani hadma-had qo‘shamiz:

$AB+BD+AD+AC+DC+AD=16+24$ yoki $AB+AC+$ $+(BD+DC)+2\cdot AD=40$.

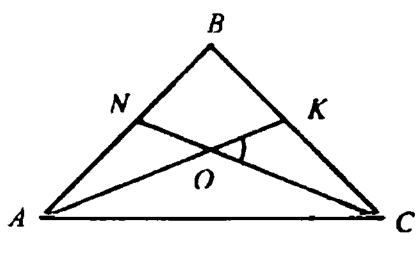
$BD+DC=BC$ yoki $AB+BC+AC=30$ bo‘lgani uchun $30+2AD=40$, $2AD=40-30$, $2AD=10$ va $AD=5$ sm ni hosil qilamiz.

Javobi: E).

4. Berilgan. ΔABC, $AB=BC$, AK, CN bissektrisalar, $AK \cup CN$, $\angle KOC<90°$, $\angle ABC=94°$.

$\angle KOC$ topilsin (2.3.4-chizma).

Yechilishi. 3-xossaga muvofiq, uchburchak ichki burchaklarining yig‘indisi 180°ga teng. 2-xossaga muvofiq, teng yonli uchburchakning asosidagi burchaklari o‘zaro teng. Shuning uchun $\angle BAC = \frac{180°-94°}{2} = \frac{86°}{2} = 43°$. AK va CN bissektrisalar, demak, $\angle KAC=\angle NCA=\frac{43°}{2}$ · ΔAOC da AK va CN bissektrisalar orasidagi burchak $\angle AOC>90°$, chunki $\angle OAC+\angle OCA<90°$. $\angle KOC$ burchak ΔAOC uchun tashqi burchak bo‘lgani sababli, uning o‘lchovi unga qo‘shni bo‘lmagan $\angle AOC$ va



2.3.4-chizma

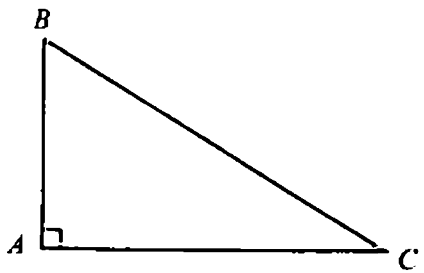
$\angle OCA$ burchaklar o'lchovlarining yig'indisiga teng, ya'ni

$$\angle KOC=\angle OAC+\angle OCA=2\angle OAC=2\cdot\frac{43^\circ}{2}=43^\circ.$$

Javobi: B).

5 Berilgan ΔABC, $\angle A:\angle B:\angle C=1:2:3$, $BC=2\sqrt{3}$ sm.

P_{ABC} perimetr topilsin (2.3.5-chizma).



2.3.5-chizma.

Yechilishi. $\angle A=x$ bo'lsin. U holda $\angle B=2x$, $\angle C=3x$. Uchburchak ichki burchaklarining yig'indisi 180°ga teng, ya'ni $x+$ $+2x+3x=180^\circ$ bo'lganligidan $6x=180^\circ$, $x=30^\circ$. Demak, $\angle A=30^\circ$, $\angle B=60^\circ$, $\angle C=90^\circ$.

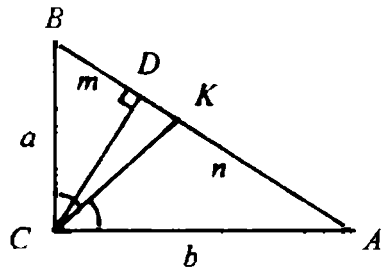
Uchburchakda kichik $\angle A$ qarshisida kichik tomon yotishi ma'lum, shuning uchun $BC=2\sqrt{3}$ sm. To'g'ri burchakli uchburchakda 30° li burchak qarshisidagi tomon gipotenuzaning yarmiga teng. Shuning uchun gipotenuza $AB=2BC=4\sqrt{3}$ sm. Ikkinchi katetni Pifagor teoremasi yordamida topamiz: $AC=\sqrt{AB^2-BC^2}=\sqrt{16\cdot3-4\cdot3}=6$ sm. U holda perimetr: $P_{ABC}=6+2\sqrt{3}+4=6+6\sqrt{3}$ sm.

Javobi: E).

6. Berilgan ΔABC, $BK:KA=1:2$, CK bissektrisa, $\angle ACK=\angle BCK=45^\circ$, CD balandlik, $CD\perp AB$.

$AD:DB$ topilsin (2.3.6-chizma).

Yechilishi. $BK=m$, $AK=n$, $BC=a$, $AC=b$, gipotenuza $AB=c$ belgilashlarni kiritamiz. U holda $m+n=c$, $x+y=c$. Shartga ko'ra $\frac{m}{n}=\frac{1}{2}$ va $n=2m$.



2.3.6- chizma.

Uchburchakning bissektrisasi qarshisidagi tomonni qolgan ikki tomonga proporsional kesmalarga ajratadi. Demak, $m:n=a:b=1:2$ va $b=2a$. Pifagor teoremasidan c gipotenuzani topamiz: $c^2=a^2+b^2=a^2+4a^2=5a^2$ va $c=a\sqrt{5}$, $c=m+n=m+2m=3m$. U holda $m=c:3=a\sqrt{5}:3$, $n==2a\sqrt{5}:3$.

ΔABC ning yuzini hisoblaymiz. Birinchidan, $S=\frac{1}{2}a\cdot b$ yoki $S=\frac{1}{2}a\cdot 2a=a^2$. Ikkinchi tomondan, $S=\frac{1}{2}c\cdot h$. Agar $CD=h$ bo'lsa, $c=a\sqrt{5}$ ni keltirib qo'yib, hosil qilingan ikkita ifodani solishtiramiz: $a^2=\frac{1}{2}\cdot a\sqrt{5}\cdot h$. U holda balandlik $h=2a:\sqrt{5}$ bo'ladi. Pifagor teoremasi yordamida ΔBCD dan $BD=x$ kesmani topamiz:

$$x^2=a^2-h^2=a^2-\frac{4}{5}\cdot a^2=\frac{(5-4)}{5}a^2=\frac{1}{5}a^2, x=\frac{a}{\sqrt{5}}.$$

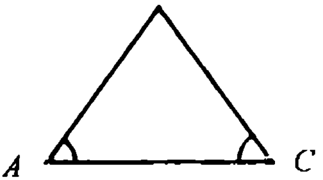
U holda $y=c-x=a\sqrt{5}-\frac{a}{\sqrt{5}}=4\frac{a}{\sqrt{5}}$ va

$$\frac{x}{y}=\frac{a}{\sqrt{5}}:\frac{4a}{\sqrt{5}}=\frac{a\sqrt{5}}{\sqrt{5}\cdot 4a}=\frac{1}{4}.$$

Javobi: A).

7. B e r i l g a n. ΔABC, $AB=BC$, $\angle ABC=\angle BAC+30°$.

$\angle A$, $\angle B$, $\angle C$ t o p i l s i n (2.3.7- chizma).



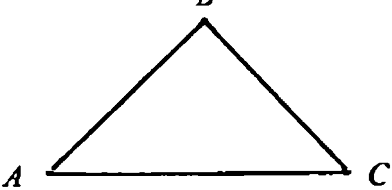
2.3.7- chizma.

Y e c h i l i s h i. Teng yonli uchburchakda 2-xossaga asosan asosidagi burchaklar o'zaro teng, demak, $\angle A=\angle C$, uchburchakda ichki burchaklar yig'indisi 180°ga teng: $2\angle A+\angle B=180°$. Shartga ko'ra $\angle B=\angle A+30°$. U holda $2\angle A+\angle B=180°$, $3\angle A=180°-30°$, $3\angle A=150°$, $\angle A=50°$, $\angle C=\angle A=50°$, $\angle B=50°+30°=80°$.

Javobi: D).

8. B e r i l g a n. ΔABC, $AB=BC$, $P_{ABC}=42$ sm, $AC=AB+6$ sm.

AB, AC t o p i l s i n (2.3.8- chizma).



2.3.8-chizma

Y e c h i l i s h i. Perimetrning ta'rifiga ko'ra:

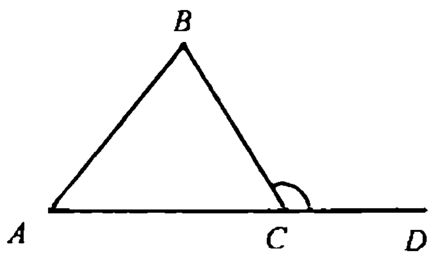
$AB+BC+AC=42$,
$2AB+AB+6=42$,
$3AB=42-6$, $AB=12$.

Demak, $AB=BC=12$ sm va $AC=12+6=18$ sm.

Javobi: B).

9. B e r i l g a n. ΔABC, $\angle BCD=120°$, $\angle A:\angle B=5:7$. $\angle A$, $\angle B$, $\angle C$ topilsin (2.3.9-chizma).

Y e c h i l i s h i. Ichki $\angle ACB$ va tashqi $\angle BCD$ qo'shni burchaklar bo'lgani uchun, ularning yig'indisi 180°ga teng. Shuning uchun $\angle ACB=180°-\angle BCD=180°-120°=60°$.



2.3.9-chizma.

Endi $\begin{cases}\angle A+\angle B=120°,\\ \angle A:\angle B=5:7\end{cases}$ sistemani yechamiz:

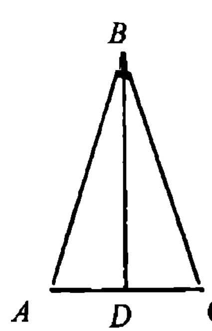
$$\begin{cases}\angle A=(5:7)\angle B,\\ (5:7)\angle B+\angle B=120°\end{cases} \Rightarrow \begin{cases}\angle A=(5:7)\angle B,\\ 12\angle B=7\cdot 120°\end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases}\angle A=(5:7)\angle B,\\ \angle B=70°\end{cases} \Rightarrow \begin{cases}\angle A=50°,\\ \angle B=70°.\end{cases}$$

Javobi: C).

10. B e r i l g a n. ΔABC, $AB=BC$, $BD\perp AC$, $BD=15$ sm, $AB=2\cdot AC$.

AB t o p i l s i n (2.3.10- chizma).



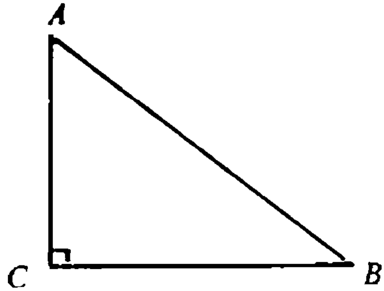
2.3.10- chizma.

Y e c h i l i s h i. BD balandlik bo'lgani uchun ΔABD to'g'ri burchakli va Pifagor teoremasidan foydalanish mumkin. $AD=x$ deb belgilaymiz. U holda $AC=2AD=2x$, $AB=4x$. ΔABD dan $AB^2=AD^2+BD^2$, $(4x)^2=x^2+15^2$,

$16x^2-x^2=15^2$, $15x^2=15^2$, $x^2=15$ va $x=\sqrt{15}$. Demak, uchburchakning yon tomoni $AB=4\sqrt{15}$ sm.

Javobi: B).

11. Berilgan. ΔABC, $\angle C=90°$, $AC:BC=3:4$, $AB=15$ sm.



2.3.11- chizma.

BC topilsin (2.3.11- chizma).

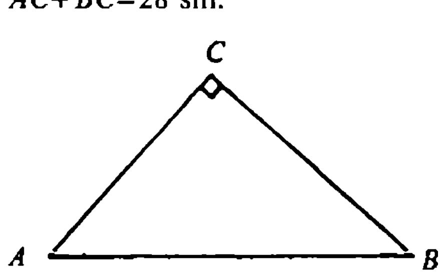
Yechilishi. Pifagor teoremasiga ko'ra, quyidagi sistemani yozamiz:

$$\begin{cases} \dfrac{AC}{BC}=\dfrac{3}{4}, \\ AC^2+BC^2=AB^2, \end{cases}$$

$$\begin{cases} AC=\dfrac{3}{4}\cdot BC, \\ \dfrac{9}{16}BC^2+BC^2=15^2, \end{cases} \quad \begin{cases} AC=\dfrac{3}{4}\cdot BC, \\ 25BC^2=15^2\cdot 16, \end{cases} \quad BC=12 \text{ sm}.$$

Javobi: B).

12. Berilgan. ΔABC, $\angle C=90°$, $AB=20$ sm, $AC+BC=28$ sm.



2.3.12- chizma.

$S_{\Delta ABC}$ hisoblansin (2.3.12- chizma).

Yechilishi. Katetlarni $AC=b$, $BC=a$, gipotenuzani $AB=c$ deb belgilaymiz. Pifagor teoremasidan foydalanib, quyidagi sistemani hosil qilamiz:

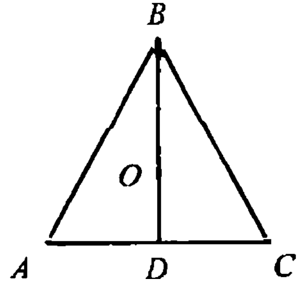
$$\begin{cases} a^2 + b^2 = 20^2, \\ a + b = 28, \end{cases} \quad \begin{cases} a^2 + b^2 = 400, \\ a + b = 28. \end{cases}$$

Lekin (2.9) formulaga muvofiq, uchburchakning yuzi $S=\frac{1}{2}\cdot a\cdot b$ ga teng. Demak, agar $a\cdot b$ ko'paytma topilsa, masala yechiladi. Ikkinchi tenglamani kvadratga ko'taramiz: $(a+b)^2=28^2$, $a^2+2ab+b^2=784$, $2ab=784-(a^2+b^2)=$ $=784-400=384$, $a\cdot b=192$. Demak, uchburchakning yuzi $S=\frac{1}{2}\cdot 192=96$ sm².

Javobi: A).

13. Berilgan. ΔABC — muntazam, $BD\perp AC$, $BD=h=6$ sm, (O, r) — ichki chizilgan aylana.

r topilsin (2.3.13-chizma).



2.3.13- chizma.

Yechilishi. Uchburchak muntazam bo'lgani uchun, $BD=h$ balandlik mediana ham bo'ladi. Shuning uchun $BO:OD=2:1$, $OD=\frac{1}{3}BD=\frac{h}{3}$. Ikkinchi tomondan, muntazam uchburchakda O nuqta ham ichki, ham tashqi chizilgan aylanalarning markazidir. Demak, $OD=r=\frac{h}{3}=\frac{6}{3}=2$ sm.

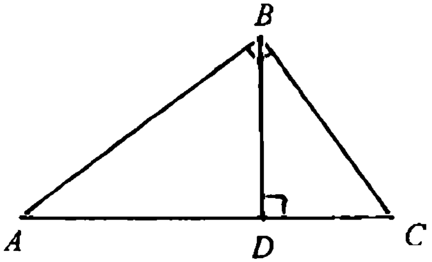
Javobi: E).

14. Berilgan. ΔABC, $\angle C=90°$, $AC=6$ sm, $BC=8$ sm, $CD\perp AB$.

$S_{\Delta ACD}$, $S_{\Delta BCD}$ hisoblansin (2.3.14- chizma).

Y e c h i l i s h i. $AB = \sqrt{AC^2 + BC^2} = \sqrt{6^2 + 8^2} =$

$= \sqrt{36 + 64} = 10\,\text{sm}.$



2.3.14- chizma.

To‘g‘ri burchakli uchburchak uchun 12-xossadan foydalanamiz. Quyidagi sistemani yozamiz:

$$\begin{cases} AC^2 = AB \cdot BD, \\ BC^2 = AB \quad BD, \\ CD^2 = AD \cdot DB, \end{cases}$$

$$\begin{cases} 6^2 = 10AD, \\ 8^2 = 10 \cdot BD, \\ CD^2 = AD \cdot DB \end{cases} \begin{cases} AD = 3{,}6, \\ BD = 6{,}4, \\ CD = \sqrt{3{,}6 \cdot 6{,}4} = 4{,}8. \end{cases}$$

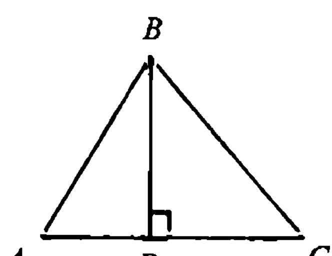
U holda $S_{\Delta ACD} = \frac{1}{2} AD \cdot CD = \frac{1}{2} \cdot 3{,}6 \cdot 4{,}8 = 8{,}64$ sm².

$S_{\Delta BCD} = \frac{1}{2} BD \cdot CD = 15{,}36$ sm².

Javobi: C).

15. B e r i l g a n. ΔABC, $BD \perp AC$, $AB = 13$ sm, $BC = 14$ sm, $AC = 15$ sm.

BD t o p i l s i n (2.3.15-chizma).



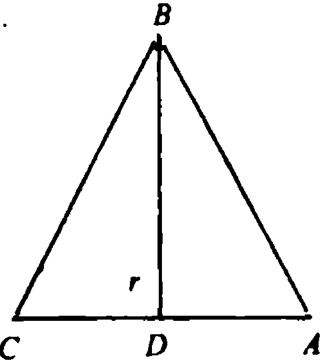
2.3.15- chizma.

Y e c h i l i s h i. Uchburchakning uchta tomoni ham ma’lum bo‘lgani uchun Geron formulasi (2.11) yordamida:

$p=\frac{13+14+15}{2}=21,\ S=\sqrt{21\cdot(21-13)(21-14)(21-15)}=$ $=7\cdot4\cdot3=84$ sm². Ikkinchi tomondan, uchburchakning yuzi (2.9) formula orqali hisoblanadi: $S=\frac{1}{2}AC\cdot BD$. Demak, BD balandlik: $BD=\frac{2S}{AC}=\frac{2\cdot84}{15}=\frac{56}{5}\approx11{,}2$ sm.

16. B e r i l g a n. ΔABC — teng yonli, $AB=BC=5$ sm, $\cos\angle A=0{,}6$, $(O,\ r)$ — ichki chizilgan aylana.

r t o p i l s i n (2.3.16-chizma).



2.3.16- chizma.

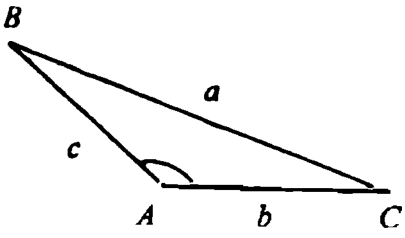
Y e c h i l i s h i. ΔBDA to‘g‘ri burchakli bo‘lganligi uchun $\cos\angle A=\frac{AD}{AB}$, bu yerdan, $AD=AB\cdot\cos\angle A=5\cdot0{,}6\ 4$, 10-xossalardan foydalansak, $\angle OAD=\frac{\angle A}{2}$. U holda to‘g‘ri burchakli ΔOAD dan $r=$ $=DO=AD\cdot\mathrm{tg}\frac{\angle A}{2}$, $r=3\cdot\mathrm{tg}\frac{\angle A}{2}$.

Yarim argumentning trigonometrik funksiyalari formulalaridan $\mathrm{tg}\frac{\alpha}{2}=\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=$ $=\sqrt{\frac{1-0{,}6}{1+0{,}6}}=\sqrt{\frac{0{,}4}{1{,}6}}=\frac{1}{2}$ ekanligini olamiz va $r=3\cdot0{,}5=1{,}5$ sm.

17. B e r i l g a n. $\Delta ABC,\ AB=c,\ AC=b,\ BC=a,$ $a^2=b^2+c^2+\sqrt{3}\,bc$.

$\angle A=\alpha$ t o p i l s i n (2.3.17-chizma).

Y e c h i l i s h i. Uchburchakning tomonlari ma’lum bo‘lgani sababli, burchakni topish uchun kosinuslar teoremasidan (6-xossa) foydalanamiz:



2.3.17- chizma.

$$a^2=b^2+c^2-2bc\cos\alpha.$$

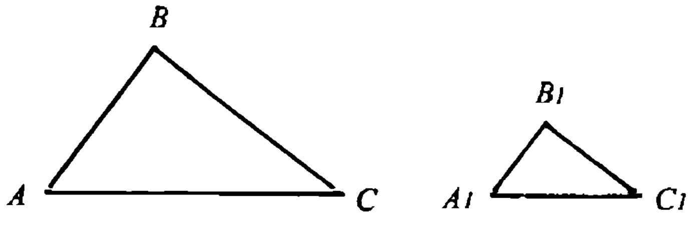
Masala shartida berilgan va bu tengliklarni solishtiramiz. Chap tomonlari teng bo'lgani uchun ularning o'ng tomonlarini tenglashtiramiz:

$$b^2+c^2-2bc\cos\alpha=b^2+c^2-\sqrt{3}\,bc,\ \cos\alpha=\frac{\sqrt{3}}{2},\ \alpha=150^\circ.$$

Javobi: D).

18. Berilgan. $\Delta ABC \sim \Delta A_1B_1C_1$, $S=32$ sm², $S_1=8$ cm², $P+P_1=48$ sm.

P_1 topilsin (2.3.18-chizma).



2.3.18-chizma.

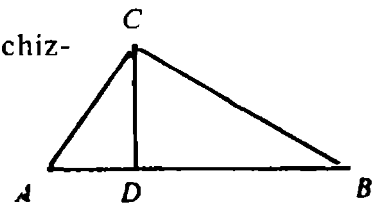
Yechilishi. O'xshash uchburchaklar yuzlarining nisbati bu uchburchaklar mos perimetrlari kvadratlarining nisbatiga tengligi bizga ma'lum, ya'ni $\frac{S}{S_1}=\left(\frac{P}{P_1}\right)^2$. Berilgan $P+P_1=48$ tenglikdan $P_1=48-P$ bo'ladi. U holda $\frac{32}{8}=\left(\frac{P}{48-P}\right)^2, \left(\frac{P}{48-P}\right)^2=4$ yoki $\frac{P}{48-P}=2$. Demak,

$P=96-2P$. Bu tenglamani yechamiz: $3P=96$, $P=32$ va $P_1=48-32=16$ sm.

Javobi: B).

19. B e r i l g a n. ΔABC, $\angle C=90°$, $AC=7$ sm, $AD=1{,}96$ sm, $CD\perp AB$.

BC t o p i l s i n (2.3.19- chizma).



2.3.19- chizma.

Y e c h i l i s h i. Katetning xossalariga ko‘ra $AC^2=AD\cdot AB$ yoki $7^2=$ $=1{,}96AB$ va gipotenuza

$$AB=\frac{49}{1{,}96}=\frac{4900}{196}=\frac{100}{4}=25 \text{ sm}$$

Pifagor teoremasidan (6-xossa) foydalanib, ikkinchi katetni topamiz:

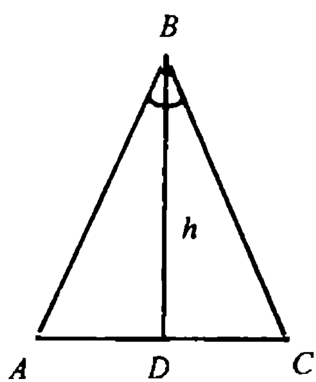
$$BC=\sqrt{AB^2-AC^2}=\sqrt{25^2-7^2},$$

$$BC=\sqrt{18\cdot 32}=\sqrt{16\cdot 36}=\sqrt{576}=24 \text{ sm}.$$

Javobi: C).

20. B e r i l g a n. ΔABC, $AB=BC$, $\angle ABC=\beta$, $AD=h\perp BC$.

AC t o p i l s i n (2.3.20-chizma).



2.3.20- chizma.

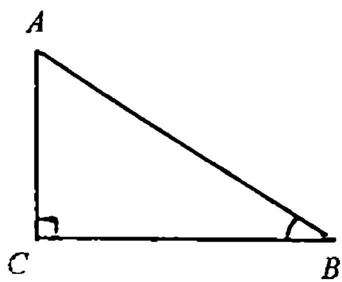
Y e c h i l i s h i. 2-xossadan foydalanib, uchburchakning asosidagi burchakning kattaligini topamiz: $\angle BAC=$ $=\angle BCA=\frac{180°-\beta}{2}=90°-\frac{\beta}{2}$.

To'g'ri burchakli ΔABD dan AD kesmani topamiz: $\frac{AD}{BD} = \text{ctg}\left(90^\circ - \frac{\beta}{2}\right)$, $AD = BD\text{tg}\frac{\beta}{2} = h\text{tg}\frac{\beta}{2}$. Demak, $AC = 2AD = 2h\text{tg}\frac{\beta}{2}$.

Javobi: E).

21. Berilgan. ΔABC, $\angle C = 90^\circ$, AC, BC, AB tomonlar o'suvchi geometrik progressiya hosil qiladi.

Kichik $\angle B$ topilsin (2.3.21- chizma).



2.3.21- chizma.

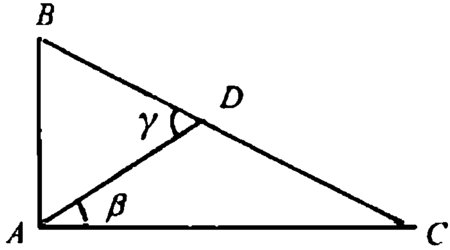
Yechilishi. $AC = b$ bo'lsin. Geometrik progressiyaning maxraji q bo'lsa, $BC = b \cdot q$, $AB = b \cdot q^2$. 8-xossaga asosan: $AB^2 = AC^2 + BC^2$, $(bq^2)^2 = b^2 + (bq)^2$, $b^2q^4 = b^2(1+q^2)$, $q^4 = 1 + q^2$, $q^4 - q^2 - 1 = 0$. Bu bikvadrat tenglamani yechamiz: $D = 1 - 4 \cdot 1(-1) = 5$, $q^2 = \frac{1}{2}(1 \pm \sqrt{5})$, $q^2 > 0$. Shuning uchun $q^2 = \frac{1}{2}(1 \pm \sqrt{5})$. To'g'ri burchakli uchburchakda katta katet qarshisida katta o'tkir burchak yotadi. Kichik o'tkir B qarshisida $AC = b$ tomon yotadi. U holda $\angle B$ uchun $\sin\angle B = \frac{AC}{AB} = \frac{b}{bq^2} = \frac{1}{q^2}$ yoki $\sin\angle B = \frac{2}{(1+\sqrt{5})}$ deb yozish mumkin. Bu yerdan $\sin\angle B = \frac{2(\sqrt{5}-1)}{(\sqrt{5})^2 - 1^2} = \frac{2(\sqrt{5}-1)}{5-1} = \frac{\sqrt{5}-1}{2}$, $\angle B = \arcsin\frac{\sqrt{5}-1}{2}$.

Javobi: E).

22. Berilgan. $\triangle ABC$, $AB=3$ sm, $AC=3$ sm, AD mediana, $\angle BAD=60^\circ$, $\angle CAD=\beta$.

$\sin\beta$ topilsin (2.3.22- chizma).

Yechilishi. AD mediana bo‘lgani uchun $BD=DC=x$ deb belgilaymiz. Sinuslar teoremasidan (8-xossa) ikki marta foydalanamiz: $\triangle ABD$ dan $\frac{x}{\sin 60^\circ}=\frac{\sqrt{3}}{\sin\gamma}$



2.3.22- chizma.

va bu yerdan $x=\frac{\sqrt{3}\sin 60}{\sin\gamma}=\frac{3}{2\sin\gamma}$. $\triangle ACD$ dan $\frac{x}{\sin\beta}=$ $=\frac{3}{\sin(180^\circ-\gamma)}$ va $x=\frac{3\sin\beta}{\sin\gamma}$. Bu munosabatlarning chap tomonlari teng bo‘lganligidan ularning o‘ng tomonlari ham tengdir, ya’ni $\frac{3}{2\sin\gamma}=\frac{3\sin\beta}{\sin\gamma}$ va $\sin\beta=\frac{1}{2}$.

Javobi: B).

2.3. Mustaqil yechish uchun masalalar

1. $\triangle ABC$ da BD mediana AC tomonning yarmiga teng. Uchburchakning B burchagi topilsin.

A) 90°; B) 75°; C) 105°; D) 70°; E) 45°.

2. Uchburchakning ikkita burchagi mos ravishda 62° va 74°ga teng. Uchburchakning bu burchaklaridan o‘tkazilgan balandliklar orasidagi o‘tmas burchak topilsin.

A) 172°; B) 126°; C) 110°; D) 104°; E) 136°.

3. Uchburchakda burchaklar kattaliklari 1:2:3 kabi nisbatda. Katta tomonning uzunligi 12 sm ga teng bo'lsa, kichik tomon uzunligi topilsin.

A) 5; B) 10; C) 7; D) 6; E) 4 sm.

4. To'g'ri burchakli uchburchakda gipotenuza va kichik katetning yig'indisi 27 sm ga teng. Agar katta katetning uzunligi $9\sqrt{3}$ sm bo'lsa, gipotenuzaning uzunligi topilsin.

A) 19; B) 18; C) 20; D) 15; E) 16 sm.

5. Teng yonli uchburchakning perimetri 25 sm, ikki tomonining ayirmasi 4 sm va tashqi burchaklaridan bittasi o'tkir burchak. Uchburchakning asosi topilsin.

A) 16; B) 17; C) 11; D) 13; E) 12 sm.

6. Uchburchakning C to'g'ri burchagi uchidan AB gipotenuzaga CD balandlik tushirilgan. Agar $\angle A=30°$ bo'lsa, gipotenuzada hosil qilingan kesmalarning $BD : AD$ nisbati topilsin.

A) $\frac{1}{3}$; B) $\frac{2}{5}$; C) $\frac{3}{5}$; D) $\frac{3}{4}$; E) $\frac{2}{3}$.

7. Teng yonli uchburchakning perimetri $2p$, asosidagi burchagi α ga teng. Uchburchakning yuzi hisoblansin.

A) $\frac{p^2 \sin 2\alpha}{1+\sin\alpha}$; B) $\frac{p^2 \sin\alpha \cos\alpha}{(1+\cos\alpha)^2}$; C) $p^2\cos2\alpha$;

D) $(1+p^2)\sin\alpha$; E) $\frac{p^2 \sin 2\alpha}{1+\sin\alpha}$.

8. To'g'ri burchakli uchburchakning yuzi 60 dm², perimetri 40 dm ga teng. Uchburchakning katetlari uzunliklari topilsin.

A) 7 va 11; B) 4 va 12; C) 8 va 15; D) 7 va 13; E) 9 dm va 12 dm.

9. To'g'ri burchakli uchburchakning balandligi gipotenuzani uzunliklari 18 va 32 sm ga teng bo'lgan kesmalarga ajratadi. Uchburchakning yuzi hisoblansin.

A) 9; B) 10; C) 5; D) 8; E) 6 dm^2.

10. Uchburchakning asosiga tushirilgan balandligi h ga teng. Uchburchakning asosiga parallel kesma uchburchakning yuzini teng ikkiga bo'ladi. Uchburchakning uchidan shu kesmagacha bo'lgan masofa topilsin.

A) $2h$; B) $h\sqrt{2}$; C) $\frac{h\sqrt{3}}{2}$; D) $\frac{h\sqrt{2}}{2}$; E) $\frac{h}{2}$.

11. Teng yonli uchburchakning yon tomoni 13 sm, yon tomoniga o'tkazilgan balandlik 5 sm ga teng. Uchburchak asosining uzunligi topilsin.

A) 6; B) $\sqrt{26}$; C) 5; D) $\sqrt{19}$; E) $\sqrt{17}$ sm.

12. Agar teng yonli uchburchakning perimetri 32 dm, o'rta chizig'i 6 dm ga teng bo'lsa, uning tomonlari uzunliklari topilsin.

A) 13, 13 va 7; B) 9, 9 va 14; C) 10, 10 va 12;
D) 12, 12 va 8; E) 10 dm, 11 dm va 11 dm.

13. To'g'ri burchakli uchburchakda katetlar 7 sm va 24 sm ga teng. To'g'ri burchakning bissektrisasi o'tkazilgan. Bu bissektrisa gipotenuzani qanday uzunlikdagi kesmalarga ajratadi?

A) $14\frac{7}{12}$ va $9\frac{5}{12}$; B) 13 va 12; C) 17 va 7.

D) $6\frac{1}{3}$ va $18\frac{2}{3}$; E) $5\frac{20}{31}$ va $19\frac{11}{31}$ sm.

14. Uchburchakning perimetri 4,5 dm ga teng, bissektrisa esa qarshi tomonni uzunliklari 6 va 9 sm ga teng bo'lgan kesmalarga ajratadi. Uchburchakning tomonlari topilsin.

A) 12, 15, 18; B) 17, 11, 18; C) 14, 15, 16;
D) 18, 17, 10; E) 12 sm, 16 sm, 17 sm.

15. O'tkir burchakli uchburchakda ikkita tomonning ayirmasi 2 sm ga, bu tomonlarning uchinchi tomondagi proeksiyalari 9 sm va 5 sm ga teng. Uchburchak tomonlari uzunliklari topilsin.

A) 12, 14, 20; B) 11, 14, 16; C) 14, 13, 17;
D) 13, 14, 15; E) 13 sm, 16 sm, 19 sm.

16. Uchburchak tomonlari uzunliklari berilgan: 7 sm, 11 sm, 12 sm. Uning eng katta medianasi topilsin.

A) $\frac{37\sqrt{3}}{2}$; B) $\frac{1}{2}\sqrt{481}$; C) $\frac{21}{2}$; D) $\frac{3\sqrt{174}}{2}$; E) $\frac{14\sqrt{2}}{3}$ sm.

17. Teng yonli ΔABC da $AB=BC=12$. BD balandlikning o'rtasidan $MP\|BC$ kesma o'tkazilgan. MP kesmaning uzunligi topilsin.

A) 7; B) 6; C) 4; D) 10; E) 9.

18. Uchburchakning asosi 60, balandligi 12, asosga tushirilgan medianasi 13 ga teng. Uchburchakning katta yon tomoni topilsin.

A) 37; B) 35; C) 32; D) 42; E) 45.

19. ΔABC da BD bissektrisa o'tkazilgan. Agar $AB=6$ sm, $BC=8$ sm va ABC uchburchakning yuzi 12 sm^2 ga teng bo'lsa, ΔABD va ΔCBD yuzlari hisoblansin.

A) $\frac{28}{11}$ va $\frac{104}{11}$; B) 8 va 4; C) $\frac{36}{7}$ va $\frac{48}{7}$;

D) $\frac{29}{7}$ va $\frac{55}{71}$; E) $\frac{31}{7}$ va $\frac{53}{7}$ sm^2.

20. ABC uchburchakning a, b, c tomonlari $a^2=b^2+c^2+$ $+\sqrt{2}\cdot b\cdot c$ munosabatni qanoatlantirsa, a tomon qarshisidagi burchak topilsin.

A) 60°; B) 135°; C) 105°; D) 75°; E) 90°.

21. Uchburchakning tomonlari 8 sm, 15 sm va 17 sm ga teng. Katta tomon qarshisidagi burchak topilsin.

A) 45°; B) 60°; C) 75°; D) 90°; E) 120°.

22. Teng yonli uchburchakning asosi a, asosidagi burchagi 75° bo'lsa, uning yuzi hisoblansin.

A) $\frac{a^2(2+\sqrt{3})}{4}$; B) $\frac{a^2(1+\sqrt{2})}{3}$; C) $\frac{a^2\sqrt{3}}{4}$;

D) $\frac{a^2(\sqrt{3+1})}{4}$; E) $\frac{a^2(\sqrt{5}+1)}{2}$.

23. ΔABC ning tomonlari uzunliklari 13 sm, 14 sm va 15 sm ga teng. Uchburchakning eng katta ichki burchagi topilsin.

A) arctg 2; B) arcsin $\frac{2}{3}$; C) arccos $\frac{5}{12}$; D) arcsin $\frac{1}{5}$;

E) arccos $\frac{5}{13}$.

24. ΔABC da AB=13 sm, AC=14 sm, BC=15 sm. Uning B uchidan o'tkazilgan balandlikning uzunligi topilsin.

A) 14; B) 15; C) 11; D) 10; E) 12.

25. ΔABC da AB=13 sm, AC=14 sm, BC=15 sm. Uning A uchidan o'tkazilgan mediananing uzunligi topilsin.

A) $\frac{\sqrt{374}}{2}$; B) $\frac{\sqrt{505}}{2}$; C) $\frac{\sqrt{481}}{2}$; D) 13; E) $\frac{\sqrt{299}}{2}$ sm.

26. Agar uchburchakning asosi a, unga yopishgan burchaklari 30° va 45° bo'lsa, uning yuzi hisoblansin.

A) $\frac{a^2\sqrt{15}}{4}$; B) $\frac{a^2(\sqrt{3}+2)}{2}$; C) $\frac{a^2(\sqrt{2}+2)}{4}$;

D) $\frac{1}{4}a^2(\sqrt{3}-1)$; E) $\frac{a^2(\sqrt{3}+1)}{2}$.

27. To'g'ri burchakli uchburchakda katetlarning nisbati 3:2 kabi, balandlik esa gipotenuzani shunday ikkita kesmaga ajratadiki, ulardan birining uzunligi ikkinchisidan 2 m katta. Gipotenuzaning uzunligi topilsin.

A) 3,8; B) 5,1; C) 6,4; D) 4,6; E) 5,2 m.

28. ABC uchburchak berilgan. Uning medianalaridan $\Delta A_1B_1C_1$ yasalgan. ΔABC va $\Delta A_1B_1C_1$ yuzlarining nisbati topilsin.

A) 4:3; B) 2:3; C) 3:1; D) 5:7; E) 3:5.

29. To'g'ri burchakli uchburchakning katetlari b va c ga teng. To'g'ri burchak bissektrisasining uzunligi topilsin.

A) $bc\sqrt{2}$; B) $\frac{(b+c)\sqrt{2}}{bc}$; C) $\frac{bc\sqrt{2}}{b+c}$; D) $\frac{c\sqrt{2}}{b+c}$; E) $\frac{b\sqrt{2}}{b+c}$.

30. ΔABC da $AB=2$ sm, BD mediana, $BD=1$ sm, $\angle BDA=30^\circ$. Uchburchakning yuzi hisoblansin.

A) $\frac{1}{2}(\sqrt{2}+\sqrt{5})$; B) $\frac{1}{2}(\sqrt{3}+\sqrt{15})$; C) $\frac{\sqrt{7}+\sqrt{8}}{4}$;

D) $\frac{\sqrt{3}+\sqrt{7}}{2}$; E) $\frac{10+\sqrt{13}}{4}$ sm².

31. ΔABC da $AB=3$ sm, $AC=5$ sm, $\angle BAC=120^\circ$. BD bissektrisaning uzunligi topilsin.

A) $\frac{3\sqrt{7}}{2}$; B) $\frac{4\sqrt{7}}{5}$; C) $\frac{3\sqrt{2}}{7}$; D) $\frac{5\sqrt{3}}{3}$; E) $\frac{4\sqrt{7}}{3}$ sm.

32. ΔABC da $\angle A$ burchak $\angle B$ dan ikki marta katta bo'lib, $AC=b$, $AB=c$. BC tomonning uzunligi topilsin.

A) $\sqrt{b^2+c^2}$; B) $\sqrt{2b+c}$; C) $\sqrt{bc}$; D) $\sqrt{b(b+c)}$;

E) $\sqrt{b+c}$.

33. ΔABC da $AC=13$ sm, $AB+BC=22$ sm, $\angle ABC=60°$. BC tomonning uzunligi topilsin.

A) 4; B) 9; C) 8; D) 6; E) 7 sm.

34. Uchburchakning yuzi S ga teng. Bu uchburchakning medianalari tashkil qilgan uchburchakning yuzi hisoblansin.

A) $\frac{3}{5}S$; B) $\frac{6}{7}S$; C) $\frac{3}{4}S$; D) $\frac{1}{2}S$; E) $\frac{4}{5}S$.

35. ΔABC ning AB tomoni o'rtasida K nuqta olingan. $AC=6$, $BC=4$, $\angle ACB=120°$ bo'lsa, CK kesmaning uzunligi topilsin.

A) 3; B) $\sqrt{7}$; C) $\sqrt{5}$; D) 4,5; E) 5.

36. Agar teng yonli uchburchakning yuzi 108 sm^2, asosi 18 sm bo'lsa, uchburchakning perimetri topilsin.

A) 36; B) 52; C) 56; D) 42; E) 48 sm.

37. Agar uchburchakning ikkita tomoni 4 sm va 6 sm va ular orasidagi burchakning tangensi 0,75 ga teng bo'lsa, uning yuzi hisoblansin.

A) 7,2; B) 7; C) 8; D) 9; E) 6,6 sm^2.

38. To'g'ri burchakli uchburchakning bir kateti gipotenuzadan 10 sm kichik, ikkinchi katetidan esa 10 sm katta. Uchburchakning yuzi hisoblansin.

A) 480; B) 640; C) 720; D) 600; E) 540 sm^2.

39. Uchburchak tomonlarining nisbati 3:6:5 kabi. Unga o'xshash uchburchakning katta tomoni 3,6 sm ga teng. Birinchi uchburchakning perimetri topilsin.

A) 5,6; B) 7,2; C) 8,4; D) 7,6; E) 9,2 sm.

40. ΔABC da AD mediana AB tomon bilan 30° li, AC tomon bilan 60° li burchaklar tashkil etadi. Agar $AB=\sqrt{3}$ sm bo'lsa, AC tomonning uzunligi topilsin.

A) 2; B) 1,5; C) 2,5; D) 3; E) 1 sm.

41. Uchburchakning a, b, c tomonlari $a^2=b^2+c^2-\sqrt{2}\,bc$ munosabatda bo'lsa, a tomon qarshisidagi burchak topilsin.

A) 45°; B) 30°; C) 60°; D) 75°; E) 90°.

42. To'g'ri burchakli uchburchakning perimetri 132, tomonlari kvadratlarining yig'indisi 6050 ga teng. Uning gipotenuzasi uzunligi topilsin.

A) 64; B) 65; C) 55; D) 60; E) 72.

43. Teng yonli uchburchakning asosi 30 sm, unga o'tkazilgan balandligi 20 sm ga teng. Yon tomonga o'tkazilgan balandlikning uzunligi topilsin.

A) 18; B) 22; C) 20; D) 24; E) 26 sm.

44. Uchburchakning asosi 60 sm, unga o'tkazilgan balandlik 12 sm va mediana 13 sm ga teng. Yon tomonlardan kattasining uzunligi topilsin.

A) 40; B) 37; C) 35; D) 42; E) 39 sm.

45. To'g'ri burchakli uchburchakning perimetri $2p$ va balandligi h ga teng. Uchburchakning uchinchi tomoni uzunligi topilsin.

A) $\frac{2p^2}{p+h}$; B) $\frac{p^2}{p+2h}$; C) $\frac{3p^2}{p+2h}$; D) $\frac{p^2}{p+h}$; E) $\frac{2p^2}{2p+h}$.

46. Uchburchakning ikkita b va c tomoni hamda uning yuzi $S=\frac{2}{5}bc$ berilgan. Uchburchakning uchinchi tomoni uzunligi topilsin.

A) $\sqrt{b^2+c^2-\frac{3}{5}bc}$; B) $\sqrt{b^2+c^2}$; C) $\sqrt{b^2+c^2-\frac{6}{5}bc}$;

D) $\sqrt{b^2-2bc}$; E) $\sqrt{b^2+c^2-\frac{4}{5}bc}$.

47. Uchburchakning ikkita tomoni AB=27 sm, AC=29 sm va BC tomonga o'tkazilgan mediana 26 sm ga teng. Uchburchakning yuzi hisoblansin.

A) 280; B) 320; C) 240; D) 270; E) 260 sm².

48. ΔABC da burchaklar kattaliklarining nisbati $\angle B:\angle A:\angle C=1:2:3$ kabi va $AC=b$, $AB=c$ bo'lsa, uning BC tomoni uzunligi topilsin.

A) $\sqrt{c^2-b^2}$; B) $\sqrt{b^2+c^2}$; C) $\sqrt{bc}$; D) $\sqrt{2b^2-c^2}$;

E) $\sqrt[4]{bc^2(b^2+c^2)}$.

49. ΔABC da AC=6, BC=4, $\angle ACB$=120° bo'lsa, uning yuzi hisoblansin.

A) $6\sqrt{5}$; B) 12; C) $3\sqrt{5}$; D) $6\sqrt{2}$; E) $6\sqrt{3}$ sm².

50. ΔABC da AC=13 sm, $AB+BC$=22 sm, $\angle ABC$=120° bo'lsa, BA tomon uzunligi topilsin.

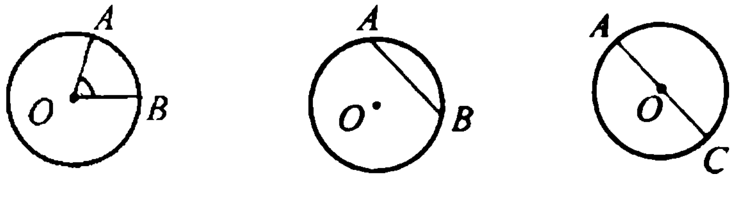
A) 18; B) 15; C) 14; D) 16; E) 12 sm.

3-§. AYLANA VA DOIRA

3.1. Asosiy tushunchalar va xossalar

Aylana tekislikdagi O nuqtadan bir xil masofada joylashgan nuqtalardan iborat geometrik shakldir. Berilgan O nuqta aylananing *markazi*, aylananing ixtiyoriy A nuqtasini uning markazi bilan tutashtiruvchi OA kesma esa aylananing *radiusi* bo'lib, u odatda $OA=R$ yoki $OA=r$ kabi belgilanadi (3.1-chizma).

Aylananing ikkita A va B nuqtasini tutashtiruvchi AB kesma aylananing *vatari* (3.2-chizma), markazdan o'tuvchi AC vatar aylananing *diametri* bo'ladi: $AC=2R$ yoki $AC=2r$. (3.3-chizma).



3.1-chizma. 3.2-chizma. 3.3-chizma.

$\angle AOB$ ning OA va OB tomonlari aylananing radiuslaridan iborat bo'lganda u *markaziy burchak*dir (3.1-chizma). Markaziy burchakning kattaligi o'zi tiralgan AB yoyning o'lchoviga teng:

$$\angle AOB = \smile AB. \qquad (3.1)$$

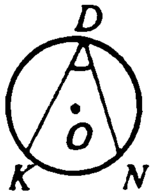
Uchi aylananing D nuqtasida bo'lib, tomonlari aylananing DK va DN vatarlaridan iborat $\angle KDN$ aylanaga *ichki chizilgan burchak* (3.4-chizma) deyilib, uning kattaligi o'zi tiralgan KN yoy o'lchovining yarmiga teng:

$$\angle KDN = \frac{1}{2} \smile KN. \qquad (3.2)$$

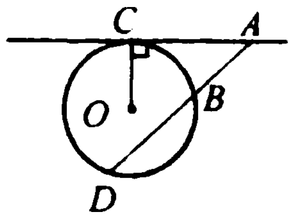
Aylanaga *urinma* shunday AC to'g'ri chiziqdan iboratki, u aylana bilan faqat bitta C umumiy nuqtaga egadir. A nuqtadan o'tib, aylana bilan ikkita B va D umumiy nuqtaga ega bo'lgan to'g'ri chiziq aylananing *kesuvchisidir* (3.5-chizma). AC urinmaning C urinish nuqtasidan aylanaga radius o'tkazilsa, u urinmaga perpendikulyar bo'ladi: $AC \perp OC$ (3.5-chizma).

Tekislikda to'g'ri burchakli xOy koordinatalar sistemasi tanlangan bo'lsin, O markazning koordinatalari (a, b), aylananing ixtiyoriy nuqtasining koordinatalari (x, y), aylana radiusi $OA = R$ bo'lsa, aylana nuqtalari uchun quyidagi tenglik bajariladi:

$$(x-a)^2 + (y-b)^2 = R^2, \qquad (3.3)$$



3.4-chizma.



3.5-chizma.

bu aylana tenglamasidir. Aylananing markazi koordinatalar sistemasining boshida bo'lsa, uning tenglamasi quyidagicha yoziladi:

$$x^2+y^2=R^2. \qquad (3.4)$$

Har qanday aylana tekislikni unga nisbatan ichki va tashqi nuqtalar to'plamlaridan iborat ikki qismga bo'ladi. Aylananing ichki qismida va aylananing o'zida joylashgan nuqtalar to'plami *doira* deyiladi.

Aylananing o'zi esa doiraning chegarasi bo'ladi.

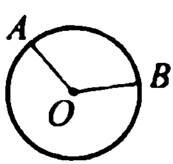
Doiraning AB yoy va OA va OB radiuslar bilan chegaralangan qismi *doiraviy sektor* bo'ladi (3.6-chizma).

Doiraning $A_1B_1C_1$ yoy va bu yoyga tiralgan A_1C_1 vatar bilan chegaralangan qismi doiraviy segmentdir (3.7-chizma).

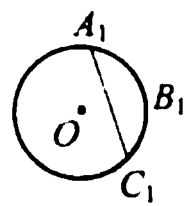
Aylana, doira, segment, sektorning ayrim xossalarini keltiramiz.

1. Bitta doirada yoki teng doiralarda:

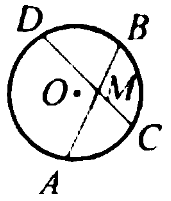
a) agar yoylar teng bo'lsa, ularga tiralgan vatarlar teng bo'lib, aylana markazidan teng masofada yotadi;



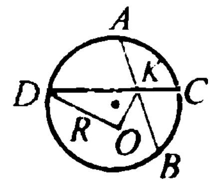
3.6-chizma.



3.7-chizma.



3.8-chizma.



3.9-chizma.

b) yarim aylanadan kichik bo'lgan ikkita yoy o'zaro teng bo'lmasa, katta yoyga tiralgan vatar ikkinchi vatardan katta va ikkinchi vatarga nisbatan aylana markaziga yaqin yotadi.

Aylananing ichida olingan M nuqtadan AB vatar va CD diametr o'tkazilgan bo'lsa, vatar qismlarining ko'paytmasi diametr qismlarining ko'paytmasiga teng (3.8-chizma):

$$MA \cdot MB = MC \cdot MD. \qquad (3.5)$$

3. Radiusi R ga teng bo'lgan aylananing ichida yotuvchi biror K nuqtadan vatarlar o'tkazilgan bo'lsa, har bir vatar qismlarining ko'paytmasi o'zgarmas miqdor va qiymati $R^2 - OK^2$ ga teng (3.9-chizma):

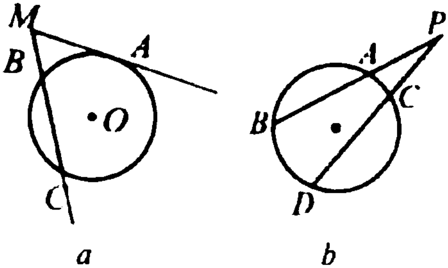
$$AK \cdot KB = CK \cdot KD = \ldots = R^2 - OK^2. \qquad (3.6)$$

4. Aylana tashqarisidagi M nuqtadan aylanaga MA urinma va MCB kesuvchi (MB — kesuvchining tashqi qismi, BC — ichki qismi) o'tkazilgan bo'lsa, urinma uzunligining kvadrati kesuvchining o'zi va uning tashqi qismining ko'paytmasiga teng (3.10-*a* chizma):

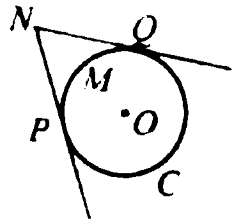
$$MA^2 = MC \cdot MB. \qquad (3.7)$$

Agar P nuqtadan aylanaga ikkita kesuvchi o'tkazilgan bo'lsa, kesuvchining uning tashqi qismiga ko'paytmasi o'zgarmas miqdor bo'ladi (3.10-*b* chizma):

$$PA \cdot PB = PD \cdot PC$$



3.10-chizma.



3.11-chizma.

5. Aylana tashqarisidagi N nuqtadan ikkita NP va NQ urinma o'tkazish mumkin, ular hosil qilgan $\angle PNQ$ burchak aylanaga *tashqi chizilgan burchak* deyiladi va uning kattaligi katta va kichik yoylar kattaliklari ayirmasining yarmiga teng (3.11-chizma):

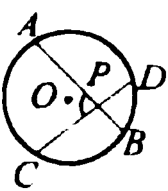
$$\angle PNQ=\frac{1}{2}(\smile QCP-\smile QMP). \qquad (3.8)$$

6. Aylananing AB va CD vatarlari uning ichidagi P nuqtada kesishsa, bu vatarlar orasidagi burchak quyidagicha topiladi (3.12-chizma):

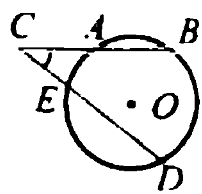
$$\angle APC=\frac{1}{2}(\smile AC+\smile DB). \qquad (3.9)$$

7. Aylananing AB va ED vatarlari uning tashqarisidagi C nuqtada kesishsa, vatarlar orasidagi $\angle ACE$ ning kattaligi quyidagicha topiladi (3.13-chizma):

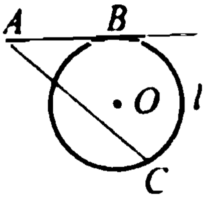
$$\angle ACE=\frac{1}{2}(\smile BD-\smile AE). \qquad (3.10)$$



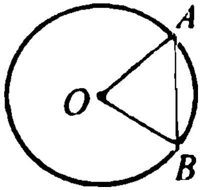
3.12-chizma.



3.13-chizma.



3.14-chizma.



3.15-chizma.

8. Aylananing urinmasi va vatari orasidagi burchakning kattaligi burchak tomonlari orasidagi aylana yoyi kattaligining yarmiga teng (3.14-chizma):

$$\angle ABC=\frac{1}{2}\smile BlC. \qquad (3.11)$$

9. Radiusi R ga teng bo'lgan aylananing uzunligi

$$L=2\pi R \qquad (3.12)$$

formula bo'yicha topiladi.

10. O'lchovi n° ga teng bo'lgan yoyning uzunligi

$$d=\frac{2\pi Rn^\circ}{360^\circ} \qquad (3.13)$$

formula orqali topiladi.

11. Radiusi R ga teng bo'lgan doiraning yuzi

$$S=\pi R^2 \qquad (3.14)$$

formula orqali hisoblanadi.

12. n° o'lchovli doiraviy sektorning yuzi (3.15- chizma)

$$S=\frac{\pi R^2 n^\circ}{360^\circ} \qquad (3.15)$$

formula bo'yicha hisoblanadi.

13. Doiraviy segmentning yuzi

$$S=S_{sekt.}-S_{\Delta OAC} \qquad (3.16)$$

formula bo'yicha hisoblanadi.

3.2. Mavzuga doir masalalar

1. Aylananing markaziy burchagi $100°$, u tiralgan yoyning uzunligi 10 sm bo'lsa, aylananing radiusi topilsin ($\pi=3$ deb qabul qilinsin).

A) 5; B) 6; C) 4; D) 3; E) 4,5 sm.

2. AB vatar aylanani ikkita yoyga ajratadi. Bu yoylarning nisbati 4:5 kabi. Katta yoyning ixtiyoriy nuqtasidan AB vatar qanday burchak ostida ko'rinadi?

A) $80°$; B) $75°$; C) $90°$; D) $85°$; E) $70°$.

3. Uzunligi $6\sqrt{3}$ ga teng bo'lgan vatar $120°$ ga teng bo'lgan yoyni tortib turadi. Aylananing uzunligi topilsin.

A) 10π; B) 8π; C) 15π; D) 9π; E) 12π.

4. Aylananing markaziy burchagi $60°$, u tiralgan yoyning uzunligi 10 sm ga teng bo'lsa, aylananing radiusi topilsin.

A) $\frac{20}{\pi}$; B) 15; C) $\frac{30}{\pi}$; D) $\frac{40}{\pi}$; E) $\frac{50}{\pi}$.

5. Aylananing AB vatari o'zi ajratgan yoylardan birining ixtiyoriy nuqtasidan $80°$ li burchak ostida ko'rinadi. A va B nuqtalar bilan chegaralangan yoylarning kattaliklari topilsin.

A) $160°$ va $200°$; B) $150°$ va $220°$; C) $140°$ va $220°$; D) $135°$ va $225°$; E) $180°$ va $120°$.

6. Aylananing $12\sqrt{2}$ ga teng vatari $90°$ li yoyga tiralgan. Aylananing uzunligi topilsin.

A) 12π; B) 18π; C) 20π; D) 24π; E) 28π.

7. Radiusi 1 ga teng aylana uchta yoyga bo'lingan, ularga mos markaziy burchaklar 1, 2 va 6 ga proporsional. Eng katta yoyning o'lchovi topilsin.

A) 5; B) 2π; C) $\frac{5\pi}{2}$; D) $\frac{3\pi}{2}$; E) $\frac{4\pi}{3}$.

8. Radiusi 5 sm ga teng bo'lgan aylanada uzunligi 8 sm ga teng bo'lgan vatar o'tkazilgan. Aylana markazidan vatargacha bo'lgan masofa topilsin.

A) 3; B) 1,5; C) 2; D) 4; E) 2,2 sm.

9. Radiusi $R=15$ sm bo'lgan doirada M nuqta olingan va ushbu nuqtadan uzunligi 18 sm ga teng bo'lgan vatar va diametr o'tkazilgan. M nuqtadan doira markazigacha bo'lgan masofa 13 sm ga teng. M nuqta vatarni qanday uzunliklardagi kesmalarga ajratadi?

A) 13 va 5; B) 7 va 11; C) 9 va 9; D) 14 va 4; E) 10 va 8 sm.

10. Aylanaga tegishli bo'lmagan A nuqtadan unga urinma va kesuvchi o'tkazilgan. A nuqtadan urinish nuqtasigacha bo'lgan masofa 16 sm, kesuvchining aylana bilan kesishish nuqtalaridan birigacha bo'lgan masofa 32 sm ga teng. Agar uning markazidan kesuvchigacha bo'lgan masofa 5 sm ga teng bo'lsa, aylananing radiusi topilsin.

A) 12; B) 13; C) 14; D) 10; E) 11 sm.

11. Bitta nuqtadan aylanaga ikkita urinma o'tkazilgan. Urinmaning uzunligi 12 sm, urinish nuqtalari orasidagi masofa 14,4 sm ga teng bo'lsa, aylananing radiusi topilsin.

A) 5; B) 8,5; C) 7; D) 8; E) 9 sm.

12. 60° ga teng bo'lgan burchakka ikkita o'zaro tashqi uringan aylana ichki chizilgan. Kichik aylananing radiusi r ga teng bo'lsa, katta aylananing radiusi topilsin.

A) $2r$; B) $\frac{r}{2}$; C) $3r$; D) $2,5r$; E) $1,5r$.

13. Burchagi 120° ga teng bo'lgan doiraviy sektorga ichki doira chizilgan. Berilgan doiraning radiusi R ga teng bo'lsa, yangi doiraning radiusi topilsin.

A) $2R$; B) $R(\sqrt{3}-\sqrt{2})$; C) $R\sqrt{3}(2-\sqrt{3})$; D) $R(3-\sqrt{2})$; E) $1,5R$.

14. Doiraning yuzini 96% orttirish uchun uning radiusini necha prosent orttirish kerak?

A) 45%; B) 15%; C) 20%; D) 35%; E) 40%.

15. Radiuslari $r_1=6$ sm, $r_2=7$ sm, $r_3=8$ sm bo'lgan aylanalar ikkitadan o'zaro urinadi. Uchlari bu aylanalar markazlarida joylashgan uchburchakning yuzini hisoblang.

A) 90; B) 78; C) 56; D) 42; E) 84 sm².

3.3. Mavzuga doir masalalarning yechimlari

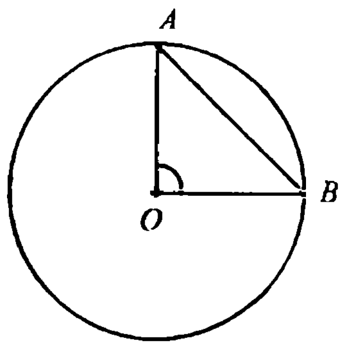
1. B e r i l g a n. (R, O) aylana, $\angle AOB=100°$, $AB=10$ sm. ($\pi=3$ deb qabul qilinsin).

R t o p i l s i n (3.3.1- chizma).

Y e c h i l i s h i. Aylana yoyining kattaligi 360°, aylananing 1° li burchagiga mos kelgan yoyning uzunligi $\frac{2\pi R}{360°}$ ga teng. Shartga ko'ra markaziy burchak 100°ga tengligidan, AB yoyning uzunligi $\frac{2\pi R}{360°}\cdot 100°$ bo'ladi. Olingan ifodalarni tenglashtirib, R ga nisbatan tenglamani yechamiz:

$$\frac{2\pi R}{360°}\cdot 100°=10,\quad \frac{2\cdot 3R}{36}=1,$$

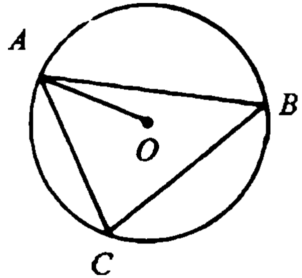
$$R=\frac{36}{6}=6 \text{ sm}.$$



3.3.1- chizma.

2. B e r i l g a n. (R,O) aylana, $\smile ADB : \smile ACB=4:5$, $C\in\smile ACB$.

$\angle ACB$ t o p i l s i n (3.3.2-chizma).



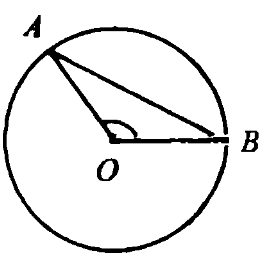
3.3.2-chizma.

Y e c h i l i s h i. Aylana yoyining kattaligi 360° ga teng yoki $\smile ABC+\smile ADB=360°$. Shartga ko‘ra, $\smile ADB=(4:5)\smile ACB$. U holda $\smile ACB+(4:5)\smile ACB=$ $=360°$, $(9:5)\ \smile ACB=360°$, $\smile ACB=(1:9)\cdot 5\cdot 360°=200°$ va $\smile ADB=(4:5)200°=160°$, $\angle ACB$ ichki chizilgan bo‘lganligidan, $\angle ACB=(1:2)\smile ADB=$ $=(1:2)\cdot 160°=80°$.

3. B e r i l g a n. (R,O) aylana, $\angle AOB=120°$, $AB=6\sqrt{3}$.

Aylana uzunligi L t o p i l s i n (3.3.3-chizma).

Y e c h i l i s h i. $OA=OB=R$. Demak, ΔAOB teng yonli va uning asosidagi burchaklar teng, ya’ni $\angle OAB=$ $=\angle ABO=\frac{1}{2}(180°-120°)=30°$. ΔAOB uchun sinuslar teoremasini (2-§, 8-xossa) yozamiz:



3.3.3- chizma.

$$\frac{OA}{\sin 30°}=\frac{AB}{\sin 120°},$$

$$\frac{R}{\sin 30°}=\frac{6\sqrt{3}}{\sin(90°+30°)},$$

bu yerdan

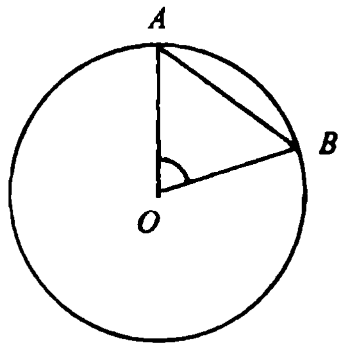
$$R=\frac{6\sqrt{3}\sin 30°}{\cos 30°}=\frac{6\sqrt{3}\cdot 1/2}{\sqrt{3}/2}=6.$$

Demak, aylananing uzunligi: $L=2\pi R=12\pi$.

Javobi: E).

4. Berilgan. (R,O) aylana, $\angle AOB=60^\circ$, $\cup AB$ uzunligi 10 sm.

$OA=R$ topilsin (3.3.4-chizma).

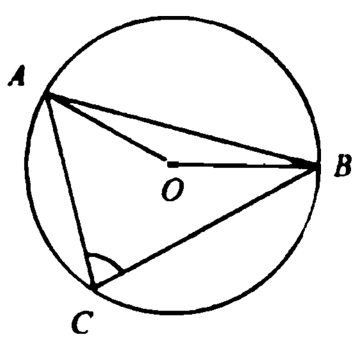


3.3.4- chizma.

Yechilishi. $OA=OB=R$ aylananing radiusi. Aylana yoyi kattaligi 360° ga teng, demak AB yoyning uzunligi aylana uzunligining 1/6 qismiga teng. Shunga asosan $10=$ $=(1/6)\cdot 2\pi R$ tenglamani tuzamiz. Bu yerdan $R=6\cdot 10/2\pi=$ $=30/\pi$.

5. Berilgan. (R,O) aylana, $\angle ACB=80^\circ$.

AB va ACB yoylar topilsin (3.3.5-chizma).

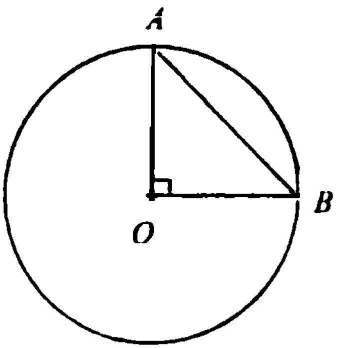


3.3.5- chizma.

Yechilishi. $\angle ACB$ ichki chizilgan burchak bo'lganligidan $\cup AB=2\cdot\angle ACB=$ $=2\cdot 80^\circ=160^\circ$. Ikkinchi yoy: $\cup ACB=360^\circ-160^\circ=200^\circ$ bo'ladi.

Javobi: A).

6. Berilgan. (R,O) aylana, $AB=12\sqrt{2}$, $\angle AOB=90°$.



3.3.6- chizma.

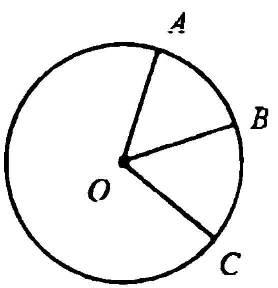
L topilsin (3.3.6- chizma).

Yechilishi. (3.12) formuladan foydalanamiz. ΔAOB to'g'ri burchakli va teng yonlidir ($OA=OB=R$), undan $R=AB\cdot\sin 45°=$ $=12\sqrt{2}\cdot\frac{\sqrt{2}}{2}=12$ va $L=$ $=2\pi\cdot 12=24\pi$ bo'ladi.

Javobi: D).

7. Berilgan. (R,O) aylana, $R=1$, $\angle AOB:\angle BOC:$ $:\angle AOC=1:2:6$.

$\cup AC$ ning uzunligi topilsin (3.3.7-chizma).



3.3.7- chizma.

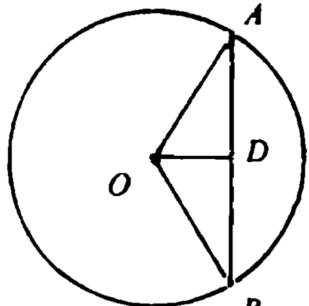
Yechilishi. Aylana uzunligi $2\pi R$ ni uning kattaligi 360°ga bo'lib, 1° li markaziy burchakka mos kelgan yoyning uzunligini topamiz: $\frac{2\pi\cdot 1}{360°}=\frac{\pi}{180}$ Berilganlardan $\angle AOB=\alpha$ bo'lsa, $\angle BOC=2\alpha$ va $\angle AOC=6\alpha$ bo'ladi. Natijada, $\alpha+2\alpha+6\alpha=360°$ tenglamani hosil qilamiz va uni yechib, $\alpha=40°$ ekanligini olamiz. Eng katta markaziy burchak $6\cdot 40°=240°$ ga teng ekan, unga mos kelgan yoyning uzunligi $\frac{\pi}{180°}\cdot 240=\frac{4\pi}{3}$ bo'ladi.

Javobi: E).

8. Berilgan. (R,O) aylana, $R=5$ sm, $AB=8$ sm.

$d(O,AB)=h$ topilsin (3.3.8-chizma).

Yechilishi. $AO=OB=R$ bo'lgani uchun, ΔAOB teng yonli. O nuqtadan AB vatarga CD perpendikulyar OD o't-kazsak, u mediana ham bo'-ladi: $AD=DB=\frac{1}{2}AB=\frac{1}{2}\cdot 8=$ $=4$ sm. Pifagor teoremasiga (2-§, 7-xossa) asosan: $h=$ $=\sqrt{OB^2-OD^2}, h=\sqrt{5^2-4^2}=3$sm.



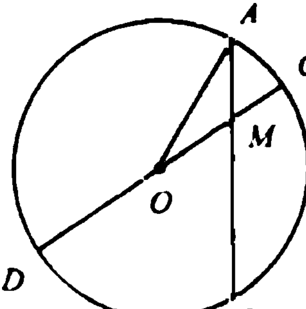
3.3.8-chizma.

Javobi: A).

9. Berilgan. (R,O) aylana, $AB=18$ sm, $OA=R=$ $=15$ sm, $M\in AB$, $MO=13$ sm.

MA, MB topilsin (3.3.9-chizma).

Yechilishi. Aylananing AB va CD vatarlari M nuqtada kesishadi va (3.5) xossaga asosan, $MA\cdot MB=CM\cdot MD$. Bu yerdan $CD=2R$ yoki $CD=$ $=2\cdot 15=30$ sm, $OC=R=$ $=15$ sm, $MO=13$ sm va shuning uchun $CM=15-13=$ $=2$ sm, $MD=30-2=28$ sm. Noma'lum MA va MB miqdorlarga nisbatan tenglamalar sistemasini yozamiz:



3.3.9- chizma.

$$\begin{cases} MA+MB=2\cdot 28, \\ MA+MB=18. \end{cases} \Rightarrow \begin{cases} (18-x)x-56, \\ MA=18-MB, MB=x, \end{cases} \Rightarrow$$

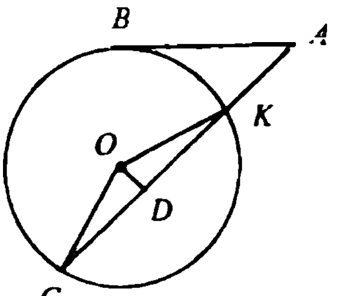
$$\Rightarrow \begin{cases} x^2 - 18x + 56 = 0, \\ MA = 18 - x. \end{cases}$$

Kvadrat tenglamaning diskriminanti $D=18^2-4\cdot 56= =324-224=100$ bo'lganligidan, u ikkita $x_1=4$, $x_2=14$ ildizga ega. Demak, $MA=14$ sm, $MB=18-4=14$ sm. ($MA=14$ sm, $MB=4$ sm).

Javobi: D).

10. B e r i l g a n. (R,O) aylana, AB urinma, $AB= =16$ sm, AKC—kesuvchi, $AC=32$ sm, $OD\perp AC$, $OD=5$ sm.

R radius t o p i l s i n (3.3.10-chizma).



3.3.10-chizma.

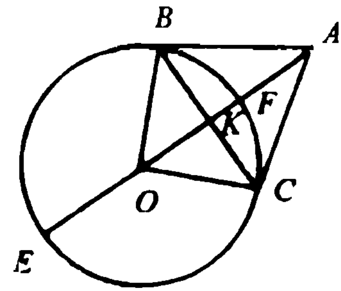
Y e c h i l i s h i. 4-xossaga asosan: $AB^2 = AC\cdot AK$. $AK=x$ deb belgilaymiz, u holda $KC=32-x$ bo'ladi. So'ngra, $16^2=32AK$ tenglamani yechamiz: $x=8$ sm. Natijada $KC = 32 - 8 = =24$ sm ekanligini olamiz. O markazni K va C nuqtalar bilan tutashtiramiz. Natijada ΔKOC teng yonli uchburchakni hosil qilamiz, unda $OK=OC=R$ va $KD=\frac{1}{2}\cdot 24=12$ sm. To'g'ri burchakli ΔKOD dan: $OK^2=KD^2+OD^2$ yoki $R^2=12^2+5^2$, $R^2=144+25=169$, $R=\sqrt{169}=13$ sm.

Javobi: B).

11. B e r i l g a n. (R,O) aylana, $AB=12$ sm, $AC=AB$, $BC=14,4$ sm.

R radius t o p i l s i n (3.3.11-chizma).

Yechilishi. O aylananing markazi bo'lsa, $OB = OC = R$ uning radiusidir. Demak, ΔOBC teng yonli bo'lganligi sababli OK balandlik mediana ham bo'ladi va $BK = KC = 7{,}2$ sm. AB va AC lar A nuqtadan berilgan aylanaga o'tkazilgan ikkita urinma bo'lganligidan, ularning uzunliklari teng bo'ladi, ya'ni



3.3.11- chizma.

$AB = AC$. To'g'ri burchakli ΔABK dan Pifagor teoremasiga (2-§, 8-xossa) asosan, $AK^2 = AB^2 - BK^2 = 12^2 - (7{,}2)^2 = 144 - 51{,}84 = 92{,}16$, $AK = 9{,}6$ sm bo'ladi.

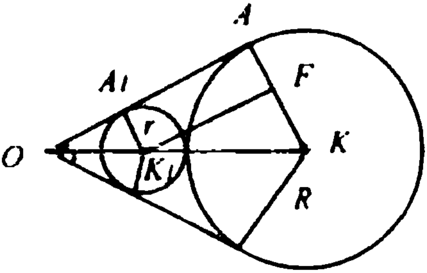
$OK = h$ deb belgilaymiz. To'g'ri burchakli ΔOBK dan $R^2 - h^2 = (7{,}2)^2$ tenglikni olamiz. (3.7) dan: $AB^2 = AE \cdot AF$ formula o'rinli, lekin $AE = AK + OK + OE = 9{,}6 + h + R$, $AF + AO - R = AK + h - R$. U holda $AB^2 = (9{,}6 + h - R)\cdot(9{,}6 + h + R) = (9{,}6 + h)^2 - R^2$, $AB = 12$, $R^2 = h^2 + 7{,}2^2$ qiymatlarni oxirgi tenglikka qo'yamiz: $144 = (9{,}6 + h)^2 - 7{,}2^2 - h^2$, $144 = 92{,}16 + 19{,}2h + h^2 - 51{,}84 - h^2$, $19{,}2h = 144 - 40{,}32 = 103{,}68$, $h = \frac{103{,}68}{19{,}2} = 5{,}4$. Aylananing radiusini topamiz: $R^2 = 5{,}4^2 + 7{,}2^2 = 51{,}84 + 29{,}16 = 81$, $R = 9$ sm.

Javobi: E).

12. Berilgan. $\angle AOB = 60°$. (r, K_1) — kichik aylana va (R, K) — katta aylana.

R topilsin (3.3.12-chizma).

Yechilishi. Burchakka ichki chizilgan aylananing markazi burchakning bissektrisasida yotganligidan, $\angle KOA = 30°$. K va K_1 mos ravishda, ichki chizilgan katta va kichik aylanalarning markazlari bo'lsin. Bu nuqtalardan burchakning OA tomoniga perpendikulyarlar



3.3.12- chizma.

o'tkazamiz: $A_1K_1 \perp OA_1$, $AK \perp OA$ va shartga ko'ra, $A_1K_1 = r$, $AK = R$ hamda $KF = R - r$, $KK_1 = R + r$, agar $K_1F \parallel AA_1$ bo'lsa. To'g'ri burchakli ΔKK_1F dan

$KF = KK_1 \cdot \sin 30° = \frac{1}{2} KK_1$

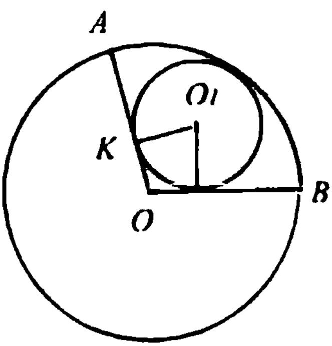
yoki $R - r = \frac{1}{2}(R + r)$,

$R + r = 2R - 2r$, $R = 3r$.

Javobi: C).

13. Berilgan. (R, OAB) doiraviy sektor, $\angle AOB = 120$, (r, O_1)—ichki chizilgan doira.

r topilsin (3.3.13-chizma).



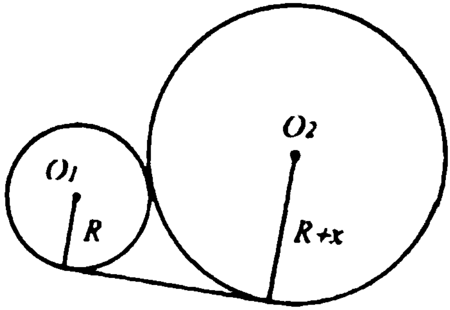
3.3.13- chizma.

Yechilishi. OA ichki chizilgan aylanaga urinma bo'lganligi uchun $OA \perp O_1K$. Shuning uchun, ΔOO_1K to'g'ri burchakli va $\angle KOA_1 = 60°$. U holda $O_1K = OO_1 \sin 60°$ yoki $r = OO_1 \cdot \frac{\sqrt{3}}{2}$ $OA = R$, $O_1K = r$ va $OO_1 = R - r$ bo'ladi. Demak,

$$r = (R - r)\frac{\sqrt{3}}{2}, \quad r + \frac{\sqrt{3}}{2} \cdot r = \frac{\sqrt{3}}{2} R, \quad r\left(1 + \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2} \cdot R$$

yoki $\frac{\sqrt{3}R}{2+\sqrt{3}} = \frac{R\sqrt{3}(2-\sqrt{3})}{4-3}$, $r = R\sqrt{3}(2-\sqrt{3})$.

Javobi: C).



3.3.14-chizma.

14. Berilgan. (O_1, R) birinchi doira, $(O_2, R+x)$ ikkinchi doira, S_1, S_2 yuzlar, $S_2 = 1{,}96 S_1$.

x topilsin (3.3.14-chizma).

Yechilishi. Berilgan doiraning radiusi R, yangi doiraning radiusi $R+x$ bo'lsa, ularning yuzlari, $S_1 = \pi R^2$, $S_2 = \pi(R+x)^2$ bo'ladi. U holda $S_1 = \pi R^2$ yuz 100% bo'lsa, $S_2 = \pi(R+x)^2$ yuz 196% ni tashkil qiladi. Proporsiya tuzamiz:

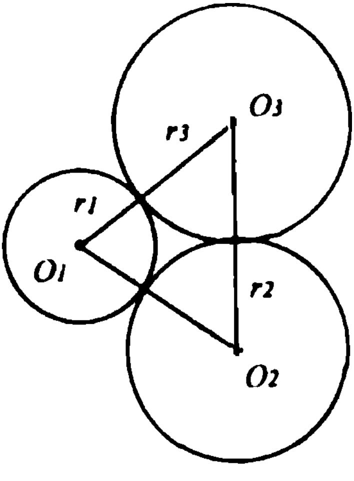
$$\pi R^2 - \quad 100\%$$
$$\pi(R+x)^2 - 196\%.$$

Bu proporsiyada o'rta hadlar ko'paytmasi chetki hadlar ko'paytmasiga teng: $196\pi R^2 = \pi(R+x)^2 \cdot 100$. Bu kvadrat tenglamani x ga nisbatan yechamiz:

$$(R+x)^2 = \frac{196R^2}{100}, \quad R+x = \frac{14R}{10}, \quad x = \frac{7}{5}R - R = \frac{2}{5}R =$$

$=0{,}4\ R$. Demak, radiusni 40% ga orttirish kerak.

15. Berilgan. (r_1, O_1), (r_2, O_2), (r_3, O_3) — o'zaro urinadigan aylanalar, $O_1O_2O_3$—uchlari aylanalar markazlarida joylashgan uchburchak.



3.3.15- chizma.

$S_{O_1O_2O_3}$ hisoblansin (3.3.15-chizma).

Yechilishi. Aylanalar o'zaro uringani uchun uchburchakning tomonlarini radiuslar yordamida topish mumkin: $O_1O_2 = r_1 + r_2 = 6 + 7 = 13$ sm, $O_1O_3 = r_1 + r_3 = 14$ sm, $O_2O_3 = r_2 + r_3 = 15$ sm. $\Delta O_1O_2O_3$ ning yuzini Geron formulasi yordamida hisoblaymiz:

$p = \frac{1}{2}(13 + 14 + 15) = 21$ sm va $S = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 7 \cdot 3 \times$ $\times 4 = 84$ sm^2.

Javobi: E).

3.4. Mustaqil yechish uchun masalalar

1. A, B, C aylanadagi nuqtalar va $\angle ABC = 30°$. Aylananing diametri 20 sm ga teng bo'lsa, AC vatarning uzunligi topilsin.

A) 8; B) 10; C) 12; D) 6; E) 9 sm.

2. AB diametrning uchidan AC vatar o'tkazilgan va bu vatar yarim aylanani kattaliklari 2:3 nisbatda bo'lgan 2 qismga bo'ladi. ABC uchburchakning burchaklari topilsin.

A) 40°, 50°, 90°; B) 30°; 60°; 90°; C) 32°, 58°, 90°; D) 36°, 54°, 90°; E) 35°; 55°; 90°.

3. Ikkita doira radiuslarining nisbati 1:2 kabi. Katta doira aylanasining uzunligi $8\sqrt{\pi}$ ga teng. Kichik doiraning yuzi hisoblansin.

A) 4; B) 6; C) 3; D) 2,5; E) 5.

4. Ikkita doira yuzlarining nisbati 1:16 kabi. Kichik doiraning radiusi $\frac{4}{\pi}$ ga teng bo'lsa, katta doira aylanasining uzunligi topilsin.

A) 40; B) 36; C) 38; D) 42; E) 32.

5. Aylananing uzunligi $8\pi\sqrt{3}$ ga teng bo'lsa, 120° ga teng bo'lgan yoyga tiralgan vatarning uzunligi topilsin.

A) 16; B) 18; C) 12; D) 10; E) 14.

6. Doiraning yuzi $6,25\pi$ ga teng. Bu doirada uzunligi 3 ga teng bo'lgan vatar o'tkazilgan. Doira markazidan vatargacha bo'lgan masofa topilsin.

A) 3; B) 2; C) 2,5; D) 1; E) 4.

7. Markazi O nuqtada bo'lgan aylanada AB vatar va OD radius o'tkazilgan va ular C nuqtada kesishadi hamda $AB \perp CD$, $OC=9$, $CD=32$. Vatarning uzunligi topilsin.

A) 60; B) 85; C) 80; D) 75; E) 90.

8. Radiusi 8 sm ga teng aylananing A nuqtasidan ikkita o'zaro teng AB va AC vatar o'tkazilgan. Vatarlar orasidagi burchak 60° ga teng bo'lsa, aylana markazidan BC vatargacha bo'lgan masofa topilsin.

A) 3; B) 4,5; C) 5; D) 4; E) 6 sm.

9. Aylanaga A nuqtada AB urinma o'tkazilgan. $AB=5$ va A nuqtadan aylananing O markazigacha masofa $5\sqrt{2}$ ga teng bo'lsa, aylananing radiusi topilsin.

A) 5; B) 4; C) 6; D) 3; E) 7.

10. Markazi O nuqtada bo'lgan aylanada AB diametr va BC vatar o'tkazilgan. Agar $\angle AOC=60°$ bo'lsa, $\angle ABC$ topilsin.

A) 60°; B) 48°; C) 36°; D) 45°; E) 30°.

11. Radiusi $\frac{7,2}{\pi}$ ga teng bo'lgan aylanada kattaligi 100° ga teng bo'lgan yoyning uzunligi topilsin.

A) 5; B) 3; C) 4; D) 6; E) 4,5.

12. Doiraning yuzi 48π ga teng. Markaziy burchak 120° ga teng bo'lsa, unga mos vatarning uzunligi topilsin.

A) 12; B) 10; C) 13; D) 15; E) 14.

13. Markazi O nuqtada bo'lgan aylanadagi B nuqtadan BA vatar, A nuqtadan aylanaga AC urinma o'tkazilgan. Agar $\angle BAC=35°$ bo'lsa, AOB topilsin.

A) 50°; B) 70°; C) 60°; D) 80°; E) 55°.

14. Aylananing AB va CD vatarlari K nuqtada kesishadi. Agar $AB=22$ sm, $CK=8$ sm, $DK=12$ sm bo'lsa, AK va BK kesmalar topilsin.

A) 3, 19; B) 5,5, 16,5; C) 4, 18; D) 6, 16; E) 5,17 sm.

15. Aylanada AB diametr, BC vatar, $AB=20$ sm, $\angle ABC=75°$ bo'lsa, markaziy AOC burchakka mos kelgan yoyning uzunligi topilsin ($\pi=3$ deb olinsin).

A) 32; B) 30; C) 26; D) 24; E) 25 sm.

16. Ikkita aylana uzunliklarining nisbati 4 ga teng bo'lsa, mos doiralar yuzlarining nisbati topilsin.

A) 16; B) 15; C) 17; D) 18; E) 19.

17. Radiusi 8 sm ga teng bo'lgan aylanada uzunligi 8 sm bo'lgan vatar o'tkazilgan. Vatar tiralgan yoyning uzunligi topilsin.

A) $\frac{14\pi}{3}$; B) 2π; C) $\frac{8\pi}{3}$; D) $\frac{7\pi}{3}$; E) 3π.

18. $x^2+y^2-4x+6y-3=0$ aylananing radiusi topilsin.

A) 3; B) 4; C) 5; D) 2; E) 6.

19. $x^2-6x+y^2-8y=0$ aylana markazining koordinatalari topilsin.

A) $(-3; -4)$; B) $(3; -4)$; C) $(-3; 4)$;
D) $(3; 4)$; E) $(-3; 0)$.

20. Berilgan $A(-1,3)$, $B(0,-2)$, $C(3,1)$ nuqtalardan qaysilari $x^2-2x+y^2+4y+4=0$ aylanaga tegishli?

A) A; B) C; C) A, C; D) A, B; E) B.

21. Aylananing bitta nuqtasidan radius va uzunligi unga teng bo'lgan vatar o'tkazilgan. Ular orasidagi burchak topilsin.

A) 60°; B) 45°; C) 75°; D) 30°; E) 90°.

22. Aylananing bitta nuqtasidan uzunligi uning radiusiga teng bo'lgan ikkita vatar o'tkazilgan. Ular orasidagi burchak topilsin.

A) 30°; B) 60°; C) 120°; D) 90°; E) 150°.

23. Aylana ichidagi nuqtadan aylanagacha eng qisqa masofa 6 sm, eng katta masofa 12 sm bo'lsa, aylananing radiusi topilsin.

A) 8; B) 9; C) 6; D) 10; E) 12 sm.

24. Aylananing tashqarisidagi nuqtadan aylanagacha bo'lgan eng qisqa masofa 7 sm, eng katta masofa 23 sm bo'lsa, aylananing radiusi topilsin.

A) 6; B) 10; C) 7; D) 8; E) 9 sm.

25. Radiusi 4 sm ga teng bo'lgan aylanada o'zaro teng bo'lgan $AB=AC=BC$ vatarlar o'tkazilgan. Aylana markazidan vatarlargacha bo'lgan masofalar topilsin.

A) 3; B) 1,5; C) 1; D) 2,5; E) 2 sm.

26. Aylana markazidan 4 sm masofada o'zaro perpendikulyar bo'lgan ikkita vatar o'tkazilgan, ulardan biri 12 sm ga teng. Kesishish nuqtasida bu vatar qanday uzunlikdagi kesmalarga ajraladi?

A) 3, 9; B) 1,5, 10,5; C) 2, 10; D) 1, 12; E) 2, 11 sm.

27. Aylananing vatari diametr bilan 30° li burchak tashkil qiladi va kesishish nuqtasi diametrni uzunliklari 2 sm va 10 sm bo'lgan kesmalarga ajratadi. Aylana markazidan vatargacha bo'lgan masofa topilsin.

A) 2; B) 3; C) 2,5; D) 4; E) 4,5 sm.

28. Radiusi 5 sm bo'lgan aylana tashqarisidagi P nuqtadan ikkita urinma o'tkazilgan va ular orasidagi burchak 60° ga teng. P nuqtadan aylana markazigacha bo'lgan masofa topilsin.

A) 12; B) 10; C) 9; D) 13; E) 8 sm.

29. O'lchovi 90° ga teng, radiusi 4 sm bo'lgan yoyning o'rtasi K dan yoyga urinma o'tkazilgan. Yoyning chetki radiuslari urinma bilan kesishguncha davom ettirilganda hosil bo'lgan kesmaning uzunligi topilsin.

A) 7; B) 14; C) 12; D) 8; E) 10 sm.

30. Aylanaga o'zaro perpendikulyar bo'lgan ikkita urinma o'tkazilgan. Urinish nuqtalarini tutashtiruvchi vatarning uzunligi 12 sm ga teng. Aylana markazidan vatargacha bo'lgan masofa topilsin.

A) 4; B) 5; C) 6; D) 8; E) 3 sm.

31. Aylana tashqarisidagi K nuqtadan KA va KB urinmalar o'tkazilgan va ular uzunliklarining yig'indisi 14,8 sm ga teng. Kichik AB yoyning ixtiyoriy C nuqtasidan aylanaga urinma o'tkazilgan bo'lib, u KA va KB urinmalarni D va E nuqtalarda kesib o'tadi. KDE uchburchakning perimetri topilsin.

A) 13,6; B) 14; C) 15; D) 15,2; E) 14,8 sm.

32. K nuqtadan aylanaga KBA va KDC kesuvchilar o'tkazilgan. AC yoyning kattaligi 106°20', BD yoyning kattaligi 42°30' bo'lsa, kesuvchilar orasidagi burchak topilsin.

A) 42°24'; B) 31°55'; C) 32°40'; D) 29°32'; E) 36°28'.

33. K nuqtadan aylanaga ikkita urinma o'tkazilgan. Urinmalar orasidagi burchak 60° bo'lsa, urinish nuqtalari orasidagi yoylarning kattaliklari topilsin.

A) 120° va 240°; B) 100° va 260°; C) 90° va 270°;
D) 130° va 230°; E) 150° va 210°.

34. Aylanaga K nuqtadan KBA va KDC kesuvchilar o'tkazilgan. Agar $KA=20$ sm, $KB=18$ sm, $KC=24$ sm bo'lsa, KD kesmaning uzunligi topilsin.

A) 16; B) 15; C) 14; D) 18; E) 17 sm.

35. P nuqtadan aylanaga PT urinma va PBA kesuvchi o'tkazilgan. Agar $PT=18$ sm va $PB : BA=4:5$ kabi bo'lsa, kesuvchining tashqi qismi uzunligi topilsin.

A) 14; B) 11; C) 12; D) 10; E) 15 sm.

36. Aylanadagi AB va CD vatarlar P nuqtada kesishadi. Agar $CP-PD=5$ sm, $AP=12$ sm, $AB=15$ sm bo'lsa, CD vatarning uzunligi topilsin.

A) 15; B) 16; C) 18; D) 13; E) 14 sm.

37. Aylananing vatari a ga teng. Mos yoyning kattaligi 120° ga teng bo'lsa, yoyning uzunligi topilsin.

A) $\frac{3\sqrt{2}a\pi}{9}$, B) $\frac{2\sqrt{2}a\pi}{9}$; C) $\frac{3a\pi}{9}$; D) $\frac{2a\pi}{7}$; E) $\frac{2\sqrt{3}a\pi}{9}$.

38. Yoyning uzunligi s ga teng va mos markaziy burchakning kattaligi 90° bo'lsa, yoyning uchlarini tutashtiruvchi vatarning uzunligi topilsin.

A) $\frac{2\sqrt{2}c}{\pi}$; B) $\frac{3\sqrt{2}c}{\pi}$; C) $\frac{\sqrt{2}c}{\pi}$; D) $\frac{\sqrt{3}c}{\pi}$; E) $\frac{3\sqrt{3}c}{\pi}$.

39. Yoyning radiusi 6 sm, unga mos markaziy burchak 120° ga teng. Bu yoydan yasalgan yangi aylananing radiusi topilsin.

A) 2,5; B) 3; C) 4; D) 2; E) 1 sm.

40. Aylananing radiusi 5 sm ga ortganda, aylananing uzunligi qancha ortadi?

A) 8π; B) 10π; C) 9π; D) 12π; E) 15π.

41. Doiraning yuzi 49π sm^2 bo'lsa, unga mos aylananing uzunligi topilsin.

A) 15π; B) 10π; C) 14π; D) 13π; E) 12π.

42. Doiraning yuzi 16 marta ortsa, mos aylananing uzunligi qanday o'zgaradi?

A) 8 marta ortadi; B) 16 marta ortadi; C) 2 marta ortadi; D) 4 marta kamayadi; E) 4 marta ortadi.

43. Umumiy markazga ega bo'lgan ikkita doiraning radiuslari 9 va 14 sm. Ular tashkil qilgan halqaning yuzi hisoblansin.

A) 115π; B) 114π; C) 110π; D) 116π; E) 112π.

44. Umumiy markazga ega bo'lgan ikkita aylananing uzunliklari mos ravishda 12π va 22π sm ga teng. Ular tashkil qilgan halqaning yuzi hisoblansin.

A) 88π; B) 72π; C) 78π; D) 85π; E) 83π sm².

45. Agar yoyning o'lchovi 120° ga va doiraviy segmentning radiusi 8 sm ga teng bo'lsa, uning yuzi hisoblansin.

A) $\frac{42\pi}{5}-8$; B) $\frac{56\pi}{3}-9$; C) $\frac{48\pi}{\sqrt{2}}-8$; D) $\frac{64\pi}{3}-16\sqrt{3}$;

E) $\frac{36\pi}{5}-8$ sm².

46. Doiraviy segmentda vatar 6 sm ga teng va mos markaziy burchakning kattaligi 60° bo'lsa, segmentning yuzi hisoblansin.

A) 16π; B) $12\pi+9\sqrt{3}$; C) $10\pi+\sqrt{3}$; D) $12\pi-\sqrt{3}$;

E) $6\pi-9\sqrt{3}$ sm².

47. Radiusi 9 sm bo'lgan doira markazining bir tomonida o'zaro parallel bo'lgan ikkita vatar o'tkazilgan. Vatarlarga mos kelgan yoylar kattaliklari 60° va 120° bo'lsa, kamarning yuzi hisoblansin.

A) 34π; B) $\frac{27}{2}\pi$; C) 36π; D) 32π; E) $\frac{29}{2}\pi$ sm².

48. Agar doiraviy sektor markaziy burchagining kattaligi 60° va radiusi 13 sm bo'lsa, uning yuzi hisoblansin.

A) $\frac{135}{8}\pi$; B) $\frac{144}{7}\pi$; C) 169π;

D) $\frac{169}{6}\pi$; E) $\frac{169}{7}\pi$ sm².

49. Radiusi 4 sm ga teng bo'lgan doira segmenti markaziy burchagining kattaligi 120° bo'lsa, segmentning yuzi hisoblansin.

A) $\frac{18\pi-8\sqrt{3}}{7}$; B) $\frac{16\pi-12\sqrt{3}}{3}$; C) $\frac{13\pi-4\sqrt{3}}{3}$;

D) $\frac{15\pi-2\sqrt{3}}{3}$; E) $\frac{16\pi-8\sqrt{3}}{5}$ sm²

50. Agar doiraning yuzi radiuslari 5 sm va 7 sm bo'lgan doiralar yuzlarining yig'indisiga teng bo'lsa, doiraning yuzi hisoblansin.

A) 74π; B) 72π; C) 64π; D) 88π; E) 84π sm^2.

4-§. TO'RTBURCHAKLAR

4.1. Asosiy tushunchalar va xossalar

I. Parallelogramm. Qarama-qarshi tomonlari parallel bo'lgan to'rtburchak *parallelogrammdir*.

U quyidagi xossalarga ega:

1. Parallelogrammning qarama-qarshi tomonlari teng: $AB=CD$, $BC=AD$.

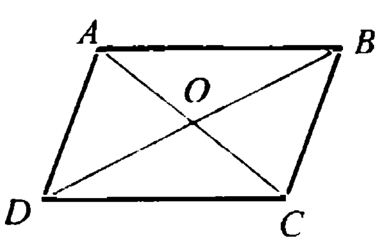
2. Parallelogrammning qarama-qarshi burchaklari teng: $\angle A=\angle C$, $\angle B=\angle D$.

3. Bir tomonga yopishgan burchaklarning yig'indisi 180° ga teng: $\angle A+\angle D=180°$, $\angle B+\angle C=180°$, $\angle C+\angle D=180°$, $\angle A+\angle B=180°$.

4. Parallelogrammning diagonali uni ikkita teng uchburchakka bo'ladi: $\Delta ABC=\Delta ADC$, $\Delta ABD=\Delta BCD$.

5. Parallelogrammning diagonallari kesishish nuqtasida teng ikkiga bo'linadi: $AO=OC$, $BO=OD$.

6. Parallelogramm diagonallarining kesishish nuqtasi O parallelogrammning simmetriya markazidir.

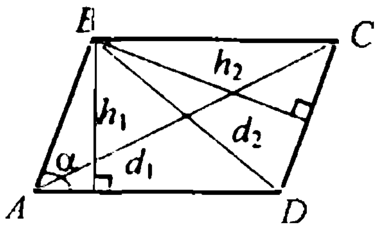


4.2-chizma.

7. Parallelogramm diagonallari kvadratlarining yig'indisi uning hamma tomonlari kvadratlarining yig'indisiga teng:

$$AC^2+BD^2=2(AB^2+AD^2).$$

8. Parallelogrammning yuzini hisoblash formulalari (4.2-chizma):



4.2-chizma.

1) $S=a\cdot h_1=b\cdot h_2$, (4.1)

h_1, h_2 — parallelogrammning balandliklari;

2) $S=a\cdot b\cdot \sin\alpha$, (4.2)

α — bu a va b qo'shni tomonlar orasidagi burchak;

3) $S=0{,}5\cdot d_1\cdot d_2\cdot \sin\gamma$, (4.3)

bunda d_1 va d_2 — diagonallar, γ — diagonallar orasidagi burchak.

II. To'g'ri to'rtburchak. *To'g'ri to'rtburchak* tomonlari o'zaro perpendikulyar bo'lgan parallelogrammdir (4.3-chizma).

To'g'ri to'rtburchak uchun parallelogrammning barcha xossalari o'rinli. Uning qo'shimcha xossalari quyidagicha:

9. To'g'ri to'rtburchakning diagonallari o'zaro teng: $AC=BD$.

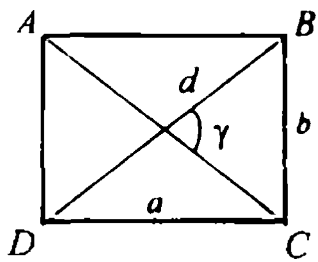
10. To'g'ri to'rtburchakning yuzini hisoblash formulalari (4.3-chizma):

$S=ab$, (4.4)

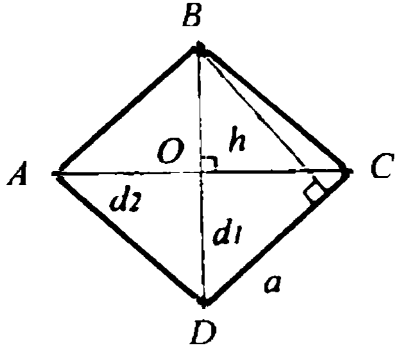
bunda a, b — to'g'ri to'rtburchakning tomonlari;

$S=0{,}5\cdot d^2\cdot \sin\gamma$, (4.5)

bunda d — diagonal, γ — diagonallar orasidagi burchak.



4.3-chizma.



4.4-chizma.

III. Romb. Tomonlari teng bo'lgan parallelogramm *rombdir* (4.4-chizma). Parallelogrammning barcha xossalari romb uchun ham o'rinli. Uning o'ziga xos xossalari quyidagilar:

11. Rombning diagonallari o'zaro perpendikulyar:

$$d_1 = AC \perp BD = d_2.$$

12. Rombning yuzini hisoblash formulalari:

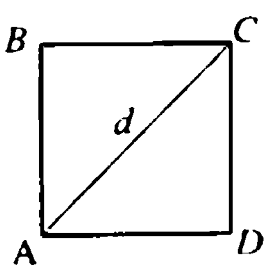
$$S = ah,$$

bunda h — rombning balandligi;

$$S = 0{,}5 d_1 d_2,$$

bunda d_1, d_2 — diagonallar.

IV. Kvadrat. Tomonlari teng bo'lgan to'g'ri to'rtburchak *kvadratdir* (4.5-chizma). Kvadrat uchun parallelogramm, to'g'ri to'rtburchak, rombning barcha xossalari o'rinli. Kvadratning yuzi: $S = a^2$, $S = \frac{1}{2} d^2$ (4.8) formulalar bo'yicha hisoblanadi.

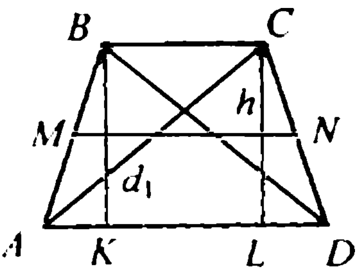


4.5-chizma.

V. Trapetsiya. Faqat ikkita tomoni parallel bo'lgan to'rtburchak *trapetsiyadir* (4.6-chizma).

Parallel bo'lgan tomonlar trapetsiyaning *asoslari*, parallel bo'lmagan tomonlar esa trapetsiyaning *yon tomonlari* deyiladi (4.6-chizmada AD va BC — asoslar, AB va CD — yon tomonlar).

Agar trapetsiyaning yon tomonlari teng bo'lsa ($AB=CD$), u *teng yonli* trapetsiya deyiladi. Trapetsiyaning uchidan qarama-qarshi asosga perpendikulyar qilib o'tkazilgan kesma trapetsiyaning *balandligi* deyiladi: BK, CL — balandliklar.



4.6-chizma.

Trapetsiyada AC, BD — diagonallardir (4.6-chizma).

Trapetsiya yon tomonlarining o'rtalarini tutashtiruvchi kesma uning *o'rta chizig'i* deyiladi. Agar $MA=MB$, $NC=ND$ bo'lsa, MN o'rta chiziqdir.

14. Trapetsiyaning o'rta chizig'i uning asoslariga parallel va ular yig'indisining yarmiga teng: $MN \| AD. MN \| BC$, $MN=\frac{BC+AD}{2}$.

15. Trapetsiyaning yuzini hisoblash formulalari:

$$1)\ S=\frac{a+b}{2}\cdot h, \qquad (4.9)$$

bunda a, b — asoslarning uzunliklari, h — balandlik uzunligi;

$$2)\ S=\frac{1}{2}d_1\ d_2\sin\gamma,$$

bunda d_1, d_2 — diagonallar uzunliklari, γ — diagonallar orasidagi burchak.

4.2. Mavzu bo'yicha masalalar

1. Parallelogrammning bir tomoni ikkinchi tomonidan 4 marta katta, perimetri $20\sqrt{2}$ sm, o'tkir burchagi 45° ga teng. Parallelogrammning yuzi hisoblansin.

A) $8\sqrt{2}$; B) $32\sqrt{2}$; C) 16; D) 8; E) $16\sqrt{2}$ sm².

2. Parallelogrammning tomonlari nisbati 3:5 kabi, perimetri 48 sm, o'tmas burchagi 120° ga teng. Parallelogrammning yuzi hisoblansin.

A) 67,5; B) $\frac{135\sqrt{2}}{2}$; C) 48; D) $67,5\sqrt{3}$; E) $48\sqrt{3}$ sm².

3. Rombning bitta diagonali 10% orttirilib, ikkinchi diagonali esa 15% kamaytirilsa, rombning yuzi qanday o'zgaradi?

A) 5% ortadi; B) o'zgarmaydi; C) 5% kamayadi;
D) 5,65% kamayadi; E) 6,5% ortadi.

4. *ABCD* rombning perimetri 14 ga teng. Romb tomonlarining o'rtalari tutashtirilsa, yangi $A_1B_1C_1D_1$ to'rtburchak hosil bo'ladi. $A_1B_1C_1D_1$ to'rtburchak tomonlarining o'rtalari yangi $A_2B_2C_2D_2$ to'rtburchakning uchlaridir. $A_2B_2C_2D_2$ to'rtburchakning perimetri topilsin.

A) 7; B) 10; C) 8; D) 6; E) 9.

5. Ikkita o'xshash romb uchun mos tomonlar nisbati 3 ga teng. Ular yuzlarining nisbati nimaga teng?

A) 7; B) 8; C) 10; D) 11; E) 9.

6. *ABCD* kvadratning *A* uchidan *AD* va *AB* to'g'ri chiziqlar o'tkazilgan. Kvadratning *C* uchidan *BD* diagonalga parallel bo'lgan *EF* to'g'ri chiziq o'tkazilgan. Agar kvadratning yuzi 3 ga teng bo'lsa, ΔAFE uchburchakning yuzi hisoblansin.

A) 5; B) 6; C) 7; D) 9; E) 8.

7. Parallelogrammning perimetri 54 sm, tomonlarining biri ikkinchisidan 3 sm katta. Parallelogrammning kichik tomoni uzunligi topilsin.

A) 10; B) 14; C) 12; D) 16; E) 15.

8. To‘g‘ri to‘rtburchakning diagonali $AC=15$ sm, tomoni $AD=12$ sm. To‘g‘ri to‘rtburchakning yuzi hisoblansin.

A) 108; B) 116; C) 100; D) 121; E) 225 sm^2.

9. Rombning tomoni 5 sm, bitta diagonali 8 sm bo‘lsa, uning ikkinchi diagonali uzunligi topilsin.

A) 14; B) 7; C) 6; D) 8; E) 5 sm.

10. Parallelogrammning yuzi 180 sm^2, balandliklari 10 sm va 15 sm bo‘lsa, uning yarimperimetri topilsin.

A) 40; B) 25; C) 45; D) 30; E) 35 sm.

11. Parallelogrammda A burchakning bissektrisasi qarshisidagi BC tomonni uzunliklari a va b bo‘lgan ikkita kesmaga ajratadi. Parallelogrammning perimetri topilsin.

A) $2(a+b)$; B) $2a+3b$; C) $2a+4b$;
D) $3a+2b$; E) $4a+2b$.

12. Rombning perimetri 16 sm, balandligi 2 sm ga teng. Rombning burchaklari topilsin.

A) 140° va 40°; B) 150° va 30°; C) 120° va 60°;
D) 100° va 80°; E) 90° va 90°.

13. Parallelogrammning diagonallari 17 sm va 19 sm, bitta tomoni esa 10 sm bo‘lsa, parallelogrammning ikkinchi tomoni uzunligi topilsin.

A) 17; B) 15; C) 16; D) 18; E) 8 sm.

14. Teng yonli trapetsiyada yon tomoni $4\sqrt{2}$ ga, kichik asos 4 ga teng. Trapetsiyaning diagonali yon tomoni bilan 30°, katta asos bilan esa α burchakni tashkil qiladi, α burchak topilsin.

A) 60°; B) 35°; C) 30°; D) 50°; E) 45°.

15. Teng yonli trapetsiyaning asoslari 4,2 va 5,4 ga, kichik asosidagi burchagi 135° ga teng. Trapetsiyaning yuzi hisoblansin.

A) 24,8; B) 9,6; C) 16,8; D) 4,8; E) 2,88.

16. Agar teng yonli trapetsiyaning asoslari 10 sm va 26 sm, diagonallari esa yon tomonlariga perpendikulyar bo'lsa, uning yuzi hisoblansin.

A) 225; B) 218; C) 216; D) 220; E) 214 sm^2

17. Trapetsiyaning bitta burchagi 30°, o'rta chizig'i 10 sm, bitta asosi 8 sm bo'lib, yon tomonlari davom ettirilganda to'g'ri burchak ostida kesishadi. Trapetsiyaning kichik yon tomoni topilsin.

A) 2; B) 3; C) 5; D) 1; E) 4 sm.

18. Parallelogrammning tomonlari a va b, o'tkir burchagi esa α ga teng. Hamma burchaklarning bissektrisalari o'tkazilganda ular kesishib, to'rtburchak hosil qiladi. Shu to'rtburchakning yuzi hisoblansin.

A) $\frac{1}{2}(a+b)^2$; B) $(a+b)^2 \sin\alpha$; C) $ab \sin\alpha$;

D) $\frac{1}{2}(a-b)^2 \sin\alpha$; E) $\frac{1}{2}(a+b)^2 \sin\alpha$.

19. Trapetsiyaning asoslariga parallel bo'lgan to'g'ri chiziq uning diagonallari kesishgan nuqtadan o'tadi. Agar trapetsiyaning asoslari m va n ga teng bo'lsa, to'g'ri chiziqning yon tomonlar orasida yotgan kesmasi uzunligi topilsin.

A) $\frac{m-n}{m+n}$; B) $\frac{2mn}{\sqrt{m^2+n^2}}$; C) $\frac{\sqrt{2mn}}{m+n}$; D) $\frac{mn}{m-n}$; E) $\frac{2mn}{m+n}$.

20. Teng yonli trapetsiyaning asoslari 15 sm va 49 sm, bitta burchagi 60° ga teng. Trapetsiyaning perimetri topilsin.

A) 130; B) 126; C) 135; D) 132; E) 128 sm.

21. Trapetsiyaning asoslari 28 sm va 64 sm ga teng. Uzunligi 42 sm bo'lgan yon tomoni katta asosi bilan 30° li burchak tashkil qiladi. Trapetsiyaning yuzi hisoblansin.

A) 900; B) 945; C) 960; D) 964; E) 966 sm^2.

22. To'g'ri burchakli trapetsiyaning kichik diagonali 15 sm va katta yon tomonga perpendikulyar. Kichik yon tomon 12 sm bo'lsa, uning katta asosi uzunligi topilsin.

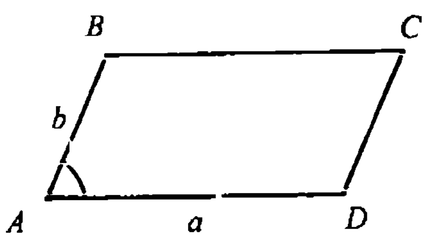
A) 20; B) 25; C) 30; D) 28; E) 32 sm.

4.3. Mavzu bo'yicha masalalarning yechimlari

1. Berilgan. $ABCD$ — parallelogramm, $AD=4AB$, $\angle BAD=45^\circ$, $P=20\sqrt{2}$ sm.

S_{ABCD} hisoblansin (4.3.1- chizma).

Yechilishi. $AB=b$, $AD=a$ bo'lsin. Perimetr formulasidan va berilganlardan foydalanib,



4.3.1-chizma.

$$\begin{cases} 2(a+b)=20\sqrt{2}, \\ a=4b \end{cases}$$

sistemani hosil qilamiz. Bu yerdan

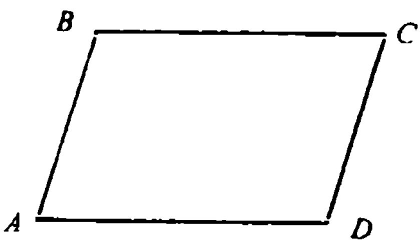
$$\begin{cases} 4a+b=10\sqrt{2}, \\ a=4b \end{cases} \Rightarrow \begin{cases} 5b=10\sqrt{2}, \\ a=4b \end{cases} \Rightarrow \begin{cases} b=2\sqrt{2}, \\ a=8\sqrt{2}. \end{cases}$$

Parallelogrammning yuzi (4.2) formula orqali hisoblanadi. Demak, $S=2\sqrt{2}\cdot 8\sqrt{2}\sin 45^\circ=16\cdot 2\frac{\sqrt{2}}{2}=$
$=16\sqrt{2}$ sm².

Javobi: E).

2. Berilgan. $ABCD$ — parallelogramm,
$AB:AD=3:5$, $P=48$ sm, $\angle ABC=120^\circ$.

S_{ABCD} topilsin (4.3.2-chizma).



4.3.2-chizma.

Yechilishi. Agar $AB=b$, $AD=a$ va $\angle BAD=\alpha$ bo'lsa, parallelogrammning yuzi (4.2) formula bo'yicha hisoblanadi. Ma'lumki, parallelogrammning bir tomoniga yopishgan burchaklari yig'indisi 180°ga teng: $\angle BAD+$ $+\angle ABC=180^\circ$. Shuning uchun, $\alpha=180^\circ-120^\circ=60^\circ$. Perimetr ta'rifidan va berilganlardan foydalanib, a va b ga nisbatan

$$\begin{cases} 2(a+b)=48, \\ b:a=3:5 \end{cases}$$

tenglamalar sistemasini hosil qilamiz. Bu yerdan

$$\begin{cases} b=3a:5, \\ a+3a:5=24, \end{cases} \Rightarrow \begin{cases} b=3a:5, \\ 8a:5=24, \end{cases} \Rightarrow \begin{cases} b=9, \\ a=15. \end{cases}$$

U holda (4.2) formulaga asosan,

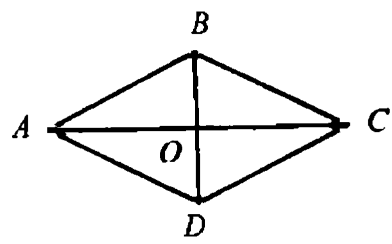
$S_{ABCD}=9\cdot 15\cdot\sin 60^\circ=\frac{135\sqrt{2}}{2}$ yoki $S=67{,}5\sqrt{3}$ sm².

Javobi: D)

3. Berilgan. $ABCD$ — romb, $AC=d_1$, $BD=d_2$ — diagonallar, d_1 10% orttirilib, d_2 15% kamaytirilsa.

S_{ABCD} o'zgarishi aniqlansin (4.3.3-chizma).

Yechilishi. Rombning yuzini (4.7) formula bo'yicha hisoblash maqsadga muvofiq, chunki uning diagonallari berilgan. 1% sonning 0,01 qismiga teng. Shuning uchun yangi rombning diagonallari $d_1+0,1d_1=1,1d_1$ va $d_2-0,5d_2=0,85d_2$ ga teng bo'ladi. Yangi rombning yuzi $S_1=\frac{1}{2}\,1,1d_1\cdot 0,85d_2=1,1\cdot 0,85S=93,5S$. Demak, rombning yuzi $100\%-93,5\%=6,5\%$ ga kamayadi.



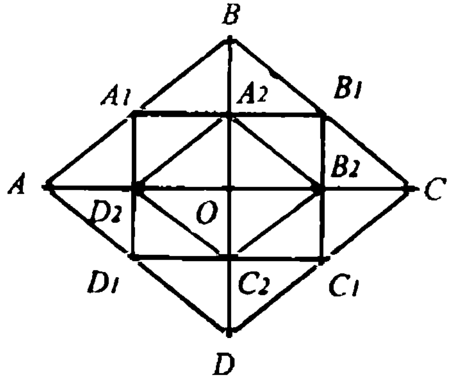
4.3.3-chizma.

Javobi: E).

4. Berilgan. $ABCD$ — romb, $P=14$ sm, A_1, B_1, C_1, D_1 — romb tomonlarining o'rtalari, A_2, B_2, C_2, D_2 — yangi rombning uchlari.

$P_{A_2B_2C_2D_2}$ topilsin (4.3.4-chizma).

Yechilishi. Rombning hamma tomonlari teng va $AB=a$ deb belgilasak, uning perimetri $4a$ ga teng bo'ladi. Shartga asosan

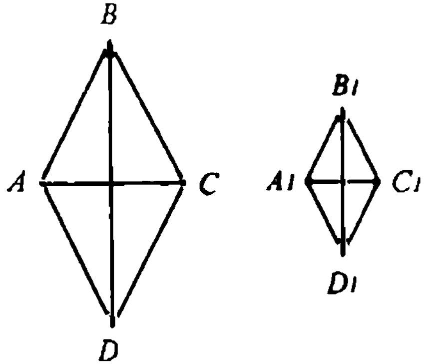


4.3.4-chizma.

$4a=14$ va $a=\frac{14}{4}=\frac{7}{2}$. A_1, B_1 o'rta nuqtalar bo'lganligidan, A_1B_1 kesma ΔABC ning o'rta chizig'idir. Rombning diagonallari — uning simmetriya o'qlaridir. Shuning uchun A_1B_1 kesmaning A_2 o'rta nuqtasi BD diagonalda yotadi va $BA_2=A_2O$. Shunga o'xshash, $CB_2=B_2O$, $DC_2=C_2O$, $AD_2=D_2O$ munosabatlarni olamiz. Demak, A_2B_2 — ΔBOC ning o'rta chizig'idir va o'rta chiziqning xossalariga ko'ra, $A_2B_2=\frac{1}{2}BC=\frac{7}{4}$ va $A_2B_2 \| BC$. Endi $A_2B_2C_2D_2$ to'rtburchakning perimetrini hisoblasak, $P_{A_2B_2C_2D_2}=4\frac{7}{4}=7$ sm bo'ladi.

Javobi: A).

5. Berilgan. $ABCD$, $A_1B_1C_1D_1$ — romblar, $ABCD\sim A_1B_1C_1D_1$, $AB:A_1B_1=3$.



4.3.5-chizma.

$S:S_1$ topilsin (4.3.5-chizma).

Yechilishi. O'xshash ko'pburchaklar yuzlarining nisbati mos tomonlar nisbatining kvadratiga teng. Shuning uchun:

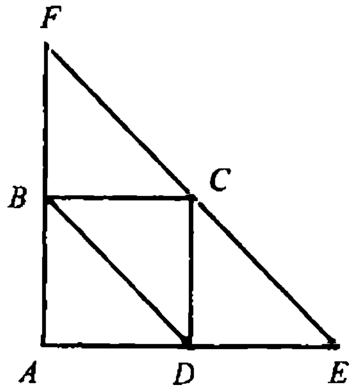
$$\frac{S}{S_1}=\left(\frac{AB}{A_1B_1}\right)^2=3^2=9.$$

Javobi: E).

6. Berilgan. $ABCD$ — kvadrat, $S_{KB}=3$, $FCE \| BD$.

$S_{\Delta FEA}$ hisoblansin (4.3.6-chizma).

Y e c h i l i s h i. Kvadratning tomoni $AB=a$ bo'lsa, uning yuzi $S=a^2$ va berilganiga ko'ra $a^2=3$. Kvadratning tomoni $a=\sqrt{3}$ va diagonali $BD=\sqrt{2a^2}=\sqrt{6}$, $FE \parallel BD$ bo'lgani uchun, BD kesma ΔAFE ning o'rta chizig'i bo'ladi va $FE=2\cdot BD=$ $=2\sqrt{6}$, $AF=2\cdot AB=2\sqrt{3}$, $AE=AF$. U holda



4.3.6-chizma.

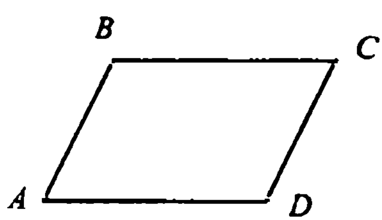
$$S_{\Delta AEF}=\frac{1}{2}AE\cdot AF=\frac{1}{2}AF^2 \text{ yoki } S_{\Delta AEF}=\frac{1}{2}\cdot 4\cdot 3=6.$$

Javobi: B).

7. B e r i l g a n. $ABCD$ — parallelogramm, $P=54$ sm, $AD=AB+3$ sm.

AB t o p i l s i n (4.3.7.-chizma).

Y e c h i l i s h i. Perimetrning ta'rifiga asosan, $P=2(AB+AD)$. AD va P ning o'rniga ma'lum miqdorlarni qo'yamiz: $54=$ $=2(AB+AB+3)$, $2AB+3=$ $=27$, $2AB=24$. Parallelogrammning kichik tomoni $AB=12$ sm.

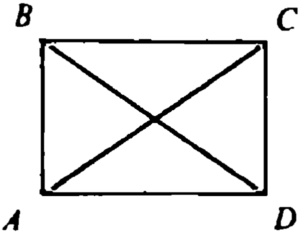


4.3.7.-chizma.

Javobi: C).

8. B e r i l g a n. $ABCD$ — to'g'ri to'rtburchak, $AC=$ $=15$ sm, $AD=12$ sm.

S_{ABCD} h i s o b l a n s i n (4.3.8-chizma).



4.3.8-chizma.

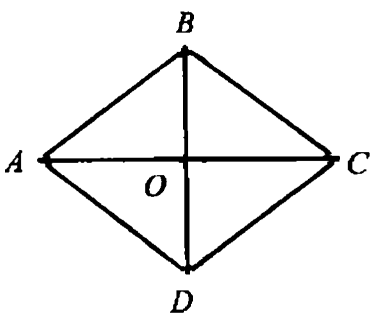
Yechilishi. Yuzni hisoblashda (4.4) formuladan foydalanamiz: $S=AD\cdot AB$. Demak, to'g'ri to'rtburchakning AD ga qo'shni bo'lgan ikkinchi AB tomonini topish kerak. ΔACD to'g'ri burchakli va $CD=AB$. Pifagor teoremasidan (2-§, 7-xossa) foydalanamiz:

$AB=\sqrt{AC^2-AD^2}=\sqrt{15^2-12^2}=\sqrt{(15-12)(15+12)}=$ $=9$ sm. Demak, $S=12\cdot 9=108$ sm^2.

Javobi: A).

9. Berilgan. $ABCD$ — romb, $AB=5$ sm, $BD=8$ sm.

AC topilsin (4.3.9-chizma).



4.3.9-chizma.

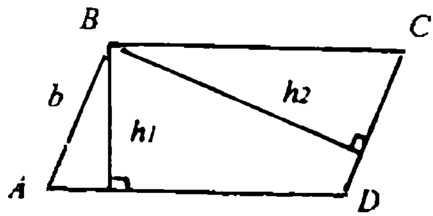
Yechilishi. Berilganlardan, $BO=4$ sm va Pifagor teoremasiga (2-§, 7-xossa) asosan, ΔAOB dan AO ni topamiz: $AO=$ $=\sqrt{AB^2-BO^2}=\sqrt{5^2-4^2}=$ $=3$ sm. Ikkinchi diagonal $AC=2\cdot AO=6$ sm bo'ladi.

Javobi: C).

10. Berilgan. $ABCD$ — parallelogramm, $h_1=10$ sm, $h_2=15$ sm, $S=180$ sm^2.

$\frac{1}{2}P_{ABCD}$ topilsin (4.3.10-chizma).

Yechilishi. Parallelogrammning tomonlari $AD=a$, $AB=b$ bo'lsa, perimetri $P=2(a+b)$. Parallelogrammning yuzi (4.1) formuladan hisoblanadi: $S=a \cdot h_1$ va $S=b \cdot h_2$. Bu tenglikdan a va b ni topamiz: $180=10 \cdot a$, $a=18$ sm, $180=15 \cdot b$, $b=12$ sm. Parallelogrammning yarimperimetrini topamiz: $\frac{1}{2}R=2(9+6)=30$ sm.



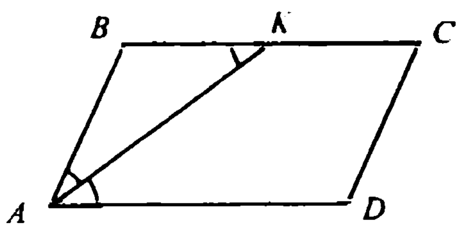
4.3.10-chizma.

Javobi: D).

11. Berilgan. $ABCD$ — parallelogramm, AK — bissektrisa, $\angle BAK=\angle KAD$, $BK=a$, $KC=b$.

P_{ABCD} topilsin (4.3.11-chizma).

Yechilishi. Perimetrning ta'rifidan, $P=2(AD+AB)$. Berilgan shartga asosan, $AD=BC=BK+$ $+KC=a+b$. AD va BC to'g'ri chiziqlar uchinchi AK to'g'ri chiziq bilan kesishgan. U holda ichki almashinuvchi burchaklar teng: $\angle BKA=\angle KAD$. AK bissektrisa bo'lgani uchun, $\angle BAK=\angle KAD$. Shuning uchun $\angle BAK=\angle BKA$ va shu sababli $\triangle ABK$ teng yonli, ya'ni $AB=BK=a$. U holda parallelogrammning perimetri $P=2(a+a+b)=4a+2b$ bo'ladi.

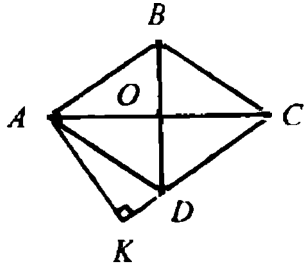


4.3.11-chizma.

Javobi: E).

12. B e r i l g a n. $ABCD$ — romb, $P=16$ sm, $AK=h=$ $=2$ sm, $AK \perp DC$.

$\angle A$, $\angle D$ t o p i l s i n (4.3.12-chizma).



4.3.12-chizma.

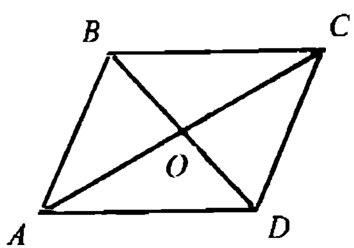
Y e c h i l i s h i. Rombning hamma tomonlari teng. $AB=a$ bo'lsa, $P=4a=16$ tenglikdan $4a=16$, $a=4$ ekanligini olamiz: ΔADK to'g'ri burchakli bo'lganligidan, $\angle ADK=\alpha$ bo'lsa, $\sin\alpha = \frac{h}{a} = \frac{1}{2}$ va $\alpha=30^\circ$ ekanligi kelib chiqadi. U holda $\angle CDA=180^\circ-30^\circ=150^\circ$. Demak, rombning o'tkir burchaklari 30°, o'tmas burchaklari esa 150° bo'ladi.

Javobi: B).

13. B e r i l g a n. $ABCD$ — parallelogramm, $AB=$ $=10$ sm, $AC=19$ sm, $BD=17$ sm.

AD t o p i l s i n (4.3.13-chizma).

Y e c h i l i s h i. 7-xossaga muvofiq, $AC^2+BD^2=2(AB^2+$ $+AD^2)$. Bu tenglikdan $AD^2 = \frac{AC^2+BD^2-2AB^2}{2}$ yoki



4.3.13-chizma.

$AD^2 = \frac{1}{2}(17^2+19^2-2\cdot10^2)=$

$=\frac{1}{2}(289+361-200)=225$

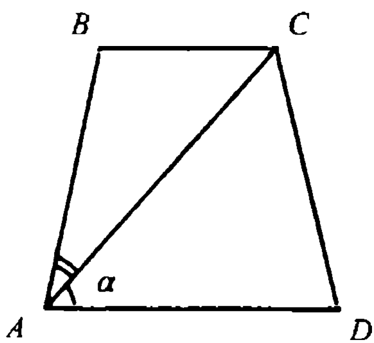
sm^2, $AD=15$ sm natijani olamiz.

Javobi: B).

14. Berilgan. $ABCD$ — trapetsiya, $BC=4$, $AB=CD=4\sqrt{2}$, $\angle CAB=30°$, $\angle CAD=\alpha$.

α topilsin (4.3.14-chizma).

Yechilishi. Trapetsiyada AD, BC asoslar o'zaro parallel, AC diagonal esa ularni kesib o'tadi. Shuning uchun hosil bo'lgan ichki almashinuvchi burchaklar o'zaro teng. $\angle BCA = \angle CAD=\alpha$. ΔABC da ikkita tomoni $BC=4$, $AB=4\sqrt{2}$ va qarshisidagi burchaklar mos ravishda $30°$ va α. Sinuslar teoremasidan (2-§, 8-xossa) foydalanamiz: $\frac{AB}{\sin\alpha}=\frac{BC}{\sin 30°}$, $\frac{4\sqrt{2}}{\sin\alpha}=\frac{4}{0{,}5}$, $\sin\alpha=\frac{\sqrt{2}}{2}$ va $\alpha=45°$.



4.3.14-chizma.

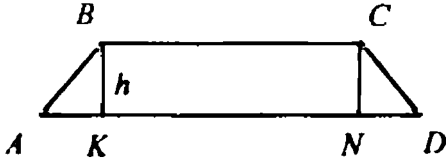
Javobi: B).

15. Berilgan. $ABCD$ — trapetsiya, $AB=CD$, $BC=4{,}2$, $AD=5{,}4$, $\angle ABC=135°$.

S_{ABCD} hisoblansin (4.3.15-chizma).

Yechilishi. Agar trapetsiyaning asoslari $AD=a$, $BC=b$, balandligi $BK=h$ ga teng bo'lsa, uning yuzi (4.9) formula bo'yicha hisoblanadi.

B va C uchlaridan trapetsiyaning asosiga perpendikulyarlar o'tkazsak, $BK=CN=h$ parallel to'g'ri chiziqlar orasidagi



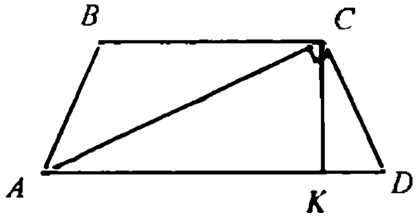
4.3.15-chizma.

masofa $KN=BC=b$ bo'ladi. $\Delta ABK=\Delta CND$, chunki ularning gipotenuzalari va bittadan katetlari teng. Shuning uchun $AK=ND$, $\angle BAK=\angle CND=45°$, chunki o'tmas burchaklar 135° ga teng. Demak, to'g'ri burchakli ABK teng yonli va $AK=BK$. Lekin $AK=0{,}5(AD-BC)=$ $=0{,}5(5{,}4-4{,}2)=0{,}6$. Shuning uchun (4.9) formulaga asosan, $S=\frac{4{,}2+5{,}4}{2}\cdot 0{,}6=2{,}88$.

Javobi: E).

16. Berilgan. $ABCD$ — trapetsiya, $AB=CD$, $AC\perp CD$, $AD=26$, $BC=10$.

S_{ABCD} hisoblansin (4.3.16-chizma).



4.3.16-chizma.

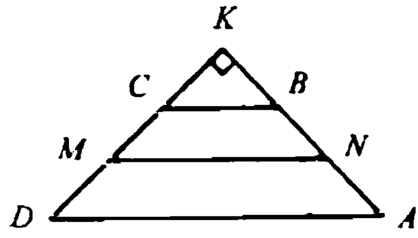
Yechilishi. Yuzni hisoblashda (4.9) formuladan foydalanamiz. Trapetsiya teng yonli bo'lgani uchun $DK=\frac{26-10}{2}=8$ va $AK=26-8=18$. ΔACD to'g'ri burchakli va to'g'ri burchak uchidan o'tkazilgan balandlikning xossasidan (2-§) foydalanamiz: $h^2=AK\cdot KD=8\cdot 18=9\cdot 16$, $h=3\cdot 4=12$. Endi trapetsiyaning yuzini hisoblaymiz: $S=\frac{26+10}{2}\cdot 12=36\cdot 6=216$.

Javobi: C).

17. Berilgan. $ABCD$ — trapetsiya, MN — o'rta chiziq, $MN=10$ sm, $BC=b=8$ sm, $\angle CDA=30°$, $(ABK)\perp(DCK)$.

AB kichik yon tomoni topilsin (4.3.17-chizma).

Yechilishi. 14-xossaga muvofiq: $MN = (a+b)/2$. Shuning uchun $a = 2 \cdot 10 - 8 = 12$. ΔAKD to'g'ri burchakli va uning bitta o'tkir burchagi 30° ga teng: $KA = 0{,}5 \cdot 12 = 6$. $BC \| AD$ bo'lgani uchun $\Delta BKC \sim \Delta AKD$ ($\angle K$ — umumiy). O'xshash uchburchaklarning xossasiga asosan (2-§, (2.4) formula):



4.3.17-chizma.

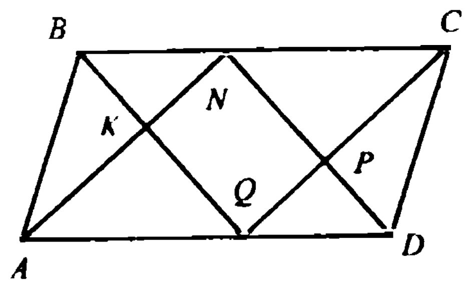
$\frac{AK}{BK} = \frac{AD}{BC}$, $\frac{AB+BK}{BK} = \frac{12}{8} = \frac{3}{2}$. Lekin ΔBKA da $\angle K = 90°$, $\angle KCB = 30°$ va shuning uchun $BK = \frac{BC}{2} = \frac{1}{2} \cdot 8 = 4$ sm. Endi AB ni topamiz: $\frac{AB+4}{4} = \frac{3}{2}$, $2AB + 8 = 12$, $2AB = 4$, $AB = 2$ sm.

Javobi: A).

18. Berilgan. $ABCD$—parallelogramm, $AD = a$, $AB = b$, $\angle BAD = \alpha$, AK, BK, CP, DP— bissektrisalar.

S_{KNPQ} hisoblansin (4.3.18-chizma).

Yechilishi. Parallelogrammning 3-xossasidan foydalanamiz, ya'ni uning bir tomoniga yopishgan burchaklarining yig'indisi 180°ga teng. Burchaklarning bissektrisalari o'tkazilsa, yarim burchaklarning yig'indisi 90° ga teng bo'ladi va shu bilan $\angle BKA = 90°$ bo'lishini ko'ramiz.



4.3.18-chizma.

Demak, $KNPQ$ to'g'ri to'rtburchak va uning yuzi (4.4) formuladan topiladi: $S=KN\cdot KQ$, $\angle BAD=\alpha$ bo'lsa, $\angle KAD=\frac{\alpha}{2}$ va $\angle CBD=90°-\frac{\alpha}{2}$.

U holda $KN=(a-b)\sin\frac{\alpha}{2}$. Haqiqatan, $BC\parallel AD$ bo'lgani uchun $\angle FAD=\angle BFA=\frac{\alpha}{2}=\angle BAF$. Demak, ΔABF teng yonli va $BF=AB=b$. Shunga o'xshash, $KD=b$ va $AK=AD-KD=a-b$ bo'lishini ko'ramiz va $MQ=(a-b)\times\sin(90°-\frac{\alpha}{2})$ ekanligini olamiz. Endi yuzini hisoblasak:

$$S_{MNPQ}=(a-b)\sin\frac{\alpha}{2}(a-b)\sin(90°-\frac{\alpha}{2})=$$

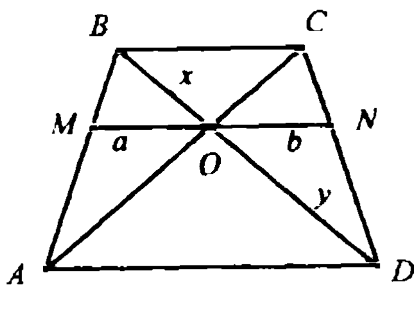
$$=\frac{1}{2}(a-b)^2\cdot 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}=\frac{1}{2}(a-b)^2\sin\alpha.$$

Javobi: D).

19. Berilgan. $ABCD$ — trapetsiya, $AD=m$, $BC=n$, $AC\cap BD=O$, $(MON)\parallel AD$.

MN topilsin (4.3.19- chizma).

Yechilishi. Trapetsiyaning AC va BD diagonallari O nuqtada kesishgan bo'lsin va $BO=x$, $OD=y$, $MO=a$, $ON=b$ deb belgilaymiz. Trapetsiyaning diagonallari kesishishi natijasida hosil bo'lgan ΔAOD va ΔBOC lar o'xshash, ya'ni $\Delta AOD\sim\Delta BOC$ ($\angle BOC=\angle AOD$ — vertikal burchaklar, $\angle CBD=\angle ADB$ — ichki almashinuvchi burchaklar bo'lgani uchun),



4.3.19- chizma.

ularning mos tomonlari proporsional bo'ladi: $m:y=n:x$ yoki $y:x=m:x$.

Ikkinchi tomondan, $\triangle ABD \sim \triangle MBO$ ($AD \parallel MO$, $\angle ABD$ — umumiy bo'lgani uchun) va ularda ham mos tomonlar proporsional, ya'ni $a:m=x:(x+y)$ yoki $a=m\cdot 1:(1+y:x)=$ $=m:(1+m:n)=mn:(m+n)$. Uchinchidan, $\triangle BCD \sim \triangle OND$ ($ON \parallel BC$, PD — umumiy bo'lgani uchun) va ularda mos tomonlar proporsional bo'ladi, ya'ni $b:n=y:x+y$, $b=$ $=n\cdot\frac{1}{x/y+1}=\frac{m\cdot n}{m+n}$ U holda, $MN=a+b==\frac{m\cdot n}{m+n}+\frac{m\cdot n}{m+n}=$ $\frac{2mn}{m+n}$.

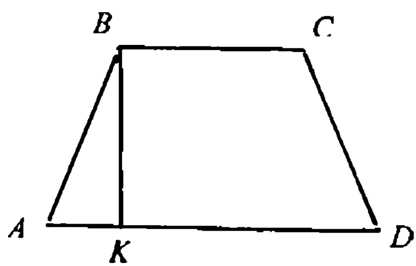
Javobi: E).

Bu yerdan trapetsiya diagonallarining kesishish nuqtasidan o'tuvchi va uning asoslariga parallel bo'lgan kesma shu kesishish nuqtasida teng ikkiga bo'linadi, degan xulosa kelib chiqadi.

20. B e r i l g a n. $ABCD$ — trapetsiya, $AB=CD$, $BC=15$ sm, $AD=49$ sm, $\angle BAD=60°$.

P_{ABCD} t o p i l s i n (4.3.20-chizma).

Y e c h i l i s h i. Ta'rifga ko'ra, $P=AD+BC+2AB$. B uchidan BK balandlik o'tkazamiz. Trapetsiya teng yonli bo'lgani uchun, $AK=$ $=\frac{49-15}{2}=17$ sm. To'g'ri burchakli $\triangle ABK$ ning bitta o'tkir burchagi 60° bo'lsa, $\angle ABK=30°$ bo'ladi. 30° li burchak qarshisidagi katet gipotenuzaning yarmiga teng bo'lganligidan, $AB=$



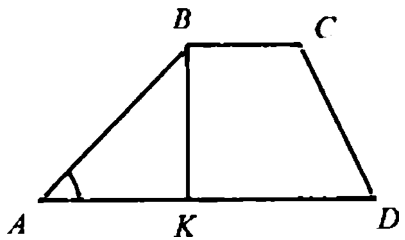
4.3.20-chizma.

$=2AK=2\cdot 17=34$ sm. Endi perimetrni hisoblaymiz: $P=15+49+2\cdot 34=132$ sm.

Javobi: D).

21. B e r i l g a n. $ABCD$ — trapetsiya, $BC=28$ sm, $AD=64$ sm, $AB=42$ sm, $\angle BAD=30°$.

S_{ABCD} h i s o b l a n s i n (4.3.21-chizma).



4.3.21-chizma.

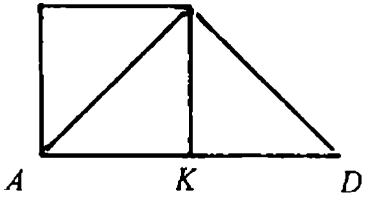
Y e c h i l i s h i. Agar $BK=h$ trapetsiyaning balandligi bo'lsa, uning yuzi (4.9) formula bo'yicha hisoblanadi:

$$S=\frac{BC+AD}{2}\cdot h=$$
$$=\frac{28+64}{2}\cdot h=46h.$$

ΔABK — to'g'ri burchakli va $\angle BAK=30°$ bo'lgani uchun $h=\frac{1}{2}AB=\frac{1}{2}\cdot 42=21$ sm va, demak, $S=46\cdot 21=$ $=966$ sm^2.

22. B e r i l g a n. $ABCD$ — trapetsiya, $\angle BAD=90°$, $AC=15$ sm, $AC\perp CD$, $AB=12$ sm.

AD t o p i l s i n (4.3.22-chizma).



4.3.22-chizma.

Y e c h i l i s h i. ΔABC to'g'ri burchakli va Pifagor teoremasi (2-§, 7-xossa) yordamida trapetsiyaning kichik BC asosi uzunligini topamiz: $BC=\sqrt{AC^2-AB^2}=\sqrt{225-144}=$ $=\sqrt{81}=9$ sm. $CK\perp AD$ o'tkazamiz, u holda $CK=AB=12$ sm, ΔACD to'g'ri burchakli bo'lganligidan C to'g'ri burchak uchidan o'tkazilgan CK balandlikning xossasidan foydalanamiz (16-masalaning yechilishiga q.): $CK^2=AK\cdot KD$, $12^2=9\cdot KD$ va $KD=$ $=\frac{12^2}{9}=\frac{144}{9}=16$ sm. U holda trapetsiyaning katta asosi $AD=AK+KD=9+16=25$ sm bo'ladi.

Javobi: B).

4.4. Mustaqil yechish uchun masalalar

1. Parallelogrammning tomonlari 3 sm va 10 sm ga teng. Katta tomoniga yopishgan ikki burchagining bissektrisalari o'tkazilgan va ular qarshisidagi tomonni uchta qismga ajratadi. Shu qismlarning uzunliklari topilsin.

A) 3, 3, 4; B) 4, 3, 3; C) 4, 4, 2;
D) 2, 4, 4; E) 3, 4, 3 sm.

2. $ABCD$ to'g'ri to'rtburchakning perimetri 24 sm ga teng. BC tomonining o'rtasidagi nuqta M bo'lib, $MA\perp MD$. To'g'ri to'rtburchakning tomonlari uzunliklari topilsin.

A) 3,5, 8,5; B) 5, 7; C) 4, 8; D) 3, 9; E) 5, 6 sm.

3. Parallelogrammning tomonlari 6 sm va 15 sm ga teng. a to'g'ri chiziq yon tomonga parallel qilib o'tkazilgan va parallelogrammni ikkita o'xshash parallelogrammga bo'ladi. Agar yon tomonda ajratilgan kesmalardan biri ikkinchisidan to'rt marta katta bo'lsa, hosil qilingan parallelogrammlar yuzlarining nisbati topilsin.

A) 3:5; B) 4:1; C) 5:2; D) 3:4; E) 4:3.

4. Rombning diagonallari 16 sm va 12 sm ga teng, uning balandligi topilsin.

A) 9,6; B) 8,8; C) 7,2; D) 10,2; E) 9,4 sm.

5. Parallelogrammning yuzi 8 sm^2, diagonallaridan biri ikkinchisidan 2 marta kichik va ular orasidagi burchak 30° ga teng. Diagonallar uzunliklari topilsin.

A) 8 va 6; B) 5 va 2,5; C) 6 va 4;
D) 4 va 8; E) 3 va 9 sm.

6. Teng yonli trapetsiyaning katta asosi 44 m, yon tomoni 17 m, diagonali 39 m ga teng. Trapetsiyaning yuzi hisoblansin.

A) 600; B) 480; C) 580; D) 560; E) 540 m^2.

7. Parallelogrammning diagonallari 14 sm va 18 sm, tomonlarining nisbati 4:7 kabi. Uning perimetri topilsin.

A) 40; B) 44; C) 42; D) 48; E) 46 sm^2.

8. Parallelogrammning yuzi 120 sm^2, balandliklari 8 va 12 sm ga teng bo'lsa, uning perimetri topilsin.

A) 50; B) 48; C) 46; D) 54; E) 58 m.

9. Parallelogrammning diagonallari 12 va 15 sm, ular orasidagi burchak 30° bo'lsa, parallelogrammning yuzi hisoblansin.

A) 100; B) 48; C) 45; D) 46; E) 58 sm^2.

10. Rombning balandligi qarama-qarshi tomonni teng ikkiga bo'ladi. Rombning o'tmas burchagi topilsin.

A) 90°; B) 110°; C) 130°; D) 120°; E) 150°.

11. Rombning yuzi 384 sm^2, diagonallari nisbati 3:4 kabi bo'lsa, uning perimetri topilsin.

A) 80; B) 90; C) 70; D) 100; E) 96 sm.

12. Teng yonli trapetsiyaning asoslari 6 sm va 10 sm, diagonali 10 sm bo'lsa, uning yuzi hisoblansin.

A) 42; B) 48; C) 44; D) 46; E) 52 sm^2.

13. To'g'ri burchakli trapetsiyaning yon tomonlari va kichik asosi mos ravishda 8, 10 va 10 sm. Trapetsiyaning katta asosi uzunligi topilsin.

A) 15; B) 12; C) 16; D) 14; E) 18 sm.

14. Teng yonli trapetsiya asoslarining ayirmasi 3 sm ga, asosidagi burchakning sinusi 0,8 ga teng. Trapetsiyaning yon tomoni uzunligi topilsin.

A) 3,5; B) 3; C) 4; D) 2; E) 2,5 sm.

15. To'g'ri to'rtburchakning perimetri 60 sm, tomonlaridan biri ikkinchisidan 10 sm katta bo'lsa, uning yuzi hisoblansin.

A) 200; B) 180; C) 225; D) 220; E) 196 sm^2.

16. *ABCD* parallelogrammda *AB* tomon va *BD* diagonal 10 sm, *AD* tomonga o'tkazilgan balandlik esa 5 sm. Parallelogrammning yuzi hisoblansin.

A) $54\sqrt{2}$; B) $54\sqrt{3}$; C) $44\sqrt{3}$; D) $50\sqrt{3}$; E) $48\sqrt{3}$ sm^2.

17. *ABCD* trapetsiyada diagonallar *P* nuqtada kesishadi. Agar $BC=10$ sm, $AP=9$ sm. $PC=6$ sm bo'lsa, uning *AD* katta asosi uzunligi topilsin.

A) 13; B) 15; C) 16; D) 18; E) 10 sm.

18. *ABCD* parallelogrammda *BK* balandlik o'tkazilgan. Agar $\angle ABK=30^\circ$, $AK=5$ dm, $KD=8$ dm bo'lsa, parallelogrammning perimetri topilsin.

A) 42; B) 45; C) 44; D) 48; E) 46 dm.

19. *ABCD* trapetsiyaning yuzi 161 sm^2, balandligi 14 sm, asoslari ayirmasi 11 sm bo'lsa, uning katta asosi uzunligi topilsin.

A) 15; B) 17; C) 18; D) 16; E) 21 sm.

20. Agar muntazam oltiburchakning yuzi $54\sqrt{3}$ sm^2 bo'lsa, uning tomoni uzunligi topilsin.

A) 2; B) 3; C) 6; D) 5; E) 4 sm.

21. Parallelogrammning katta tomoni 5 sm, balandliklari 2 sm va 2,5 sm bo'lsa, uning ikkinchi tomoni uzunligi topilsin.

A) 5; B) 4; C) 3; D) 2; E) 3,5 sm.

22. *ABCD* parallelogrammda $AB=12$ dm, $\angle A=30°$ bo'lsa, *C* nuqtadan *AD* to'g'ri chiziqqacha va *AD* kesmagacha bo'lgan masofalar topilsin.

A) 6 va 13; B) 3 va 15; C) 3 va 14;
D) 5 va 13; E) 6 va 12 dm.

23. Parallelogramm burchagining bissektrisasi qarshisidagi tomonni uzunliklari 5 sm va 3 sm bo'lgan kesmalarga ajratadi. Parallelogrammning perimetri topilsin.

A) 30 yoki 24; B) 26 yoki 24; C) 28 yoki 26;
D) 26 yoki 22; E) 24 yoki 24 sm.

24. To'g'ri to'rtburchakning kichik tomoni 7 sm, diagonallari esa 60° li burchak ostida kesishadi. To'g'ri to'rtburchakning yuzi hisoblansin.

A) $49\sqrt{3}$; B) $56\sqrt{3}$; C) $42\sqrt{3}$; D) $48\sqrt{3}$; E) $54\sqrt{3}$ sm^2.

25. *ABCD* to'g'ri to'rtburchakda *A* va *B* burchaklarning bissektrisalari *CD* tomonni uchta teng kesmaga ajratadi va har bir kesmaning uzunligi 3 sm ga teng. To'g'ri to'rtburchakning perimetri topilsin.

A) 16 yoki 18; B) 18 yoki 28; C) 24 yoki 30;
D) 26 yoki 28; E) 20 yoki 32 sm.

26. Rombning tomoni *a* ga, burchagi 150° ga teng. Rombning qarama-qarshi tomonlari orasidagi masofa topilsin.

A) 3*a*; B) 0,5*a*; C) *a*; D) 1,5*a*; E) 2*a*.

27. Agar kvadrat tomonlari 7 sm va 28 sm bo'lgan to'g'ri to'rtburchakka tengdosh bo'lsa, uning perimetri topilsin.

A) 56; B) 48; C) 52; D) 64; E) 60 sm.

28. Agar to'g'ri to'rtburchakning tomonlari nisbati 2:3 kabi, yuzi 54 sm^2 bo'lsa, uning perimetri topilsin.

A) 42; B) 40; C) 30; D) 28; E) 32 sm.

29. Birinchi kvadratning diagonali ikkinchi kvadratning tomonidan iborat bo'lsa, ikkinchi va birinchi kvadratlar yuzlarining nisbati topilsin.

A) 5:2; B) 4:3; C) 2:3; D) 3:1; E) 2:1.

30. Birinchi kvadratning diagonali ikkinchi kvadratning tomonidan, ikkinchi kvadratning diagonali esa uchinchi kvadratning tomonidan iborat bo'lsa, uchinchi va birinchi kvadratlar perimetrlarining nisbati topilsin.

A) 2:7; B) 1:3; C) 2:3; D) 2:1; E) 2:5.

31. Trapetsiyaning asoslari nisbati 3:5 kabi, o'rta chizig'i esa 32 sm bo'lsa, uning katta asosi uzunligi topilsin.

A) 38; B) 40; C) 42; D) 36; E) 34 sm.

32. Trapetsiyaning diagonallari uning o'rta chizig'ini uchta teng kesmaga bo'ladi. Trapetsiyaning katta va kichik asoslari nisbati topilsin.

A) 2:1; B) 3:1; C) 4:3; D) 3:2; E) 1:5.

33. Teng yonli trapetsiyaning asoslari 22 sm va 42 sm, yon tomoni 26 sm bo'lsa, uning diagonali uzunligi topilsin.

A) 42; B) 50; C) 40; D) 30; E) 36 sm.

34. Teng yonli trapetsiyaning asoslari 5 sm va 11 sm, perimetri 28 sm bo'lsa, uning yuzi hisoblansin.

A) 24; B) 19; C) $26\sqrt{3}$; D) $24\sqrt{3}$; E) $18\sqrt{3}$ sm².

35. Agar teng yonli trapetsiyaning katta asosi 22 sm, yon tomoni 8,5 sm va diagonali 19,5 sm bo'lsa, uning yuzi hisoblansin.

A) 124; B) 136; C) 118; D) 120; E) 135 sm².

36. Teng yonli trapetsiyaning asosi $AD=36$ sm, $\angle BAC=\angle CAD$, perimetri 90 sm bo'lsa, uning yon tomoni uzunligi topilsin.

A) 18; B) 16; C) 14; D) 12; E) 10 sm.

37. AB kesmaning uchlari a to'g'ri chiziqdan 9 sm va 13 sm masofada joylashgan. Kesmaning o'rtasidagi C nuqtadan a to'g'ri chiziqqacha bo'lgan masofa topilsin.

A) 12; B) 11; C) 10; D) 13; E) 17 sm.

38. Agar parallelogrammning balandliklari 7 sm va 5 sm bo'lib, katta balandligi uzunligi 10 sm bo'lgan tomonga o'tkazilgan bo'lsa, uning perimetri topilsin.

A) 36; B) 46; C) 48; D) 42; E) 45 sm.

39. $ABCD$ parallelogrammning diagonali $BD=14$ sm va AD tomonga perpendikulyar. Agar $\angle A=45°$ bo'lsa, parallelogrammning yuzi hisoblansin.

A) 184; B) 180; C) 200; D) 192; E) 196 sm².

40. Rombning yuzi 24 sm^2, diagonallari nisbati 3:4 kabi bo'lsa, uning perimetri topilsin.

A) 19; B) 16; C) 22; D) 20; E) 18 sm.

41. Teng yonli trapetsiyaning diagonali 6 dm va yon tomonlari bilan 38° va 112° li burchaklar tashkil qiladi. Uning yuzi hisoblansin.

A) 10; B) 12; C) 16; D) 8; E) 9 dm^2.

42. O'xshash to'rtburchaklar perimetrlarining nisbati 2:3 kabi, yuzlarining yig'indisi 260 dm^2 bo'lsa, to'rtburchaklardan har birining yuzi hisoblansin.

A) 82 va 108; B) 76 va 100; C) 80 va 180;
D) 64 va 196; E) 84 va 180 dm^2.

43. Ikkita o'xshash to'rtburchakning yuzlari 50 sm^2 va 32 sm^2, perimetrlarining yig'indisi 117 sm. Har bir to'rtburchakning perimetri topilsin.

A) 48 va 64; B) 52 va 65; C) 50 va 60;
D) 54 va 62; E) 58 va 60 sm.

44. Rombning perimetri 4 dm, diagonallarining nisbati 3:4 kabi bo'lsa, uning yuzi hisoblansin.

A) 9,6; B) 8,8; C) 10,4; D) 10,2; E) 9,8 dm^2.

45. Rombning perimetri $2p$ sm, diagonallarining yig'indisi m sm bo'lsa, uning yuzi hisoblansin.

A) $\sqrt{m^2+n^2}$; B) $\frac{1}{2}mp$; C) $\frac{m^2+p^2}{4}$;
D) $\frac{m^2-p^2}{4}$; E) $\frac{3mp}{4}$ sm^2.

46. Trapetsiyaning asoslari a va b ga teng. Asoslarga parallel bo'lgan va trapetsiyani ikkita tengdosh trapetsiyaga bo'luvchi kesmaning uzunligi topilsin.

A) $\sqrt{a^2+2b^2}$; B) $\sqrt{\frac{a^2+b^2}{2}}$; C) $\sqrt{a^2+b^2}$; D) $\sqrt{ab}$;
E) $\sqrt{2a^2+b^2}$.

47. Parallelogrammning o'tkir burchagi 60°, diagonallari kvadratlarining nisbati 19:7 kabi bo'lsa, parallelogrammning tomonlari nisbati topilsin.

A) 3:2; B) 2:1; C) 3:4; D) 5:2; E) 5:3.

48. Trapetsiyaning asoslari a va b ga teng, yon tomonlari katta asos bilan α va β o'tkir burchaklar tashkil qilsa, uning yuzi hisoblansin.

A) $\frac{(a^2+b^2)\cos 2\alpha}{\sin\alpha\cdot\cos\beta}$; B) $\frac{(a^2-b^2)\sin 2\alpha}{\sin(\alpha+\beta)}$; C) $\frac{(a^2-b^2)\sin\alpha\sin\beta}{2\sin(\alpha+\beta)}$;

D) $\frac{(a^2-b^2)\cos 2\alpha}{\sin(\alpha-\beta)}$; E) $\frac{(a^2-b^2)\sin\alpha\cos\beta}{\sin(\alpha-\beta)}$.

49. Trapetsiyaning yuzi 36 sm², balandligi 10 sm, asoslaridan biri ikkinchisidan 3 marta katta bo'lsa, uning katta asosi uzunligi topilsin.

A) 4,2; B) 6,0; C) 5,8; D) 6,2; E) 5,4 sm.

5-§. ICHKI VA TASHQI CHIZILGAN KO'PBURCHAKLAR

5.1. Asosiy tushunchalar va xossalar

Hamma uchlari aylanada yotgan ko'pburchak aylanaga *ichki chizilgan ko'pburchak* deyiladi.

Hamma tomonlari aylanaga uringan ko'pburchak aylanaga *tashqi chizilgan ko'pburchak* deyiladi.

Har qanday uchburchakka ichki aylana chizish mumkin va agar uchburchakning tomonlari a, b, c, yuzi S_Δ bo'lsa, bu aylananing radiusi

$$r = \frac{2S_\Delta}{a+b+c} \qquad (5.1)$$

formula orqali topiladi.

Har qanday uchburchakka tashqi aylana chizish mumkin bo'lib, uning radiusi

$$R = \frac{a \cdot b \cdot c}{4 \cdot S_{\Delta}} \qquad (5.2)$$

ga teng.

Quyidagi xossalarni eslatib o'tamiz:

1. Uchburchakka ichki chizilgan aylananing markazi uchburchak burchaklari bissektrisalarining kesishish nuqtasidir.

Uchburchakka tashqi chizilgan aylananing markazi uchburchak tomonlarining o'rtalaridan shu tomonlarga o'tkazilgan perpendikulyarlarning kesishish nuqtasidir.

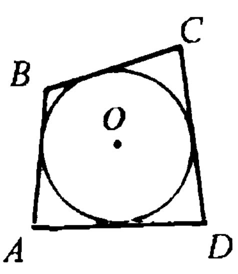
2. Agar to'rtburchakning qarama-qarshi tomonlari yig'indisi o'zaro teng bo'lsa, ya'ni $AB+CD=BC+AD$ bo'lsa (5.1-chizma), to'rtburchakka ichki aylana chizish mumkin.

3. Agar to'rtburchak qarama-qarshi burchaklarining yig'indisi 180° ga teng, ya'ni $\angle A+\angle C=\angle B+\angle D=180°$ bo'lsa, to'rtburchakka tashqi aylana chizish mumkin.

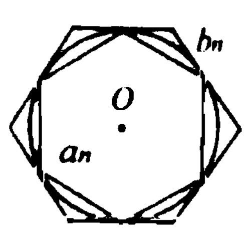
4. To'g'ri burchakli uchburchakka tashqi chizilgan aylananing markazi gipotenuzaning o'rtasidir.

Muntazam ko'pburchak aylanaga ichki chizilgan bo'lsin. Uning a_n tomoni

$$a_n = 2R\sin\frac{180°}{n} \qquad (5.3)$$



5.1-chizma.



5.2-chizma.

formula bilan hisoblanadi. Bu formuladan foydalanib, aylanaga ichki chizilgan muntazam uchburchak, muntazam to'rtburchak va muntazam oltiburchakning tomonlarini aniqlash mumkin:

$$n=3, \quad a_3=2R\sin 60°=R\sqrt{3}; \tag{5.4}$$

$$n=4, \quad a_4=2R\sin 45°=R\sqrt{3}; \tag{5.5}$$

$$n=6, \quad a_6=2R\sin 30°=R. \tag{5.6}$$

Endi aylanaga tashqi chizilgan muntazam ko'pburchakni qaraymiz. Uning tomonini b_n deb belgilasak, u ichki chizilgan aylananing r radiusi orqali

$$b_n = 2r\,\mathrm{tg}\,\frac{180°}{n} \tag{5.7}$$

formuladan aniqlanadi. Xususiy hollar:

$$n=3, \quad b_3=2r\,\mathrm{tg}60°=2r\sqrt{3}; \tag{5.8}$$

$$n=4, \quad b_4=2r\,\mathrm{tg}45°=2r; \tag{5.9}$$

$$n=6, \quad b_6=2r\,\mathrm{tg}30°=\frac{2r\sqrt{3}}{3}. \tag{5.10}$$

5.2. Mavzuga doir masalalar

1. Bir tomoni 10, unga yopishgan burchaklari 105° va 45° bo'lgan uchburchakka tashqi chizilgan aylananing radiusi topilsin.

A) 8; B) 10; C) 12; D) 14; E) 7.

2. Doiraning yuzi 36 π ga teng. Unga tashqi chizilgan kvadratning yuzi hisoblansin.

A) 100; B) 169; C) 128; D) 130; E) 144.

3. Tomoni 81 bo'lgan teng tomonli uchburchakka tashqi chizilgan aylananing radiusi topilsin.

A) $27\sqrt{3}$; B) $16\sqrt{2}$; C) $16\sqrt{3}$; D) 18; E) $9\sqrt{5}$.

4. Doiraning radiusi 40% ortsa, uning yuzi qanday o'zgaradi?

A) 20 % ortadi; B) 96 % ortadi; C) 80 % ortadi; D) 38 % ortadi; E) o'zgarmaydi.

5. Teng yonli uchburchakning yon tomoni 3, uchidagi burchagi 120° ga teng. Shu uchburchakka tashqi chizilgan aylananing radiusi topilsin.

A) 1; B) 5; C) 2; D) 3; E) 4.

6. Rombning kichik diagonali va tomoni $18\sqrt{3}$ ga teng. Rombga ichki chizilgan aylananing radiusi topilsin.

A) 13,5; B) 14; C) 16; D) 9; E) 20.

7. Doiraga ichki chizilgan to'g'ri to'rtburchakning tomonlari 12 va 16 ga teng. Doiraning yuzi hisoblansin.

A) 80π; B) 100π; C) 96π; D) 24π; E) 64π.

8. Radiusi $\sqrt{3}$ ga teng bo'lgan doiraga o'tkir burchagi 60° bo'lgan teng yonli trapetsiya tashqi chizilgan. Trapetsiya o'rta chizig'ining uzunligi topilsin.

A) 5; B) 3; C) 4; D) 2; E) 1.

9. To'g'ri burchakli uchburchakning kateti $\sqrt{3}$ va unga yopishgan burchagi 30° ga teng. Uchburchakka tashqi chizilgan aylananing radiusi topilsin.

A) 12; B) 10; C) $\sqrt{7}$; D) 4; E) 1.

10. To'g'ri burchakli uchburchakka aylana ichki chizilgan va urinish nuqtasida gipotenuza uzunliklari 5 sm va 12 sm bo'lgan ikkita kesmaga ajratilgan. Uchburchakning katta kateti uzunligi topilsin.

A) 10; B) 17; C) 20; D) 15; E) 14 sm.

11. Aylananing uzunligi 6 π ga teng bo'lsa, unga ichki chizilgan kvadratning yuzi hisoblansin.

A) 18; B) 14; C) 13; D) 12; E) 22.

12. Rombning tomoni $a=4$, unga ichki chizilgan aylana radiusi $r=1{,}5$ bo'lsa, rombning yuzi hisoblansin.

A) 10; B) 12; C) 13; D) 11; E) 8.

13. To'g'ri to'rtburchakka tashqi chizilgan aylananing radiusi 15 sm bo'lsa, uning qo'shni tomonlari o'rtalari orasidagi masofa topilsin.

A) 12; B) $\frac{24}{7}$; C) 15; D) 14; E) 13 sm.

14. Teng yonli uchburchakning asosiga tushirilgan balandlik 25 sm, ichki chizilgan aylananing radiusi 8 sm bo'lsa, uchburchak asosining uzunligi topilsin.

A) 12; B) $\frac{24}{7}$; C) $\frac{32}{5}$; D) $\frac{40}{3}$; E) $\frac{80}{3}$ sm.

15. Uchburchakning tomonlari 13, 14 va 15 sm. Unga tashqi va ichki chizilgan doiralar yuzlarining nisbati topilsin.

A) $\left(\frac{65}{32}\right)^2$; B) 12; C) $\frac{64}{15}^2$; D) 4; E) $\left(\frac{2\pi}{5}\right)^2$.

16. Muntazam o'nikkiburchakning ichki burchagi α bo'lsa, $\sin\alpha$ topilsin.

A) 0,8; B) 0,6; C) 0,75; D) 0,5; E) 0,25.

17. Muntazam uchburchakka aylana ichki chizilgan, shu aylanaga esa muntazam oltiburchak ichki chizilgan. Uchburchak va oltiburchak yuzlarining nisbati topilsin.

A) 1; B) 3; C) 4; D) 2; E) 7.

18. Uchburchakning ikkita burchagi $\frac{\pi}{3}$ va $\frac{\pi}{4}$ ekanligi ma'lum va u radiusi 2 sm bo'lgan aylanaga ichki chizilgan. Uchburchakning yuzi hisoblansin.

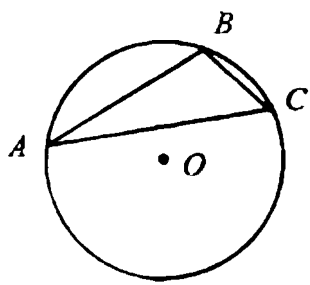
A) $3+\sqrt{3}$; B) $2+\sqrt{3}$; C) 5; D) 4,5; E) 4 sm².

5.3. Mavzuga oid masalalarning yechimlari

1. Berilgan. $(O,\ R)$ — aylana, ΔABC, $A,B,C\in(O,\ R)$, $BC=10$, $\angle B=105°$, $\angle C=45°$.

R topilsin (5.3.1- chizma).

Yechilishi. Uchburchak ichki burchaklarining yig'indisi 180° ga teng. Ikkita B va C burchaklari berilgan. Uchinchi burchakni topamiz: $A=180°-150°=30°$. Sinuslar teoremasining (2-§, 8-xossa) natijasidan foydalanamiz: $\frac{BC}{\sin\angle A}=2R$ yoki $\frac{10}{\sin 30°}=2R$. U holda $R=\frac{10}{2\sin 30°}=\frac{10}{2\cdot 1/2}=10$.



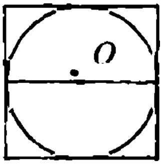
5.3.1- chizma.

Javobi: B).

2. Berilgan. $(O,\ R)$ — doira, $S_D=36\pi$, $ABCD$ — kvadrat.

S_{ABCD} topilsin (5.3.2-chizma).

Yechilishi. Aylananing O markazidan kvadratning AB va CD tomonlariga radiuslar o'tkazamiz. Lekin urinish nuqtasidan o'tkazilgan radius urinmaga perpendikulyar, ON va MO parallel AB va CD to'g'ri chiziqlarga perpen-



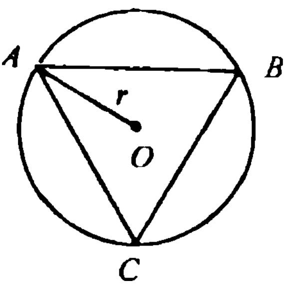
5.3.2-chizma.

dikulyar bo'lgani uchun bitta MN to'g'ri chiziqda yotadi va $MN=BC=a$ bo'ladi. Demak, MN aylananing diametridan iborat, ya'ni $MN=2R$ bo'lib, kvadratning yuzi $S=(2R)^2=4R^2$ bo'ladi.

Berilgan shartga ko'ra, $S_D=\pi R^2$, $36\pi=\pi R^2$ va $R^2=36$. U holda kvadratning yuzi $S=4\cdot 36=144$.

3. Berilgan. (O, R) — aylana, ΔABC, $AB=AC=$ $=BC$, $A,B,C\in(O, R)$, $AB=81$.

R topilsin (5.3.3-chizma).



5.3.3-chizma.

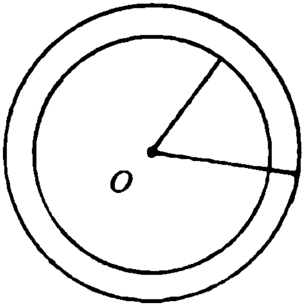
Yechilishi. Uchburchak teng tomonli bo'lgani uchun uning ichki burchaklari o'zaro teng va $\angle A=\angle B-\angle C=60^\circ$. Sinuslar teoremasi (2-§, 8-xossa) ning natijasidan foydalanamiz:

$$\frac{AB}{\sin 60^\circ} = 2r,\ r = \frac{81}{\sqrt{3}} = 27\sqrt{3}\,.$$

Javobi: A).

4. Berilgan. (O, r) — aylana, $R=1{,}4r$.

$S_1 - S$ topilsin (5.3.4-chizma).



5.3.4-chizma.

Yechilishi. Aylananing radiusi 40% ortganligi va 1% sonning 0,01 qismiga teng bo'lgani uchun yangi aylananing radiusi $R=r+0{,}4r=1{,}4r$ bo'ladi.

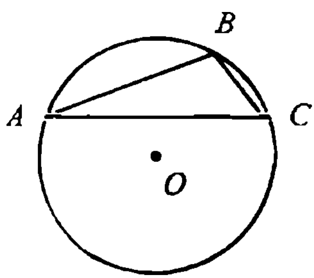
Doiralarning yuzlari, mos ravishda, $S=\pi r^2$ va $S_1=\pi R^2=$ $=\pi(1,4r)^2=1,96\pi r^2$ bo'ladi. U holda o'zgarish miqdorini topsak, $S_1-S=(1,96-1)\pi r=0,96\pi$. Demak, doiraning yuzi 96% ortadi.

Javobi: B).

5. Berilgan. (O, R) — aylana, ΔABC, $A, B, C\in(O, R)$, $\angle ABC=120°$, $AB=BC=3$.

R topilsin (5.3.5-chizma).

Yechilishi. ΔABC teng yonli bo'lgani uchun asosdagi burchaklar o'zaro teng va $\angle A=\angle C=\frac{180°-120°}{2}=30°$. Sinuslar teoremasining (2-§, 8-xossa) natijasiga ko'ra $\frac{AB}{\sin 30°}=2R$ va $R=\frac{3}{2\cdot 1/2}=3$.



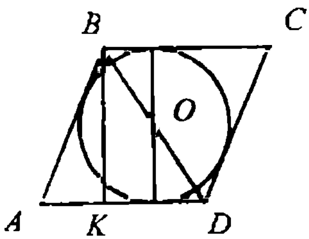
5.3.5-chizma.

Javobi: D).

6. Berilgan. $ABCD$ — romb, $AB=BD=18\sqrt{3}$, (O, R) — ichki chizilgan aylana.

R topilsin (5.3.6-chizma).

Yechilishi. Rombning tomoni va diagonali teng bo'lgani uchun ΔABD teng tomonlidir. Demak, $\angle BAD=60°$. Ichki chizilgan aylananing O markazidan rombning BC va AD tomonlariga perpendikulyarlar o'tkazamiz. Ular parallel to'g'ri chiziqlarga perpendikulyar

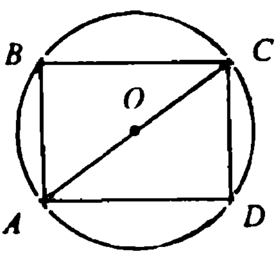


5.3.6-chizma.

bo'lgani uchun bir to'g'ri chiziqda yotadi va romb uchun balandlik bo'ladi: $2R=H$. Rombning balandligini B nuqtadan tushiramiz va ΔABK ni hosil qilamiz. U holda $H=AB\cdot\sin 60^\circ=18\sqrt{3}\cdot\frac{\sqrt{3}}{2}=9\cdot 3=27$ va $R=\frac{1}{2}H=13{,}5$.

7. Berilgan. $ABCD$ — to'g'ri to'rtburchak, (O, R) — tashqi chizilgan aylana, $AD=16$, $CD=12$.

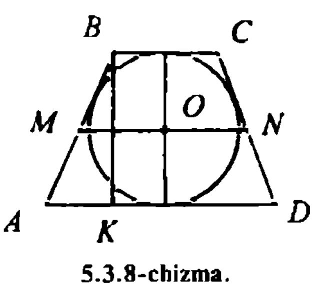
S_D topilsin (5.3.7-chizma).



5.3.7-chizma.

Yechilishi. Doiraning yuzi (3-§) $S=\pi R^2$ formula bilan hisoblanadi. $ABCD$ to'rtburchakda AC diagonalni o'tkazamiz. Uning o'rtasidagi O nuqta to'rtburchakning simmetriya markazi bo'lgani uchun $AC=2R$. To'g'ri burchakli ΔACD dan Pifagor teoremasi (2-§, 7-xossa) ga asosan, $AC^2=AD^2+CD^2=16^2+12^2=256+$ $+144=400$, $AC=20$ va $R=\frac{1}{2}AC=\frac{1}{2}\cdot 20=10$. Doiraning yuzi $S=100\pi$.

8. Berilgan. (O, R) — aylana, $R=\sqrt{3}$, $\angle A=60^\circ$, $ABCD$ — trapetsiya, $ABCD$ — tashqi chizilgan.



5.3.8-chizma.

MN o'rta chiziq topilsin (5.3.8-chizma).

Yechilishi. Aylananing O markazidan BC va AD ga perpendikulyarlar o'tkazamiz. U holda aylananing diametri trapetsiyaning balandligiga teng bo'ladi: $H=2R=2\sqrt{3}$. To'g'ri burchakli ΔABK ($BK\perp AD$) dan topamiz:

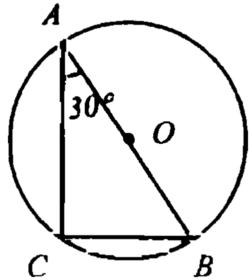
$$\frac{BK}{AB}=\sin 60^\circ,\ AB=\frac{BK}{\sin 60^\circ}=\frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}}=4.$$

Aylanaga trapetsiya tashqi chizilgani uchun qarama-qarshi tomonlar yig'indisi $AB+CD=BC+AD$, $2AB=$ $=BC+AD$, $MN=\frac{AD+BC}{2}=AB=4$.

Javobi: C).

9. Berilgan. (O, R) — aylana, $AC=\sqrt{3}$, $A, B, C\in(O, R)$, ΔABC, $\angle C=90^\circ$, $\angle A=30^\circ$.

R topilsin (5.3.9-chizma).



5.3.9-chizma.

Yechilishi. To'g'ri burchakli uchburchakka aylana tashqi chizilgan bo'lsa, aylananing diametri gipotenuzaning uzunligiga teng. Shuning uchun berilgan katet va burchak orqali gipotenuzani topamiz:

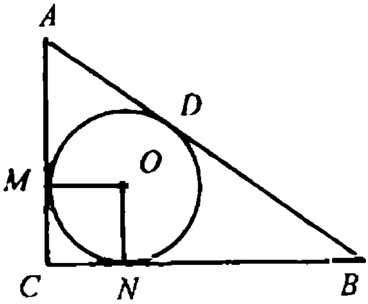
$$\cos 30^\circ=\frac{AC}{AB} \text{ va } AB=\frac{AC}{\cos 30^\circ}=\frac{\sqrt{3}}{\frac{\sqrt{3}}{2}}=2.$$

Tashqi chizilgan aylananing radiusi esa $R=\frac{1}{2}\cdot AB=$ $=1$ sm.

Javobi: E).

10. Berilgan. ΔABC, $\angle C=90^\circ$, (O, r) — ichki chiz. aylana, D — urinish nuqtasi, $AD=5$ sm, $BD=12$ sm.

BC topilsin (5.3.10-chizma).



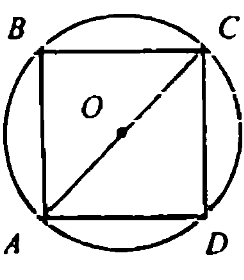
5.3.10-chizma.

Yechilishi. Ichki chizilgan aylananing radiusi r bo'lsin: $ON=OM=OD=r$. 3-§, 4-xossaga muvofiq aylanadan tashqaridagi nuqtadan aylanaga urinmalar o'tkazilgan bo'lsa, shu nuqtadan urinish nuqtasigacha bo'lgan kesmalarning uzunliklari o'zaro tengdir: $MA=AD=5$ sm, $BD=BN=12$ sm, $CM=CN=r$. ΔABC da gipotenuza $AB=AD+BD=5+12=17$ sm, $AC=5+r$, $BC=12+r$. Pifagor teoremasi (2- §, 7-xossa) ga ko'ra, $AC^2+BC^2=AB^2$, $(r+5)^2+(r+12)^2=17^2$, $r^2+10r+25+r^2+24r+144=289$, $2r^2+34r-120=0$, $r^2+17r-60=0$, $D=17^2+4\cdot60=529=23^2$, $r_1=\frac{-17-23}{2}$, $r_2=\frac{-17+23}{2}$, $r_2=3$, $r_1=-20$. Bu yerda, katta katet $BC=12+3=15$ sm bo'ladi.

Javobi: D).

11. Berilgan. (O, R) — aylana, $ABCD$ — ichki chizilgan kvadrat, $L_0=6\pi$.

S_{ABCD} hisoblansin (5.3.11- chizma).



5.3.11- chizma.

Yechilishi. Kvadratning tomonı $AB=a$ bo'lsa, uning yuzi $S=a^2$. Agar radiusi R bo'lgan aylanaga kvadrat ichki chizilgan bo'lsa, (5.5) formulaga ko'ra uning tomoni $a=R\sqrt{2}$ ga teng.

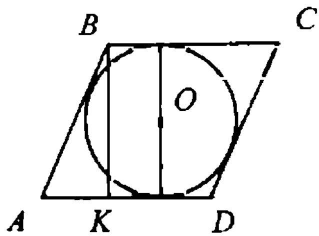
Demak, aylananing radiusini topish kerak. Aylana uzunligi ma'lum bo'lgani uchun $2\pi R=6\pi$ tenglama-dan radiusni topamiz: $R=3$. Demak, kvadratning tomoni $a=3\sqrt{2}$ va uning yuzi $S=(3\sqrt{2})^2=18$ bo'ladi.

Javobi: A).

12. B e r i l g a n. $ABCD$ — romb, $AB=a=4$, (O, r) — ichki chizilgan aylana, $r=1{,}5$.

S_{ABCD} h i s o b l a n s i n (5.3.12-chizma).

Y e c h i l i s h i. Rombning A uchidan $AK=h$ balandlik ($AK\perp DC$) o'tkazamiz. Rombning yuzi (4.6) formula bo'yicha hisoblanadi: $S=ah$. O nuqtadan AB tomondagi urinish nuqtasiga radius o'tkazamiz. U holda $h=2r=$ $=2\cdot 1{,}5=3$ va rombning yuzi $S=4\cdot 3=12$ bo'ladi.



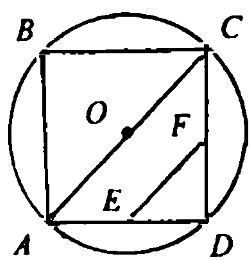
5.3.12-chizma.

Javobi: B).

13. B e r i l g a n. (O, R) — aylana, $R=15$, $ABCD$ — to'g'ri to'rtburchak, $AE=ED$, $CF=FD$.

EF t o p i l s i n (5.3.13-chizma).

Y e c h i l i s h i. A, B, C, D nuqtalar aylanaga tegishli. Diagonallarning kesishish nuqtasi O to'g'ri to'rtburchakning simmetriya markazi bo'lganligidan, AC diagonal O nuqtadan o'tadi va $AC=2R=2\cdot 15=30$. ΔACD da E va F nuqtalar tomonlarning o'rtalari bo'lgani uchun, EF kesma



5.3.13-chizma.

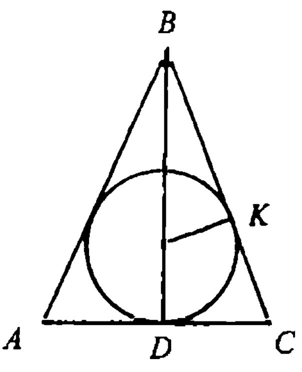
ACD uchburchakning o'rta chizig'i bo'ladi va asosining yarmiga teng:

$$EF = \frac{1}{2} \cdot AC = \frac{1}{2} \cdot 30 = 15.$$

Javobi: C).

14. Berilgan. ΔABC, $AB=BC$, $BD \perp AC$, $BD=25$ sm, (O, r) — ichki chizilgan aylana, $r=8$ sm.

AC topilsin (5.3.14-chizma).



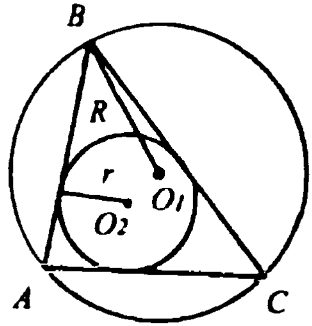
5.3.14-chizma.

Yechilishi. O nuqta uchburchakka ichki chizilgan aylananing markazi bo'lsin. Shartga ko'ra, $BD=25$ sm, $OD=8$ cm. U holda $BO=25-8=17$ sm. O nuqtadan urinish nuqtasi K ga $OK=8$ sm radiusni o'tkazamiz. $OK \perp BC$ va ΔOBK to'g'ri burchakli. Pifagor teoremasiga (2-§, 7-xossa) asosan $KC=DC=a$, $BC=15+a$. ΔOBK dan: $BK=\sqrt{BO^2+OK^2}=15$ sm, $KC=KD=a$, u holda $BC=15+a$. To'g'ri burchakli ΔBDC uchun Pifagor teoremasidan (2-§, 7-xossa): $BC^2=BD^2+DC^2$, $(15+a)^2=25^2+a^2$, $225+30 \cdot a+a^2=625+a^2$, $30 \cdot a=400$, $a=\frac{40}{3}$ ekanligini olamiz. Bu yerdan, $AC=2a=\frac{80}{3}$ sm.

15. Berilgan. ΔABC, $AC=13$ sm, $BC=14$ sm, $AB=15$ sm, (O_1, R) — tashqi chizilgan aylana, (O_2, r) — ichki chizilgan aylana, S_1, S_2 — doiralar yuzlari.

S1:S2 topilsin (5.3.15-chizma).

Y e c h i l i s h i. Uchburchakka ichki chizilgan aylananing radiusi r, tashqi chizilgan aylana radiusi R bo'lsin. Uchburchakning yuzi quyidagi: $S=\frac{abc}{4R}$ yoki $S=pr$, $p=\frac{a+b+c}{2}$ formuladan topiladi. Uchburchak yuzini Geron formulasidan (2-§, (2.14) formula) topamiz:



5.3.15-chizma.

$$S=\sqrt{p(p-a)(p-b)(p-c)}.$$

Bizda $a=13$, $b=14$, $c=15$, $p=\frac{13+14+15}{2}=21$ bo'ladi.

U holda $S=\sqrt{21(21-13)(21-14)(21-15)}$,

$S=\sqrt{21\cdot 8\cdot 7\cdot 6}=7\cdot 3\cdot 4=84$ sm².

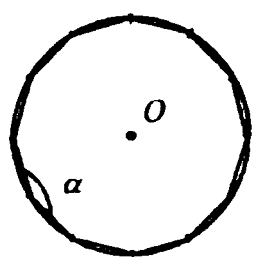
$R=\frac{abc}{4S}=\frac{13\cdot 14\cdot 15}{4\cdot 84}=\frac{65}{8}$ sm, $r=\frac{S}{p}=\frac{84}{21}=4$ cm. Tashqi va ichki chizilgan doiralar yuzlarining nisbatini topamiz:

$$\frac{S_1}{S_2}=\frac{\pi R^2}{\pi r^2}=\frac{65^2}{8^2\cdot 4^2}=\left(\frac{65}{32}\right)^2.$$

Javobi: A).

16. B e r i l g a n. AB — muntazam o'nikkiburchak, $\alpha=\angle ABC$.

sinα t o p i l s i n (5.3.16-chizma).



5.3.16-chizma.

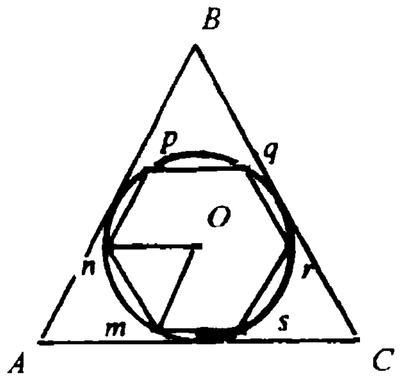
Y e c h i l i s h i. Muntazam n-burchak ichki burchaklarning yig'indisi (1-§, (1.2) formula) $180°(n-2)$ ga teng. Bizda $n=12$ va ichki burchaklar bir-biriga teng, shu sababli,

$\alpha = \frac{180^\circ(12-2)}{12} = 150^\circ$. Unda $\sin 150^\circ = \sin(180^\circ - 30^\circ) =$
$= \sin 30^\circ = \frac{1}{2}$.

Javobi: D).

17. Berilgan. ΔABC, (O, r) — ichki chizilgan aylana, $(mnpqrs)$ — muntazam oltiburchak.

$S_\Delta : S_6$ topilsin (5.3.17-chizma).



5.3.17-chizma.

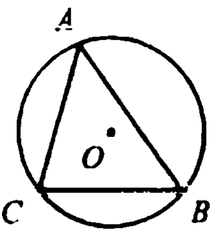
Yechilishi. Uchburchakning tomonini $AB=a$ deb belgilaymiz. Muntazam uchburchakning har bir burchagi 60° ga teng va uning yuzi (2-§, (2.10) formulaga muvofiq), $S_\Delta =$ $= \frac{1}{2} a^2 \sin 60^\circ = \frac{2a^3\sqrt{3}}{4 \cdot 3a} =$ $= \frac{2a\sqrt{3}}{12} = \frac{a\sqrt{3}}{6}$. Aylanaga muntazam oltiburchak ichki chizilgan bo'lsa, uning a_6 tomoni aylananing radiusiga teng: $a_6 = r = \frac{a\sqrt{3}}{6}$. U holda ΔmnO teng tomonli va uning yuzi $S_1 =$ $= \frac{r^2\sqrt{3}}{4} = \frac{3a^2}{36} \cdot \frac{\sqrt{3}}{4}$ va $S_6 = 6 \cdot S_1 = \frac{6 \cdot 3a^2\sqrt{3}}{36 \cdot 4} = \frac{a^2\sqrt{3}}{8}$ bo'ladi. Izlanayotgan nisbatni hisoblaymiz:

$$\frac{S_\Delta}{S_6} = \frac{a^2\sqrt{3}}{4} : \frac{a^2\sqrt{3}}{8} = \frac{a^2\sqrt{3} \cdot 8}{4 \cdot a^2\sqrt{3}} = 2$$

Javobi: D).

18. Berilgan. ΔABC, $\angle A=\frac{\pi}{3}$, $\angle B=\frac{\pi}{4}$, (O, R) — tashqi chizilgan aylana, $R=2$ sm.

S_Δ hisoblansin (5.3.18-chizma).



5.3.18-chizma.

Yechilishi. Agar $AC=b$, $AB=c$ bo'lsa, uchburchakning yuzi (2-§, (2.10) formula) dan $S_\Delta=\frac{1}{2}bc\sin\frac{\pi}{3}=$ $=\frac{bc\sqrt{3}}{4}$. Sinuslar teoremasi (2-§, 8-xossa) ga asosan, $\frac{b}{\sin 45^\circ}=\frac{c}{\sin(180^\circ-45^\circ-60^\circ)}$, $\frac{b}{\sin 45^\circ}=\frac{c}{\sin 75^\circ}=2R$ munosabat o'rinli. U holda $b=2R\sin 45^\circ=2R\cdot\frac{\sqrt{2}}{2}=$ $R\sqrt{2}$, $c=2\sin 75^\circ=\frac{\sqrt{2}(\sqrt{3}+1)}{4}$. Uchburchakning yuzi (2-§, (2.10) formuladan) $S=\frac{1}{2}bc\sin 60^\circ=\frac{\sqrt{2}(\sqrt{3}+1)}{4}\cdot\frac{4}{2}\sqrt{6}=$ $=3+\sqrt{3}$ bo'ladi.

Javobi: A).

5.4. Mustaqil yechish uchun masalalar

1. To'g'ri burchakli uchburchakka ichki chizilgan aylananing radiusi 4 sm, gipotenuzasi esa 26 sm. Uchburchakning perimetri topilsin.

A) 64; B) 54; C) 60; D) 45; E) 70 sm.

2. Teng yonli uchburchakning uchidagi burchagi 120° va yon tomoni 2 sm bo'lsa, unga tashqi chizilgan aylananing diametri topilsin.

A) 2; B) 3; C) 2,5; D) 4; E) 3,5 sm.

3. Aylanaga teng yonli trapetsiya tashqi chizilgan. Trapetsiyaning bitta burchagi 30°, o'rta chizig'i 2 m. Aylananing radiusini toping.

A) 2; B) 2,5; C) 1,5; D) 1; E) 0,5 sm.

4. Agar to'g'ri burchakli uchburchakka ichki chizilgan aylananing radiusi 3 m, uning kichik kateti esa 10 m bo'lsa, uchburchakka tashqi chizilgan aylananing radiusini toping.

A) 7; B) 7,5; C) 8; D) 7,25; E) 8,25 m.

5. To'g'ri burchakli uchburchakda gipotenuzaga o'tkazilgan balandlik 4 sm. Gipotenuzada ajratilgan kesmalar uzunliklarining ayirmasi 6 sm ga teng. Uchburchakning kichik kateti uzunligi topilsin.

A) 6; B) $2\sqrt{5}$; C) $3\sqrt{5}$; D) 4; E) $4\sqrt{5}$ sm.

6. Muntazam o'nburchakning ichki burchagi topilsin.

A) 110°; B) 122°; C) 150°; D) 144°; E) 136°.

7. Muntazam o'nbeshburchak uchun markaziy burchak topilsin.

A) 20°; B) 22°; C) 18°; D) 36°; E) 24°.

8. Qanday ko'pburchakning ichki burchagi uning markaziy burchagidan 10 marta katta?

A) 16; B) 22; C) 24; D) 18; E) 15 sm.

9. Teng yonli uchburchakning balandligi 20 sm, asos va yon tomonining nisbati 4:3 kabi. Uchburchakka ichki chizilgan aylananing radiusi topilsin.

A) 5; B) 6; C) 9; D) 8; E) 7 sm.

10. Teng yonli uchburchakning yon tomoni 2 dm, asosi 2,4 dm. Uchburchakka aylana ichki chizilgan va uchburchak asosiga parallel qilib, unga urinma o'tkazilgan. Ushbu urinma yordamida ajratilgan uchburchakning perimetri topilsin.

A) 2,4; B) 1,8; C) 1,6; D) 2,1; E) 3,2 dm.

11. Teng yonli uchburchakning asosi 12 sm, asosiga tushirilgan balandlik 8 sm bo'lsa, uchburchakka tashqi chizilgan aylananing diametri topilsin.

A) 12,5; B) 12; C)13; D) 13,5; E) 11,5 sm.

12. Qavariq to'rtburchakka aylana ichki chizilgan. Agar to'rtburchakning tomoni 12 sm, unga yopishgan burchaklari esa 60° va 120° bo'lsa, uning radiusi topilsin.

A) $3\sqrt{2}$; B) $4\sqrt{2}$; C) $4\sqrt{3}$; D) $2\sqrt{3}$; E) $3\sqrt{3}$ sm.

13. Trapetsiya aylanaga ichki chizilgan. Trapetsiyaning uchlari aylanani 2:3:2:5 nisbatda bo'ladi. Agar aylananing radiusi 6 sm bo'lsa, trapetsiyaning yuzi hisoblansin.

A) $4(2\sqrt{3}+3)$; B) $4\sqrt{3}+7$; C) $4(\sqrt{3}+5)$;
D) $6(\sqrt{3}+2)$; E) $8+5\sqrt{3}$ sm².

14. Radiusi 14 dm ga teng bo'lgan aylanaga muntazam uchburchak ichki chizilgan va uchburchakka yana aylana ichki chizilgan. Hosil bo'lgan halqaning yuzi hisoblansin.

A) 18π; B) 10π; C) 12π; D) 16π; E) 15π dm².

15. Aylananing radiusi $\sqrt{3}$ sm ga teng. Uning atrofida teng yonli trapetsiya tashqi chizilgan va uning o'tkir burchagi 60° ga teng. Trapetsiyaning yuzi hisoblansin.

A) $6\sqrt{5}$; B) $8\sqrt{5}$; C) $8\sqrt{2}$; D) $6\sqrt{3}$; E) $8\sqrt{3}$ sm².

16. Muntazam to'rtburchak aylanaga ichki chizilgan bo'lib, uning tomoni $4\sqrt{2}$ sm. Aylanaga tashqi chizilgan muntazam uchburchakning yuzi hisoblansin.

A) $56\sqrt{2}$; B) 48; C) $45\sqrt{3}$; D) $48\sqrt{3}$; E) 45 sm².

17. Aylanaga tashqi chizilgan muntazam oltiburchakning tomoni $4\sqrt{2}$. Aylanaga ichki chizilgan kvadratning yuzi hisoblansin.

A) 64; B) 48; C) 52; D) 50; E) 60 sm^2.

18. Rombning tomoni $10\sqrt{3}$ sm ga, o'tkir burchagi 60° ga teng. Rombga ichki chizilgan doiraning yuzi hisoblansin.

A) $56{,}25\pi$; B) $48{,}75\pi$; C) $52{,}25\pi$;
D) $50{,}6\pi$; E) $48{,}5\pi$ sm^2.

19. O'tmas burchagi 120° bo'lgan rombga ichki chizilgan doiraning yuzi 36π sm^2. Rombning yuzi hisoblansin.

A) $92\sqrt{2}$; B) $96\sqrt{2}$; C) $88\sqrt{3}$; D) $96\sqrt{3}$; E) $92\sqrt{3}$ sm^2.

20. Aylanaga muntazam oltiburchak ichki chizilgan va uning kichik diagonali 12 sm. Oltiburchakning yuzi hisoblansin.

A) $64\sqrt{2}$; B) 64; C) $72\sqrt{2}$; D) $64\sqrt{3}$; E) $72\sqrt{3}$ sm^2.

21. To'g'ri burchakli uchburchak aylanaga tashqi chizilgan va gipotenuza aylanaga urinish nuqtasida 3 sm va 2 sm bo'lgan kesmalarga ajraladi. Uchburchakka ichki chizilgan doiraning yuzi hisoblansin.

A) 2π; B) $1{,}44\pi$; C) π; D) 4π; E) $2{,}25\pi$ sm^2.

22. Teng yonli $\triangle ABC$ ning asosi $AC = 12$ sm, balandligi $DB = 8$ sm. Uchburchakka ichki chizilgan aylana markazidan uning B uchigacha bo'lgan masofa topilsin.

A) 8; B) 5; C) 6; D) 9; E) 4 sm.

23. Kvadratning tomoni 8 sm bo'lsa, unga tashqi chizilgan aylananing uzunligi topilsin.

A) $8\sqrt{2}\pi$; B) $6\sqrt{2}\pi$; C) 8π; D) $10\sqrt{2}\pi$; E) $7\sqrt{5}\pi$ sm.

24. Radiusi $R=6$ bo'lgan aylanaga uchburchak ichki chizilgan va uning ichki burchaklari kattaliklari 3:4:5 kabi nisbatda. Eng katta yoyning uzunligi topilsin.

A) 3π; B) 8π; C) 4π; D) 6π; E) 5π.

25. Aylanaga tashqi chizilgan teng yonli trapetsiyaning o'rta chizig'i 5 sm. Trapetsiyaning perimetri topilsin.

A) 21; B) 24; C) 22; D) 20; E) 18 sm.

26. Muntazam to'rtburchakning tomoni 8 sm bo'lsa, unga tashqi chizilgan doiraning yuzi hisoblansin.

A) 28π; B) 24π; C) 32π; D) 30π; E) 16π sm².

27. Rombning tomoni 15 sm, o'tkir burchagi 30° bo'lsa, rombga ichki chizilgan aylananing uzunligi topilsin.

A) 6π; B) $7{,}5\pi$; C) $8{,}5\pi$; D) 7π; E) 12π sm.

28. Trapetsiyaning tomonlari a, a, a va $2a$ bo'lsa, unga tashqi chizilgan aylananing uzunligi topilsin.

A) $7a\pi$; B) $3a\pi$; C) $6a\pi$; D) $2a\pi$; E) $4a\pi$.

29. Teng yonli trapetsiyaga aylana ichki chizilgan. Trapetsiyaning yon tomoni 4 sm, katta asosidagi o'tkir burchagi 30° bo'lsa, trapetsiyaning yuzi hisoblansin.

A) 8; B) 6; C) 9; D) 12; E) 13 sm².

30. Aylanaga ichki chizilgan uchburchakning uchlari aylanani uzunliklari 2:3:4 kabi nisbatda bo'lgan uchta qismga ajratadi. Uchburchakning ichki burchaklari kattaliklari topilsin.

A) 30°, 60°, 90°; B) 40°, 50°, 90°; C) 50°, 60°, 70°; D) 60°, 65°, 40°; E) 40°, 60°, 80°.

31. To'g'ri burchakli uchburchak to'g'ri burchagi uchidan o'tkazilgan mediana va bissektrisa orasidagi burchak 10°. Uchburchakning burchaklari kattaliklari topilsin.

A) 20° va 70°; B) 45° va 50°; C) 35° va 55°;
D) 30° va 60°; E) 40° va 50°.

32. Uchburchakning bitta uchidan o'tkazilgan balandlik, bissektrisa va mediana shu burchakni to'rtta teng burchakka bo'ladi. Uchburchakning burchaklari kattaliklari topilsin.

A) 90°, 35°, 55°; B) 90°, 22°, 30°; C) 90°, 40°, 50°;
D) 90°, 36°, 54°; E) 90°, 30°, 60°.

33. To'g'ri burchakli uchburchakning to'g'ri burchagi uchidan va ichki hamda tashqi chizilgan aylanalar markazlaridan nurlar o'tkazilgan bo'lib, ular orasidagi burchak 7°. Uchburchakning o'tkir burchaklari kattaliklari topilsin.

A) 30° va 60°; B) 40° va 50°; C) 45° va 45°;
D) 38° va 52°; E) 36° va 54°.

34. Muntazam oltiburchakning tomoni $a=12$ sm bo'lsa, oltiburchakka ichki chizilgan doiraning yuzi hisoblansin.

A) 112π; B) 96π; C) 98π; D) 120π; E) 108π sm^2.

35. Radiusi 5 sm bo'lgan aylanaga muntazam o'nikkiburchak ichki chizilgan. Markaziy AOB burchakka mos kelgan yoyning uzunligi topilsin.

A) $\frac{5\pi}{6}$; B) $\frac{2\pi}{3}$; C) $\frac{3\pi}{4}$; D) $\frac{5\pi}{8}$; E) $\frac{6\pi}{7}$ sm.

36. Teng yonli uchburchakning uchidagi burchagi 2α, unga tashqi chizilgan aylananing radiusi p ga teng. Uchburchakning yuzi hisoblansin.

A) $p^2\sin2\alpha$; B) $4p^2\cos^3\alpha\cdot\sin\alpha$; C) $p^2\sin3\alpha$; D) $p^2\cos2\alpha$;
E) $(1+p^2)\sin2\alpha$.

37. r radiusli aylanaga teng yonli trapetsiya tashqi chizilgan. Trapetsiyaning o'tkir burchagi α bo'lsa, uning yuzi hisoblansin.

A) $2r^2\sin\alpha$; B) $r^2\mathrm{tg}\alpha$; C) $\frac{4r^2}{\sin\alpha}$; D) $\frac{2r^2}{\cos\alpha}$; E) $\frac{3r^2}{\mathrm{tg}\alpha}$.

38. Aylanaga muntazam uchburchak ichki chizilgan va uning yuzi S. So'ngra uchburchakka aylana ichki chizilgan. Hosil bo'lgan kamarning yuzi hisoblansin.

A) $1{,}5S$; B) $0{,}5S$; C) $0{,}75S$; D) $3\pi\frac{\sqrt{2}}{5}$; E) $S\pi\sqrt{3}/3$.

39. Aylanaga muntazam oltiburchaklar ichki va tashqi chizilgan. Ikkinchi oltiburchakning yuzi birinchisining yuzidan $8\sqrt{3}$ sm^2 ortiq. Aylananing radiusi topilsin.

A) $5\sqrt{3}$; B) 6; C) $4\sqrt{3}$; D) 4; E) 8 sm.

40. Uchburchakning tomonlari $AB=29$ sm, $AC=25$ sm, $BC=6$ sm. Uchburchakka tashqi chizilgan aylananing radiusi topilsin.

A) $\frac{145}{8}$; B) 16; C) $\frac{140}{9}$; D) $\frac{139}{7}$; E) $\frac{152}{7}$ sm.

41. Uchburchakning ikki tomoni $a=11$, $b=24$ sm va ular orasidagi burchagi 120°. Uchburchakning tashqi chizilgan aylananing radiusi topilsin.

A) $3\sqrt{2}$; B) $\frac{31}{\sqrt{3}}$; C) $\frac{40}{\sqrt{3}}$; D) $\frac{53}{\sqrt{3}}$; E) $4\sqrt{3}$ sm.

42. Teng yonli ABC uchburchakda asos $AC=4$ sm, $\angle ADC=135°$ va AD uchburchakning bissektrisasi bo'lsa, uning uzunligi topilsin.

A) $\sqrt{13}$; B) $2\sqrt{7}$; C) $2\sqrt{5}$; D) $2\sqrt{3}$; E) $2\sqrt{2}$ sm.

43. Muntazam oltiburchakning yuzi $12\sqrt{3}$ sm^2. Oltiburchakka ichki chizilgan doiraning yuzi hisoblansin.

A) 7π; B) 4π; C) 6π; D) 5π; E) 8π sm^2.

44. Tomoni $3\sqrt{2}$ sm bo'lgan muntazam to'rtburchakka aylana tashqi chizilgan. Shu aylanaga tashqi chizilgan muntazam uchburchakning tomoni uzunligi topilsin.

A) $4\sqrt{2}$; B) $5\sqrt{3}$; C) $6\sqrt{2}$; D) $6\sqrt{3}$; E) $7\sqrt{2}$ sm.

45. Aylanaga muntazam oltiburchak tashqi chizilgan va uning tomoni $2\sqrt{3}$ sm bo'lsa, aylanaga ichki chizilgan kvadratning yuzi hisoblansin.

A) 15; B) 16; C) 20; D) 19; E) 18 sm^2.

46. $ABCD$ to'rtburchak doiraga ichki chizilgan va $CB=4$, $CD=5$, $\angle A=60°$ bo'lsa, BD diagonalning uzunligi topilsin.

A) $\sqrt{61}$; B) $\sqrt{59}$; C) $\sqrt{57}$; D) $\sqrt{71}$; E) $\sqrt{65}$.

47. Radiusi $\sqrt{3}$ bo'lgan doiraga o'tkir burchagi 60° bo'lgan teng yonli trapetsiya tashqi chizilgan. Trapetsiyaning o'rta chizig'i uzunligi topilsin.

A) 3; B) 4; C) 5; D) 8; E) 6.

6-§. VEKTORLAR

6.1. Asosiy tushunchalar

Boshlanish nuqtasi A va oxirgi nuqtasi B tanlangan AB kesma yo'nalgan kesma deyiladi, bunda A nuqta yo'nalgan kesmaning *boshi*, B nuqta *oxiri* deyiladi.

Geometriyada yo'nalgan kesma *vektor* deb ataladi. Vektorlar quyidagicha belgilanadi: AB, AC, BC yoki a, $\bar{b}$, c.

AB vektorning *uzunligi* deb AB kesmaning uzunligini aytiladi va u $|AB|$ yoki $|a|$ kabi belgilanadi.

Boshi va oxiri ustma-ust tushgan vektor 0 *nol vektor* deb ataladi. Uning uzunligi nolga teng.

Agar ikki a va b vektorning uzunliklari teng, yo'nalishlari esa qarama-qarshi bo'lsa, ular *qarama-qarshi vektorlar* deyiladi va quyidagicha yoziladi: $a = -b$.

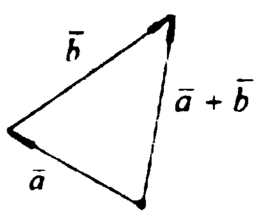
Vektorlarni qo'shishning ikkita qoidasi mavjud.

1. **UCHBURCHAK QOIDASI**. Ikkita a va b vektorni qo'shish uchun birinchi a vektorning oxiriga ikkinchi vektorning boshini joylashtiramiz. Birinchi vektorning boshini ikkinchi vektorning oxiri bilan tutashtiruvchi vektor a va b vektorlarning yig'indisi deyiladi va u $a + b$ kabi belgilanadi (6.1-chizma).

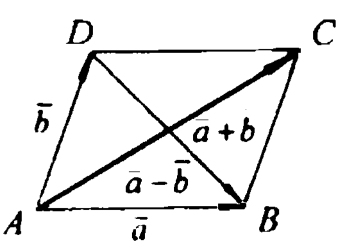
2. **PARALLELOGRAMM QOIDASI.** a va b vektorlarning boshini umumiy A nuqtaga keltiramiz. Har bir vektorning uchidan ikkinchi vektorga parallel to'g'ri chiziq o'tkazib, $ABCD$ parallelogramm yasaymiz. a va b vektorlarning umumiy A nuqtasidan chiqqan diagonaldagi AC vektor a va b vektorlarning yig'indisi bo'ladi (6.2-chizma).

Bu qoidalar yordamida vektorlarning ayirmasini aniqlaymiz.

3. Agar $\bar{b}$ va p vektorlarning yig'indisi $\bar{a}$ vektorga teng bo'lsa, $\bar{p}$ vektor a va b vektorlarning *ayirmasi* deb ataladi va $a - b = \bar{p}$ kabi belgilanadi (6.2-chizma).



6.1-chizma.



6.2-chizma.

Demak, a va b vektorlar yordamida yasalgan parallelogrammning A uchidan chiqqan diagonalida $a+b$ vektor, bu vektorlarning oxirida yotgan uchlaridan o'tuvchi diagonalida esa $a-b$ vektor yotadi (6.2-chizma).

Endi ba'zi ta'riflar va xossalarni keltiramiz.

4. Agar vektorlar bitta to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotsa, ular *kollineur* deyiladi.

5. a *vektor* va *k sonning ko'paytmasi* deb, shunday b vektorga aytiladiki, uning uchun $a \parallel b$ va $|a|=|k||a|$ shartlar bajariladi.

6. Berilgan a va b vektorlarni umumiy O nuqtaga keltiramiz. Ushbu vektorlar orasidagi burchak φ bo'lsa, $pr_{\bar{b}} = |\bar{a}| \cdot \cos\varphi$ con berilgan a vektorning $\bar{b}$ vektor yo'nalishidagi *proyeksiyasidir*. φ burchak 0 dan π gacha o'zgarganligi uchun proyeksiya musbat, manfiy va nolga teng qiymatlar qabul qilishi mumkin.

7. Teng vektorlarning proyeksiyalari ham o'zaro teng.

8. Vektorlar yig'indisining proyeksiyasi qo'shiluvchi vektorlarning proyeksiyalari yig'indisiga teng, ya'ni $c=a+b$ bo'lsa, $c_{pr}=a_{p}+b_{pr}$.

9. Ikkita a va $\bar{b}$ vektorning skalyar ko'paytmasi shu vektorlarning uzunliklari va ular orasidagi burchak kosinusining ko'paytmasiga teng:

$$(a \cdot b)=|a| \cdot |b| \cdot \cos\varphi. \qquad (6.2)$$

Vektorning proyeksiyasi tushunchasidan foydalanib, ikkita a va b vektorning skalyar ko'paytmasini quyidagicha ham yozish mumkin:

$$(a \cdot b)=|a| \cdot pr_{\bar{a}}\,\bar{b}=|b| \cdot pr_{\bar{b}}\,\bar{a}. \qquad (6.3)$$

Ckalyar ko'paytma quyidagi xossalarga ega:

9.1. $(a \cdot b)=(b \cdot a)$ — o'rin almashtirish xossasi.

9.2. $p(a \cdot b)=((p \cdot a) \cdot b)=(a \cdot (p \cdot b))$ — guruhlash xossasi, r — haqiqiy son.

9.3. $a \cdot (b+c)=(a \cdot b)+(a \cdot c)$ — taqsimot xossasi.

9.4. Agar $\bar{a}$ va b lardan biri nol vektor yo $\bar{a}$ va b vektorlar o'zaro perpendikulyar bo'lsa, $(a \cdot b)=0$ bo'ladi.

9.5. (6.2) da $a = b$ bo'lsa, $(a \cdot a)=|a|^2$ bo'ladi. Natijada vektorning uzunligi

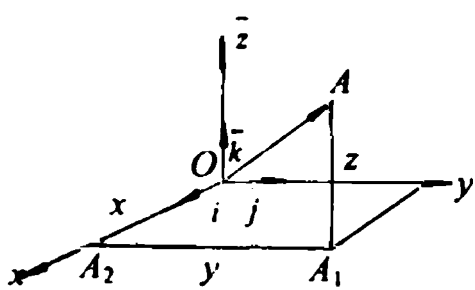
$$|a|=\sqrt{\bar{a}^2}=\sqrt{(a \cdot a)}. \qquad (6.4)$$

9.6. Ikki vektor orasidagi burchak

$$\cos\varphi = \frac{(a \cdot b)}{|a| \cdot |b|} \qquad (6.5)$$

formuladan topiladi.

10. Vektorning fazodagi koordinatalari. Fazoda to'g'ri burchakli $Oxyz$ koordinatalar sistemasi tanlangan bo'lsa, $\bar{a} = OA$ vektorni O nuqtaga keltiramiz va koordinatalar o'qlariga proyeksiyalaymiz. Proyeksiyalarning algebraik qiymatlari a vektorning koordinatalaridir. Koordinatalar o'qlarining har birida birlik vektorlarni tanlaymiz: (Ox o'qda i, Oy o'qda j, Oz o'qda k vektorlar). Berilgan A nuqtani Oxy tekislikka proyeksiyalaymiz. (Proyeksiya A_1 nuqta bo'lsa, uni Ox o'qqa proyeksiyalaymiz va uning proyeksiyasi A_2 bo'lsin). So'ngra OA_2A_1A yopiq siniq chiziqni hosil qilamiz. U holda



6.3-chizma.

$$\overline{OA} = \overline{OA_2} + \overline{A_2A_1} + \overline{A_1A} \tag{6.6}$$

$OA_2 \parallel i$, $A_2A_1 \parallel j$, $A_1A \parallel k$ bo'lgani uchun, $OA_2 = xi$, $A_1A_2 = yj$, $A_1A = zk$ deb yozish mumkin, natijada vektorning i, j, k vektorlar orqali yoyilmasi deb ataladigan

$$OA = xi + yj + zk \tag{6.7}$$

tenglikni hosil qilamiz. Bu yoyilmadagi i, j, k vektorlar oldidagi koeffitsientlar berilgan a vektorning koordinatalaridir: $a(x, y, z)$. $a = AB$ vektorning uchlari A (x_1, y_1, z_1) va B (x_2, y_2, z_2) nuqtalarda bo'lsa, A va B nuqtalarni O nuqta bilan tutashtiramiz va (6.7) formuladan: $OA = x_1 i + y_1 j + z_1 k$, $OB = x_2 i + y_2 j + z_2 k$ hamda

$$AB = OB - OA = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k \tag{6.8}$$

munosabatlarni hosil qilamiz.

Demak, ikki nuqta bilan aniqlangan vektorning koordinatalari shu nuqtalar mos koordinatalarining ayirmasiga teng:

$$x = x_2 - x_1,\ y = y_2 - y_1,\ z = z_2 - z_1. \tag{6.9}$$

$a(x_1, y_1, z_1)$, $\bar{b}(x_2, y_2, z_2)$ vektorlar va p son berilgan bo'lsin. U holda

$$\begin{aligned}
&a + b = (x_1 + x_2)i + (y_1 + y_2)j + (z_1 + z_2)k,\\
&(a + b)(x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2);\\
&a - b = (x_1 - x_2)i + (y_1 - y_2)j + (z_1 - z_2)k,\\
&(a - b)(x_1 - x_2,\ y_1 - y_2,\ z_1 - z_2);\\
&pB = px_2\bar{i} + py_2\bar{j} + pz_2k,\ pB(px_2,\ py_2,\ pz_2).
\end{aligned} \tag{6.10}$$

$a(x_1, y_1, z_1)$ va $b(x_2, y_2, z_2)$ vektorlarning skalyar ko'paytmasi ularning koordinatalari orqoli bunday ifodalanadi:

$$(a\ b)=x_1x_2+y_1y_2+z_1z_2. \qquad (6.11)$$

a va b vektorlar kollinear bo'lsa, $a = kb$ va $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = k$ bo'ladi. Vektorning uzunligini hisoblash formulasi

$$|\bar{a}|= \sqrt{x_1^2 + y_1^2 + z_1^2} \qquad (6.12)$$

yoki

$$|a|=|AB|= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

ko'rinishni oladi.

$a(x_1, y_1, z_1)$ va $b(x_2, y_2, z_2)$ vektorlarning perpendikulyarlik sharti quyidagicha yoziladi:

$$x_1x_2+y_1y_2+z_1z_2=0 \qquad (6.14)$$

Ikki vektor orasidagi burchak formulasi quyidagichadir:

$$\cos\varphi = \frac{x_1x_2+y_1y_2+z_1z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\cdot\sqrt{x_1^2+y_1^2+z_1^2}}. \qquad (6.15)$$

6.2. Mavzu bo'yicha masalalar

1. $|\bar{a}|=2$, $|b|=3$ vektorlar orasidagi burchak $(\bar{a}\,\hat{,}\,\bar{b}) = \frac{\pi}{3}$ bo'lsa, a) $(\bar{\bar{a}} \cdot b)$; b) $(a - b)^2$; d) $(2a - b)\cdot(a + 3b)$ skalyar ko'paytmalar hisoblansin.

a): A) 2; B) 3; C) 7; D) 5; E) 6.

b): A) 8; B) 9; C) 10; D) 7; E) 6.

d): A) 4; B) −3; C) −4; D) 9; E) 3.

2. e_1 va e_2 o'zaro perpendikulyar ($e_1 \perp e_2$) birlik vektorlar bo'lsa, $a = 2e_1 - e_2$ vektorning uzunligi hisoblansin.

A) $\sqrt{5}$; B) 2; C) $\sqrt{6}$; D) $\sqrt{7}$; E) $\sqrt{11}$.

3. $\bar{a}$ va b vektorlarning uzunliklari $|a|=3$, $|b|=1$ va$(\hat{a, b}) = \frac{\pi}{3}$ bo‘lsa, $\bar{p} = a - b$ va $q = \bar{a} + b$ vektorlar orasidagi burchak$(\hat{\bar{p}, q})$ topilsin.

A) $\arccos\frac{5}{\sqrt{41}}$; B) $\arcsin\frac{8}{\sqrt{91}}$; C) $\frac{\pi}{4}$; D) arctg2; E) $\arccos\frac{8}{\sqrt{91}}$.

4. $\bar{a} = 3i + 4j$ va $\bar{b} = 5i + 12j$ vektorlar orasidagi burchakning kosinusi hisoblansin.

A) $-\frac{11}{35}$; B) $\frac{13}{65}$; C) $-\frac{17}{65}$; D) $-\frac{33}{65}$; E) $-\frac{23}{65}$.

5. Uchlari $A(4\sqrt{3}, -1)$, $B(0, 3)$, $C(8\sqrt{3}, 3)$ nuqtalarda bo‘lgan ΔABC ning B burchagi topilsin.

A) 45°; B) 30°; C) 75°; D) 60°; E) 15°.

6. $a(2, 1, 0)$ va $b\ (0, -1, 1)$ vektorlar yordamida yasalgan parallelogrammning diagonallari orasidagi burchakning kosinusi hisoblansin.

A) $\frac{1}{\sqrt{6}}$; B) $\frac{1}{3}$; C) $\frac{1}{\sqrt{5}}$; D) $\frac{1}{\sqrt{7}}$; E) $\frac{1}{4}$.

7. $\bar{a}(-2, 1)$, $\bar{b}(0, 2)$, $c(3, -1)$ vektorlar berilgan bo‘lsa, $2a - b + \bar{c}$ vektorning koordinatalari topilsin.

A) $(-1; -1)$; B) $(0,1)$; C) $(2, -1)$;
D) $(-1,3)$; E) $(-2, -2)$.

8. $\bar{a}(-1, 3)$ va $b(4, -7)$ vektorlar berilgan bo‘lsa, $a + b$ vektorning uzunligi hisoblansin.

A) 6; B) 3,5; C) 4; D) 7; E) 5.

9. Fazoda $a(2, 4, 0)$, $b(0, -3, 1)$, $c(5, -1, 2)$ vektorlar berilgan. $2\bar{a} - b + c$ vektorning koordinatalari topilsin.

A) (4, 7, −2); B) (9, 16, −1); C) (4, 16, −2);
D) (6, −4; 12); E) (8, 12, 3).

10. $\bar{a}(-3, p, 9)$ va $\bar{b}(2, -8, r)$ vektorlar o‘zaro parallel bo‘lsa, p va r topilsin.

A) $p=6$, $r=-12$; B) $p=-6$, $r=-12$; C) $p=4$, $r=-6$;
D) $p=12$, $r=-6$; E) $p=-12$, $r=6$.

6.3. Mavzu bo‘yicha masalalarning yechimlari

1. Yechilishi. a) (6.2) formuladan foydalanamiz: $(\bar{a}\cdot\bar{b})=2\cdot3\cdot\cos\frac{\pi}{3}=2\cdot3\cdot\frac{1}{2}=3.$

Javobi: B).

b) Uchinchi va beshinchi xossalardan foydalanamiz:

$(\bar{a}-\bar{b})^2=\bar{a}^2-2(\bar{a}\cdot\bar{b})+\bar{b}^2=2^2-2\cdot3+3^2=7.$

d) $(2\bar{a}-\bar{b})\cdot(\bar{a}+3\bar{b})=2\bar{a}^2+6(\bar{a}\cdot\bar{b})-(\bar{b}\cdot\bar{a})-3\bar{b}^2=$
$=2\cdot4+5\cdot3\cdot2-3\cdot9=11.$

Javobi: C).

2. Yechilishi. Skalyar ko‘paytmaning 9.3 va 9.5-xossalaridan foydalanamiz:

$$|\bar{a}|=\sqrt{(2\bar{e}_1-\bar{e}_2)^2}=\sqrt{4\bar{e}_1^2-4(\bar{e}_1\cdot\bar{e}_2)+\bar{e}_1^2}=$$
$$=\sqrt{4-4\cdot0+1}=\sqrt{5}.$$

Javobi: A).

3. Yechilishi. Dastlab p va q vektorlarning skalyar ko‘paytmasi va uzunliklarini hisoblaymiz:

$$(\bar{p}\cdot\bar{q})=(\bar{a}-\bar{b})\cdot(\bar{a}+\bar{b})=\bar{a}^2-\bar{b}^2=3^2-1^2=8.$$

$$|\bar{p}|=|\bar{a}-\bar{b}|=\sqrt{(\bar{a}-\bar{b})^2}=\sqrt{\bar{a}^2-2\bar{a}\bar{b}+\bar{b}^2}=$$

$$=\sqrt{|a|^2-2|a|\cdot|b|\cos 60^\circ+|b|^2}=\sqrt{3^2-2\cdot 3\cdot 1\cdot \frac{1}{2}+1}=\sqrt{7}.$$

$$|q|=|a+b|=\sqrt{(a-b)^2}=\sqrt{3^2+2\cdot 3\cdot \frac{1}{2}+1^2}=\sqrt{13}.$$

Endi (6.5) formuladan foydalansak,

$$\cos\varphi=\frac{(p\,q)}{|p|\cdot|q|}=\frac{8}{\sqrt{7}\sqrt{13}}=\frac{8}{\sqrt{91}}\text{ va }\varphi=\arccos\frac{8}{\sqrt{91}}.$$

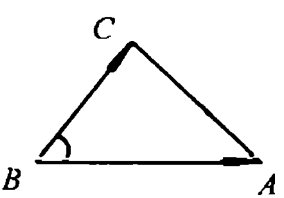
Javobi: E).

4. Y e c h i l i s h i. $\bar{a}$ va b vektorlarning yoyilmalaridan ularning koordinatalarini yozib olamiz: $a(3,-4)$ va $b(5,\ 12)$. So'ngra (6.5) formuladan foydalansak,

$$\cos\varphi=\frac{x_1x_2+y_1y_2}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}},$$

$$\cos\varphi=\frac{3\cdot 5-4\cdot 12}{\sqrt{3^2+16}\cdot\sqrt{5^2+12^2}}=\frac{-33}{13\cdot 5}=\frac{-33}{65}.$$

Javobi: D).



6.3.1-chizma.

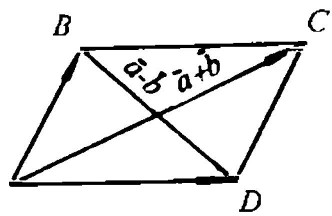
5. Y e c h i l i s h i. $\angle B$ berilishiga ko'ra, $\overline{BA}$ va BC vektorlar yordamida hosil qilingan (6.3.1-chizma). Shu sababli ularning koordinatalarini topamiz: $BA(4\sqrt{3}-0,-1-3)=(4\sqrt{3},-4)$, $BC(8\sqrt{3}-0,3-3)=(8\sqrt{3},0)$. Natijada,

$$\cos\angle B=\frac{(\overline{BA}\ \overline{BC})}{|\overline{BA}|\cdot|\overline{BC}|}=\frac{4\sqrt{3}\cdot 8\sqrt{3}-4\cdot 0}{\sqrt{48+16}\cdot\sqrt{(8\sqrt{3})^2}}=\frac{32\cdot 3}{8\cdot 8\sqrt{3}}=\frac{\sqrt{3}}{2}.$$

va $\cos\angle B = \frac{\sqrt{3}}{2}$, bu yerdan $\angle B=30°$.

Javobi: B).

6. Y e c h i l i s h i. $\bar{a}$ va b vektorlar umumiy bitta nuqtaga keltirilib, parallelogramm yasalganligidan, uning diagonallari ustida $a+\bar{b}$ va $a-b$ vektorlar yotadi (6.3.2-chizma). Ularning koordinatalarini (6.9) formuladan topamiz:



6.3.2-chizma.

$$a-\bar{b}=(2+0,\ 1-1,\ 0+1)=(2,\ 0,\ 1),$$

$$a-\bar{b}=(2-0,\ 1+1,\ 0-1)=(2,\ 2,\ -1).$$

Endi bu vektorlar orasidagi burchakning kosinusini (6.5) formula bo‘yicha hisoblaymiz:

$$\cos\varphi = \frac{(a+\bar{b})\cdot(a-\bar{b})}{|(a+\bar{b})|\cdot|(\bar{a}-\bar{b})|};$$

$$\cos\varphi = \frac{2\cdot 2+0\cdot 2+1\cdot(-1)}{\sqrt{2^2+0^2+1^2}\sqrt{2^2+2^2+(-1)^2}} = \frac{1}{\sqrt{5}}.$$

Javobi: C).

7. Y e c h i l i s h i. Ma’lumki, vektor songa ko‘paytirilganda uning har bir koordinatasi shu songa ko‘paytiriladi:

$$2\bar{a}=(2\cdot(-2),\ 2\cdot 1)=(-4,\ 2).$$

Endi $2a-b+c$ ifodaning koordinatalarini topamiz:

$$2a-b+c=(-4-0+3, 2-2+(-1))=(-1,\ -1).$$

Javobi: A).

8. Yechilishi. Avvalo $a-b$ vektorning koordinatalarini topamiz:

$$a+b=(-1+4,\ 3-7)=(3,\ -4).$$

So'ngra uning uzunligini hisoblaymiz:

$$|a+b|=\sqrt{x^2+y^2}=\sqrt{3^2+(-4)^2}=\sqrt{9+16}=\sqrt{25}=5.$$

Javobi: E).

9. Yechilishi. Dastlab $2\bar{a}$ va $3b$ vektorlarning koordinatalarini topamiz:

$$2a=(2\cdot 2,\ 2\cdot 4,\ 2\cdot 0)=(4,\ 8,\ 0),$$

$$3b=(3\cdot 0,\ 3\cdot(-3),\ 3\cdot 1)=(0,\ -9{,}3).$$

U holda, $2a-3\bar{b}+c=(4-0+5,\ 8+9-1,\ 0-3+2)=$ $=(9,\ 16,\ -1)$.

Javobi: B).

10. Yechilishi. Parallel vektorlarning mos koordinatalari proporsional bo'lganligidan, quyidagi $-\frac{3}{2}=\frac{-p}{-8}=\frac{9}{r}$ formula o'rinlidir. Bu yerdan, $-\frac{3}{2}=\frac{-p}{-8}\Rightarrow p=\frac{24}{2}=12$, $-\frac{3}{2}=\frac{9}{r}\Rightarrow r=-6$.

Javobi: D).

6.4. Mustaqil yechish uchun masalalar

1. $\overline{AB}=c$ va $AC=b$ vektorlar yordamida ΔABC yasalgan. AK medianadagi $\overline{AK}$ vektorni b va c vektorlar orqali ifodalang.

A) $\frac{\bar{b}-\bar{c}}{2}$; B) $\frac{\bar{c}-2\bar{b}}{2}$; C) $\frac{2\bar{b}-\bar{c}}{3}$; D) $\frac{\bar{b}+\bar{c}}{2}$; E) $\bar{c}+2\bar{b}$.

2. ABC uchburchakda $\overline{AB} = c$, $AC = b$, $B\bar{C} = a$ va O uning medianalarining kesishish nuqtasi bo'lsa, $OA + OB + OC$ yig'indini hisoblang.

A) $a + 2b$; B) 0; C) $a - \bar{b}$; D) $2a$; E) $\bar{a} + \bar{b}$.

3. $ABCDEF$ muntazam oltiburchakning markazi O nuqta bo'lsin, $OA, \overline{OB}, OC, OD, OE, OF$ vektorlarning yig'indisi hisoblansin.

A) 0; B) $2AC$; C) AB; D) $\overline{AE}$; E) BC.

4. $ABCD$ parallelogramm $\overline{AB} = a$ va $\overline{AD} = c$ vektorlar yordamida yasalgan va uning diagonallari kesishish nuqtasi O bo'lsin. OD vektor a va c orqali ifodalansin.

A) $\frac{\bar{a}}{2}$; B) $2\bar{a}\, c$; C) $\frac{\bar{c} - \bar{a}}{2}$; D) $\frac{\bar{a} - \bar{c}}{2}$; E) $\bar{a} + 2c$.

5. $ABCD$ parallelogrammda $\overline{AC} = a$ va $BD = c$ bo'lsa, BC vektor $\bar{a}$ va $\bar{c}$ vektorlar orqali ifodalansin.

A) $\frac{\bar{a} - \bar{c}}{2}$; B) $2\bar{a} + c$; C) $a - 2\bar{c}$; D) $\frac{\bar{a} + \bar{c}}{2}$; E) $\frac{\bar{c} - \bar{a}}{2}$.

6. $\bar{a}$ va b vektorlar o'zaro perpendikulyar hamda $|a| = 3$, $|b| = 4$ bo'lsa, $|a + b|$ topilsin.

A) 5; B) 4; C) 6; D) 6; E) 7.

7. ΔOAB $OA = a$ va $OB = \bar{b}$ vektorlar yordamida yasalgan. AOB burchakning bissektrisasidagi OK vektor a va b vektorlar orqali ifodalansin.

A) $\frac{\overline{ba}}{|\bar{a} + \bar{b}|}$; B) $\frac{\overline{ab}}{|\bar{a} + \bar{b}|}$; C) $\frac{\bar{a}}{|\bar{a} + \bar{b}|}$; D) $\frac{\bar{a} + \bar{b}}{|\bar{a} + \bar{b}|}$; E) $\frac{\overline{ab} + \overline{ba}}{|\bar{a} + \bar{b}|}$.

8. Agar: 1) $|\bar{a}|=6$, $|b|=1$, $(a, b) = \frac{\pi}{3}$ bo'lsa; 2) $|\bar{a}|=3$, $|b|=2\sqrt{2}$, $(\bar{a} \hat{,} \bar{b})=135°$ bo'lsa; 3) $|\bar{a}|=2$, $|b|=3$, $a \uparrow\uparrow b$ bo'lsa; 4) $|a|=2$, $|b|=3$, $a \uparrow\downarrow b$ bo'lsa, a va b vektorlarning skalyar ko'paytmasi topilsin.

1) A) 2; B) 1; C) 3; D) 4; E) 2,5.
2) A) 4; B) 3; C) −2; D) −6; E) 1.
3) A) 4; B) 6; C) 2; D) 3; E) 12.
4) A) −6; B) 6; C) −3; D) −12; E) 4.

9. Agar $|a|=2$, $|b|=3$, burchak $(\bar{a} \hat{,} \bar{b}) = \frac{\pi}{3}$ bo'lsa, quyidagilar hisoblansin:

a) $(\bar{a}, b)$; b) a^2; d) b^2; e) $(a-b)^2$; f) $(2a-b)(a-2b)$; g) $(a-b)(a+b)$.

a) A) 4; B) 3; C) 5; D) 2; E) 6.
b) A) 3; B) 5; C) 4; D) 2; E) 1.
d) A) 9; B) 7; C) 6; D) 11; E) 10.
e) A) 6; B) 5; C) 4; D) 7; E) 8.
f) A) 11; B) 12; C) 13; D) 15; E) 14.
g) A) −5; B) −14; C) −2; D) −1; E) −3.

10. $|a|=\frac{1}{2}$, $|\bar{b}|=4$, $(a, b) = \frac{2\pi}{3}$ bo'lsa, $2|\bar{a}|(a\ b) - 3(b\ a) - 5b^2$ ifodaning qiymati hisoblansin.

A) −78; B) −36; C) 42; D) 56; E) −64.

11. Agar: 1) $(a\ b)=40$, $|a|=5$, $|b|=16$; 2) $(a\ b)=-24$, $|a|=6$, $|b|=4$; 3) $(a\ b)=4\sqrt{3}$, $|a|=5$, $|b|=20$ bo'lsa, $(\bar{a} \hat{,} \bar{b})$ topilsin.

1) A) 45°; B) 30°; C) $\arccos\frac{2}{5}$; D) 60°; E) 90°.

2) A) 30°; B) 90°; C) 120°; D) 180°; E) 60°.

3) A) $\arccos\frac{\sqrt{3}}{25}$; B) $\arccos\frac{13}{15}$; C) 60°; D) 45°; E) 90°.

12. $\bar{a} \perp b$, $|a|=5$, $|b|=2$ bo'lsa, quyidagi ifodalar hisoblansin:

1) $(a-b)b$; 2) $(a+b)(a-b)$; 3) $(2a-3b)(a-2b)$:

1) A) 8; B) 6; C) −2; D) 1; E) −4.
2) A) 25; B) 4; C) 21; D) 29; E) 16.
3) A) 65; B) 74; C) 68; D) 72; E) 70.

13. Agar $|a|=2$, $|b|=1$ va $(a, b) = \frac{\pi}{3}$ bo'lsa, $\bar{p} = a - 2\bar{b}$ vektorning uzunligi topilsin.

A) 2; B) 3; C) 1; D) 5; E) 6.

14. $a \perp b$, $|a|=5$, $|b|=2$ berilgan bo'lsa, 1) $(a-b)^2$; 2) $|2a-3b|^2$; 3) $|a-5b|^2$ ifodalar hisoblansin.

1) A) 25; B) 26; C) 27; D) 28; E) 29.
2) A) 136; B) 137; C) 138; D) 139; E) 140.
3) A) 120; B) 125; C) 126; D) 135; E) 121.

15. $AB = 2a + b$ va $AD = a - 3b$ vektorlar yordamida $ABCD$ parallelogramm yasalgan. Agar $a \perp b$, $|a|= =|b|=1$ bo'lsa, AC va BD diagonallarning uzunliklari hisoblansin.

A) $\sqrt{17}$ va $\sqrt{19}$; B) $\sqrt{19}$ va $\sqrt{21}$; C) $\sqrt{11}$ va $\sqrt{15}$;
D) $\sqrt{13}$ va $\sqrt{17}$; E) 4 va 6.

16. Agar a va b birlik vektorlar bo'lib, $(\bar{a} \hat{,} \bar{b})=90°$ bo'lsa, $\bar{p} = 2\bar{a} - \bar{b}$, $q = 2b + a$ vektorlar orasidagi burchak topilsin.

A) 75°; B) 60°; C) 45°; D) 30°; E) 90°.

17. Agar $|p|=2$, $|r|=1$ va ular orasidagi burchak 60° ga teng bo'lsa, $a = p - r$ va $b = 5p - 2r$ vektorlar orasidagi burchak topilsin.

A) $\arccos\frac{2}{5}$; B) $\arccos\frac{5}{\sqrt{7}}$; C) $\arccos\frac{5}{2\sqrt{7}}$; D) $\arccos\frac{1}{4}$;

E) $\arccos\frac{2}{5}$.

18. Agar $\bar{a} = p - r$ va $b = 4p - 5r$ vektorlar o'zaro perpendikulyar bo'lsa, p va r birlik vektorlar orasidagi burchak topilsin.

A) 0°; B) 15°; C) 30°; D) 45°; E) 60°.

19. $ABCD$ parallelogramm $\overline{AB} = 2a - \bar{b}$ va $\overline{AD} = a - 3b$ vektorlar yordamida yasalgan. Agar $\bar{a}$ va b o'zaro perpendikulyar birlik vektorlar bo'lsa, AC va $\overline{BD}$ vektorlar orasidagi burchak topilsin.

A) $\arccos\frac{2}{\sqrt{5}}$; B) $\arccos\frac{1}{\sqrt{5}}$; C) $\arccos\frac{3}{\sqrt{5}}$; D) 75°;
E) 60°.

20. $\bar{p} = 2\bar{a} - 3\bar{b}$ va $r = 4a - kb$ vektorlar o'zaro perpendikulyar, $|a| = |\bar{b}| = 1$, a va b yo'nalishdosh bo'lsa, k ning qiymati topilsin.

A) 3; B) −2; C) −3; D) 4; E) −4.

21. $a(-4, 2, 1)$ va b (3, −1, 1) vektorlar berilgan bo'lsa, $\bar{a} + \bar{b}$ vektorning koordinatalari topilsin.

A) (0, 2, 1); B) (1, −1, −2); C) (−1; 1, 2);
D) (−1, −1, 2); E) (1, −1, 2).

22. $a = 21i - 3j + \bar{k}$, $b = -5i - k$ vektorlar ma'lum bo'lsa, $2a$, $3\bar{b}$ vektorlarning koordinatalari topilsin.

A) (0, 2, 1) va (15, 2, −3); B) (42, −6, 2) va (−15, 0, −3);
C) (24, 6, 8) va (5, 0, 1); D) (32, −4, 1) va (−5, 1, −3);
E) (12, 4, 8) va (0, −10, 3).

23. $\bar{a} = 3i - 4j - 2k$ va $b = -2i + 2j$ vektorlar berilgan. $\bar{a} + b$ vektorning uzunligi hisoblansin.

A) 2; B) 1; C) 4; D) 5; E) 3.

24. $a(2, -4, 5)$ va $b(4, -3, 5)$ vektorlar orasidagi burchakning kosinusi topilsin.

A) $\frac{2}{\sqrt{11}}$; B) $\frac{3}{\sqrt{10}}$; C) $\frac{5}{\sqrt{11}}$; D) $\frac{12}{\sqrt{145}}$; E) $\frac{3}{5}$.

25. ΔABC ning $A(-1, 4, 1)$, $B(3, 4, -2)$, $C(5, 2, -1)$ uchlari berilgan. Uchburchakning B burchagi topilsin.

A) π-arccos $\frac{1}{3}$; B) $\arccos\left(-\frac{2}{3}\right)$;
C) 60°; D) 120°; E) 45°.

26. $\bar{a}(-2, -y, 1)$ va $b(3, -1, 2)$ vektorlar perpendikulyar bo'lsa, y ning qiymati topilsin.

A) 5; B) −3; C) 4; D) 5; E) 1.

27. $\bar{a}(1, -2, 2)$ va $b(2, -2, -1)$ vektorlar berilgan bo'lsa, $2a^2 - 4(ab) + 5b^2$ ifodaning qiymati hisoblansin.

A) 43; B) 44; C) 45; D) 46; E) 47.

28. $a(3, -1, 4)$ vektor berilgan bo'lib, c vektor a vektor bilan kollinear va $(ac) = -52$ shartni qanoatlantirishi ma'lum bo'lsa, c vektorning koordinatalari topilsin.

A) (6, −3, 2); B) (5, −3, 4); C) (8, −6, 4);
D) (−6, 2, −8); E) (4, 1, −4).

29. Uchlari $A(-4, -3, -2)$, $B(2, -2, -3)$, $C(-8, -5, 1)$, $D(4, -3, -1)$ bo'lgan $ABCD$ to'rtburchak berilgan. Uning AC va BD diagonallari orasidagi burchak topilsin.

A) 45°; B) 90°; C) 60°; D) 75°; E) 0°.

30. $a(2, p, 6)$ va $c(1, 1, r)$ vektorlar kollinear bo'lsa, p va r ning qiymatini toping.

A) $p=12$, $r=12$; B) $p=3$, $r=14$; C) $p=-2$, $r=7$;
D) $p=3$, $r=8$; E) $p=-2$, $r=10$.

31. $a(2, 3, -1)$ va $b(0, 1, 4)$, $c(1, 0, -3)$ vektorlar berilgan, $a+2b+3c$ vektorning koordinatalari topilsin.

A) $(-5, 5, -2)$; B) $(5, 5, -2)$; C) $(7, -3, 4)$;
D) $(6, -3, 4)$; E) $(12, 14, -1)$.

32. $a(l, -2, 5)$ va $b(l, m, -3)$ vektorlar kollinear bo'lsa, l va m lar topilsin.

A) $l = \frac{5}{3}$, $m = \frac{6}{5}$; B) $l = -\frac{2}{3}$, $m = \frac{4}{5}$;

C) $l = -\frac{5}{3}$, $m = -\frac{1}{2}$; D) $l = -\frac{5}{3}$, $m = \frac{6}{5}$;

E) $l = 2$, $m = \frac{4}{5}$.

33. $\bar{a}$ va b birlik vektorlar bo'lib, ular orasidagi burchak 30° bo'lsa, $(a+b)^2$ ni hisoblang.

A) $4+\sqrt{3}$; B) $2+\sqrt{3}$; C) $3+\sqrt{2}$; D) $5+\sqrt{2}$; E) 13.

34. $a = 2m + n$ va $b = m - 2n$ vektorlar bo'yicha parallelogramm yasalgan. Agar m va n birlik vektorlar va ular orasidagi burchak 60° bo'lsa, parallelogramm diagonallarining uzunliklari topilsin.

A) $\sqrt{5}$ va $\sqrt{7}$; B) $\sqrt{10}$ va $\sqrt{11}$; C) $\sqrt{7}$ va $\sqrt{13}$;
D) $\sqrt{11}$ va $\sqrt{13}$; E) $\sqrt{7}$ va $\sqrt{11}$.

35. $|a|=2$, $|b|=1$ va a va b vektorlar orasidagi burchak 60° bo'lsa, b va $a-b$ vektorlar orasidagi burchakning kosinusi topilsin.

A) $\frac{\pi}{3}$; B) $\frac{\pi}{6}$; C) $\frac{\pi}{2}$; D) $\frac{\pi}{4}$; E) $\frac{\pi}{8}$.

36. $|\bar{a}|=2$, $|b|=1$ va ular orasidagi burchak $\varphi = \frac{\pi}{3}$ bo'lsa, $\bar{c} = 2a - 3b$ vektorning uzunligi topilsin.

A) $\sqrt{10}$; B) $\sqrt{7}$; C) 7; D) $\sqrt{13}$; E) $\sqrt{15}$.

37. $a(1, -2, 2)$ va $b(-1, 1, 0)$ vektorlarning skalyar ko'paytmasi hisoblansin.

A) -4; B) 4; C) 3; D) -3; E) 0.

38. $a(1, 3, -1,)$ va $b(-1, 1, 2)$ vektorlar berilgan bo'lsa, $2a^2 - 4ab + 5b^2$ hisoblansin.

A) 52; B) 44: C) 42; D) 60; E) -24.

39. $a(4, m, -6,)$ va $b(m, 2, -7)$ vektorlar o'zaro perpendikulyar bo'lsa, m ning qiymati topilsin.

A) -4; B) -5; C) -7; D) 2; E) 4.

40. Agar $|a|=3, |\bar{b}|=1$, $a \perp b$ bo'lsa, $(3a-5b)(2a+7b)$ ko'paytma hisoblansin.

A) -17; B) 12; C) 14; D) 16; E) 19.

41. $a = i + 5j - 6k$ va $b = 2i - j + \lambda k$ vektorlar o'zaro perpendikulyar bo'lsa, λ ning qiymati topilsin.

A) $\frac{1}{2}$; B) $-\frac{1}{2}$; C) $\frac{1}{5}$; D) $\frac{3}{4}$; E) $-\frac{3}{4}$.

42. Agar $(\bar{a} \cdot \bar{b})=3$ bo'lsa, $a(1, 1, -2)$ vektorga parallel bo'lgan b vektor topilsin.

A) $\left(-\frac{1}{2}, 1, -1\right)$; B) $\left(\frac{1}{2}, -1, \frac{1}{2}\right)$; C) $(1, -1, 2)$;

D) $(3, -2, 1)$; E) $\left(\frac{1}{2}, \frac{1}{2}, -1\right)$.

43. $\bar{a}(2, \cos 10°, \sin 10°)$ va $b\left(\frac{\sqrt{2}}{2}, \sin 10°, \cos 10°\right)$ vektorlar orasidagi burchakning kosinusi hisoblansin.

A) $\frac{\sqrt{15}}{13}$; B) $\frac{3\sqrt{2}}{14}$; C) $\frac{2\sqrt{2}}{\sqrt{15}}$; D) $\frac{\sqrt{21}}{7}$; E) $\frac{13}{15}$.

44. $\bar{a}(1, 1, -1)$ va $b(2, 0, 0)$ vektorlar berilgan bo'lsa, $2a+3b$ vektorning uzunligi hisoblansin.

A) $4\sqrt{3}$; B) $4\sqrt{2}$; C) $5\sqrt{3}$; D) $6\sqrt{2}$; E) 12.

45. $a(-2, 2, 4k)$ vektorning uzunligi $b(3, 3k, 0)$ vektorning uzunligidan 2 marta kichik bo'lsa, k ning qiymati topilsin.

A) -1; B) 3; C) 2; D) $\frac{2}{3}$; E) yechim yo'q.

46. $\bar{a}(3, 1, -2)$ va $b(-2, 3, 4)$ vektorlar orasidagi burchak topilsin.

A) $\pi-\arccos\frac{3}{\sqrt{29}}$; B) $\pi-\arccos\frac{11}{\sqrt{406}}$; C) $\pi-\arccos\frac{11}{12}$;

D) 75°; E) 45°.

47. Agar $\bar{b}(-2, 3, 4)$, $(\bar{a}\cdot\bar{b})=29$ va $a\parallel b$ bo'lsa, a vektorning uzunligi hisoblansin.

A) $\sqrt{23}$; B) $\sqrt{29}$; C) $\sqrt{25}$; D) $\sqrt{27}$; E) $\sqrt{22}$.

48. $a=2i+mj-3k$ va $b=i-2j+k$ vektorlar perpendikulyar ekanligi ma'lum bo'lsa, m ning qiymati topilsin.

A) $\frac{3}{4}$; B) $\frac{1}{4}$; C) $-\frac{1}{3}$; D) $\frac{2}{5}$; E) $-\frac{1}{2}$.

49. Uchlari $A(1, -1, 1)$, $B(1, 3, 1)$, $D(4, -1, 1)$ nuqtalarda yotgan ΔABD berilgan bo'lsin. Uchburchakning AB va AD tomonlari orasidagi burchak topilsin.

A) 180°; B) 30°; C) 60°; D) 75°; E) 90°.

50. Agar $\bar{a}(-4, 2, 4)$ va $\bar{b}(\sqrt{2}, -\sqrt{2}, 0)$ berilgan bo'lsa, $2\bar{a}$ va $0{,}5b$ vektorlar orasidagi burchak topilsin.

A) $\frac{\pi}{4}$; B) $\frac{\pi}{2}$; C) $-\frac{3\pi}{4}$; D) π; E) $-\frac{3\pi}{8}$.

7-§. ARALASH MASALALAR

1. Uchburchak bir tomonining uzunligi 10 sm, bu tomonga yopishgan burchaklari esa 60° va 30°. Uchburchakning yuzi hisoblansin.

A) $12{,}5\sqrt{3}$; B) $16\sqrt{3}$; C) $15\sqrt{3}$; D) $18\sqrt{3}$;
E) $14\sqrt{3}$ sm^2.

2. Radiusi 4 sm bo'lgan aylanaga yuzi 80 sm^2 bo'lgan teng yonli trapetsiya tashqi chizilgan. Trapetsiyaning yon tomoni topilsin.

A) 9; B) 7; C) 10; D) 8; E) 11 sm.

3. Rombning balandligi 4 sm, diagonallaridan biri 5 sm ga teng. Rombning yuzi hisoblansin.

A) $\frac{45}{4}$; B) $\frac{50}{3}$; C) $\frac{47}{3}$; D) 16,4; E) 16,5 sm^2.

4. Agar to'g'ri to'rtburchakning yuzi $12\sqrt{3}$ dm^2, diagonallari hosil qilgan burchaklardan biri 60° bo'lsa, uning perimetri topilsin.

A) $3\sqrt{48+24\sqrt{3}}$; B) $\sqrt{48+24\sqrt{3}}$; C) $5\sqrt{48+24\sqrt{3}}$;

D) $2\sqrt{48+24\sqrt{3}}$; E) $6\sqrt{48+24\sqrt{3}}$ dm.

5. 60° ga teng bo'lgan o'tkir burchakka bir-biriga tashqi urinuvchi ikkita aylana ichki chizilgan. Kichik aylananing radiusi 2 sm bo'lsa, katta aylananing radiusi topilsin.

A) 5; B) 7; C) 8; D) 4; E) 6 sm.

6. Katta asosi AD bo'lgan $ABCD$ teng yonli trapetsiyaning AC diagonali CD tomoniga perpendikulyar va $\angle BAC=\angle CAD$. Agar trapetsiyaning perimetri 20 sm, $\angle D=60°$ bo'lsa, AD tomon uzunligi topilsin.

A) 7; B) 8; C) 9; D) 10; E) 6 sm.

7. Agar aylana diametrining uchlari uning biror urinmasidan 18 va 12 sm uzoqlikda ekanligi ma'lum bo'lsa, shu aylana diametrining uzunligi topilsin.

A) 28; B) 27; C) 29; D) 30; E) 26 sm.

8. Agar $(\bar{a}, \bar{c}) = (\bar{b}, \bar{c}) = 60°$, $|a| = 1$, $|b| = |c| = 2$ bo'lsa, $(a + b)c$ hisoblansin.

A) 2; B) 5; C) 3; D) 4; E) 1.

9. Agar $A_1A_4 = 2.24$ bo'lsa, muntazam $A_1A_2A_3A_4A_5A_6$ olti-burchakning perimetri topilsin.

A) 6,72; B) 6,75; C) 6,77; D) 6,43; E) 6,47.

10. Radiusi 10 sm bo'lgan aylanaga teng yonli uchburchak ichki chizilgan. Uchburchakning uchidagi burchagi 120° ga teng bo'lsa, uning yuzi hisoblansin.

A) $16\sqrt{3}$; B) $18\sqrt{3}$; C) $15\sqrt{3}$; D) $26\sqrt{3}$; E) $25\sqrt{3}$ sm².

11. Uchburchak asosidagi burchaklarning kattasi 45° ga teng, balandligi asosini 24 sm va 7 sm uzunlikdagi kesmalarga ajratadi. Shu uchburchakning katta yon tomoni uzunligi topilsin.

A) 23; B) 25; C) 24; D) 26; E) 27 sm.

12. Aylananing 90° li markaziy burchagiga tiralgan yoyning uzunligi 15 sm. Aylanaga tashqi chizilgan muntazam uchburchakning tomoni topilsin.

A) $73\sqrt{3}$; B) $74\sqrt{3}$; C) $77\sqrt{3}$; D) $60\sqrt{3}$; E) $71\sqrt{3}$ sm.

13. *ABCD* parallelogrammda $AB = 7$ sm, $AC = 11$ sm, $BD = 13$ sm bo'lsa, uning AD tomoni uzunligi topilsin.

A) $2\sqrt{6}$; B) $3\sqrt{6}$; C) $4\sqrt{6}$; D) $5\sqrt{6}$; E) 2.

14. R radiusli aylanaga o'tkir burchaklari 15° va 60° bo'lgan uchburchak ichki chizilgan. Shu uchburchakning yuzi hisoblansin.

A) $\frac{R^2\sqrt{3}}{3}$; B) $\frac{R^2\sqrt{3}}{5}$; C) $\frac{R^2\sqrt{3}}{6}$; D) $\frac{R^2\sqrt{3}}{8}$; E) $\frac{R^2\sqrt{3}}{4}$.

15. Agar kvadratning ikki uchi R radiusli aylanada, qolgan ikki uchi esa aylanaga urinmada yotsa, kvadrat diagonalining uzunligi topilsin.

A) $\frac{8\sqrt{2}R}{5}$; B) $\frac{8\sqrt{3}R}{5}$; C) $\frac{8\sqrt{3}R}{3}$; D) $\frac{8\sqrt{2}R}{7}$; E) $\frac{6\sqrt{2}R}{7}$.

16. To'g'ri burchakli uchburchakning katetlaridan biri 15 sm bo'lib, unga ichki chizilgan aylananing radiusi 3 sm bo'lsa, uchburchakning yuzi hisoblansin.

A) 62; B) 61; C) 60; D) 58; E) 59 sm^2.

17. Katetlari 3 m va 4 m bo'lgan to'g'ri burchakli uchburchakka u bilan umumiy to'g'ri burchakka ega bo'lgan kvadrat ichki chizilgan. Kvadratning yuzi hisoblansin.

A) $\frac{139}{49}$; B) $\frac{138}{49}$; C) $\frac{137}{49}$; D) $\frac{144}{49}$; E) $\frac{143}{49}$ m^2.

18. Agar $|a|=2\sqrt{2}$, $|b|=3$ va $a, b = 45°$ bo'lsa, $5a-2b$ va $a-3b$ vektorlar yordamida yasalgan parallelogrammning diagonallari uzunliklari topilsin.

A) $\sqrt{165}$ va $\sqrt{151}$; B) $\sqrt{163}$ va $\sqrt{153}$; C) $\sqrt{165}$ va $\sqrt{155}$; D) $\sqrt{163}$ va $\sqrt{155}$; E) $\sqrt{185}$ va $\sqrt{153}$.

19. Agar noldan farqli a va b vektorlarning uzunliklari teng bo'lib, $\bar{P} = a - 2b$ va $Q = 5a - 4b$ vektorlar o'zaro perpendikulyar bo'lsa, a va b vektorlar orasidagi burchak topilsin.

A) $\arccos\frac{13}{14}$; B) $\arccos\frac{11}{12}$; C) $\arccos\frac{11}{13}$; D) $\arccos\frac{12}{13}$;

E) $\arccos\frac{11}{14}$.

20. Radiuslari 1 m va 3 m bo'lgan aylanalar bir-biriga tashqi urinadi. Urinish nuqtasidan aylanalarning umumiy urinmasigacha bo'lgan masofa topilsin.

A) 1,8; B) 1,6; C) 1,4; D) 1,5; E) 1,3 m.

21. Yon tomoni 4 sm bo'lgan teng yonli uchburchak yon tomonining medianasi 5 sm. Shu uchburchakka tashqi chizilgan aylana radiusi topilsin.

A) $\frac{6\sqrt{22}}{15}$; B) $\frac{8\sqrt{22}}{11}$; C) $\frac{8\sqrt{22}}{15}$; D) $\frac{8\sqrt{3}}{25}$; E) $\frac{6\sqrt{33}}{13}$.

22. Parallelogrammning perimetri 90 sm bo'lib, uning o'tkir burchagi 60° ga teng. Agar parallelogrammning diagonali uning o'tmas burchagini 1:3 kabi nisbatda bo'lsa, parallelogrammning yuzi hisoblansin.

A) 227; B) 226; C) $225\sqrt{2}$; D) $226\sqrt{2}$; E) $225\sqrt{3}$ sm².

23. To'g'ri burchakli uchburchakka ichki va tashqi chizilgan aylanalarning radiuslari mos ravishda 2 sm va 5 sm. Uchburchakning katetlari topilsin.

A) 5 va 7; B) 6 va 7; C) 7 va 8; D) 6 va 8; E) 8 va 10 sm.

24. Rombning diagonallaridan biri uning tomoniga teng. Rombga ichki chizilgan aylananing radiusi 2 sm bo'lsa, rombning yuzi hisoblansin.

A) $\frac{32\sqrt{3}}{5}$; B) $\frac{33\sqrt{3}}{5}$; C) $\frac{32\sqrt{3}}{3}$; D) $\frac{32\sqrt{3}}{7}$; E) $\frac{32\sqrt{5}}{4}$ sm².

25. Yuzi 9 m² bo'lgan to'g'ri to'rtburchakning diagonallari o'zaro 120° li burchak tashkil qiladi. To'rtburchakning tomonlari topilsin.

A) $3\sqrt[3]{3}$ va $3\sqrt{3}$; B) $3\sqrt[4]{3}$ va $3\sqrt{3}$; C) $\sqrt[3]{3}$ va $\sqrt{3\sqrt{3}}$; D) $3\sqrt[4]{3}$ va $\sqrt[3]{3}$; E) $3\sqrt[4]{3}$ m va $\sqrt{3\sqrt{3}}$ m.

26. Uchburchakda medianalar kvadratlari yig'indisining tomonlar kvadratlari yig'indisiga nisbati topilsin.

A) 0,75; B) 0,5; C) 4; D) $\frac{2}{3}$; E) 0,8.

27. To'g'ri burchakli uchburchakning gipotenuzasida teng tomonli uchburchak yasalgan va uning yuzi berilgan uchburchak yuzidan 2 marta katta. To'g'ri burchakli uchburchak katetlarining nisbati topilsin.

A) $\sqrt{5}$; B) $\sqrt{3}$; C) $\sqrt{6}$; D) $\sqrt{7}$; E) 2.

28. Teng yonli uchburchakning asosidagi burchak 45°, yon tomoni esa $3\sqrt{2}$. Uchburchakning uchidan medianalar kesishish nuqtasigacha bo'lgan masofa topilsin.

A) 3; B) 4; C) 2; D) 5; E) 6 sm.

29. Uchburchakning perimetri 4,5 dm bo'lib, ichki burchagining bissektrisasi qarama-qarshi tomonni 6 sm va 4 sm uzunlikdagi kesmalarga ajratadi. Uchburchakning tomonlari topilsin.

A) 12, 18, 15; B) 13, 19, 13; C) 16, 18, 11;
D) 10, 14, 21; E) 15, 13, 17 sm.

30. ABC uchburchakda $\angle C=90°$. AB gipotenuzaning davomida BC katetga teng bo'lgan BD kesma ajratilgan hamda C va D nuqtalar tutashtirilgan. Agar $BC=7$ sm, $AC=24$ sm bo'lsa, CD kesmaning uzunligi topilsin.

A) 11,3; B) 11,4; C) 11,1; D) 11; E) 11,2 sm.

31. Aylanaga ichki chizilgan teng yonli uchburchak asosining uzunligi 10 sm, yon tomonining uzunligi 12 sm. Uchburchak balandligining o'rtasidan asosga parallel bo'lgan vatar o'tkazilgan. Vatarning uzunligi topilsin.

A) 13; B) 14; C) 12; D) 11; E) 10 sm.

32. Radiusi $7\sqrt{3}$ bo'lgan aylanaga uchburchak ichki chizilgan. Uchburchakda o'tkir burchak qarshisidagi tomon 21 sm, qolgan ikkita tomonlarning nisbati 5:8 kabi. Shu tomonlar topilsin.

A) 15 va 23; V) 15 va 25; S) 14 va 24; D) 15 va 24; E) 16 va 23 sm.

33. $30°$ ga teng bo'lgan burchakning bitta tomoni uchidan o'zaro teng 10 ta kesma ajratilgan. Bo'linish nuqtalaridan o'tkazilgan perpendikulyarlar burchakning ikkinchi tomoni bilan kesishguncha davom ettirilgan. Agar ulardan eng kattasining uzunligi 10 sm ga teng bo'lsa, ajratilgan kesmaning uzunligi topilsin.

A) $\sqrt{2}$; B) $2\sqrt{3}$; C) $3\sqrt{2}$; D) 2; E) $\sqrt{3}$ sm.

34. $ABCD$ teng yonli trapetsiya va AD uning katta asosidir. Trapetsiyaning o'rta chizig'i 12 dm, ACD va ABC uchburchaklar perimetrlarining ayirmasi 6 dm. Trapetsiyaning katta asosi topilsin.

A) 14; B) 13; C) 15; D) 12; E) 11 dm.

35. Aylanaga tashqi chizilgan to'rtburchakning ikkita qo'shni tomoni 5 va 12 sm bo'lib, o'zaro perpendikulyar. To'rtburchakning boshqa ikkita tomonlari orasidagi burchak $60°$ bo'lsa, shu tomonlar topilsin.

A) 7 va 15; B) 8 va 15; C) 8 va 13;
D) 7 va 13; E) 8 va 14 sm.

36. Asosi a ga teng bo'lgan teng yonli uchburchakka radiusi R ga teng bo'lgan doira ichki chizilgan. Uchburchak va doira orasida joylashgan qismning yuzi hisoblansin.

A) $\frac{a^3R}{a^2-4R^2}-\pi R^2$; B) $\frac{a^3R}{a^2-2R}-\pi R^2$; C) $\frac{a^3R}{a^2-3R}-\pi R^2$;

D) $\frac{a^3R^2}{a^2-4R^2}-\pi R^2$; E) $\frac{a^3R^2}{a^2-2R^2}-\pi R^2$.

37. Doira va unga ichki chizilgan to'g'ri burchakli uchburchak yuzlarining nisbati $\frac{2\pi}{\sqrt{3}}$ ga teng. Uchburchakning katta o'tkir burchagi topilsin.

A) 60°; B) 30°; C) 50°; D) 45°; E) 70°.

38. Aylanaga muntazam uchburchak va muntazam oltiburchak ichki chizilgan. Oltiburchak va uchburchak yuzlarining nisbati topilsin.

A) 3:2; B) 4:3; C) 4:1; D) 3:1; E) 2:1.

39. Radiusi 1 ga teng bo'lgan aylanaga teng yonli uchburchak ichki chizilgan va uning yon tomoni asosidan 2 marta katta. Shu uchburchakka aylana ichki chizilgan bo'lsa, uning radiusi topilsin.

A) $\frac{4}{5}$; B) $\frac{3}{8}$; C) $\frac{6}{7}$; D) $\frac{2}{3}$; E) $\frac{1}{4}$.

40. Kichik asosi 1 ga teng bo'lgan teng yonli trapetsiyaga radiusi 1 ga teng bo'lgan aylana ichki chizilgan. Trapetsiyaning yuzi hisoblansin.

A) 8; B) 6; C) 5; D) 4; E) 3.

41. To'g'ri burchakli uchburchakning gipotenuzasi c ga, katetlari a va b ga teng. Uchburchakka ichki chizilgan aylananing diametri topilsin.

A) $a+b-c$; B) $a-b+c$; C) $a-b-c$; D) $b-b-c$;
E) $c-(a+b)$.

42. Birinchi uchburchakning medianalari ikkinchi uchburchakning tomonlariga teng bo'lsa, ular yuzlarining nisbati topilsin.

A) 3:2; B) 5:2; C) 2:1; D) 4:3; E) 3:2.

43. O'xshash ko'pburchaklarning yuzlari mos ravishda 121 va 225 sm^2 ga teng. Agar ko'pburchaklardan ikkinchi-

sining perimetri birinchisinikidan 16 sm katta bo'lsa, ularning perimetrlari topilsin.

A) 44 va 58; B) 43 va 60; C) 43 va 58; D) 45 va 62; E) 44 va 60 sm.

44. *ABC* uchburchakda $AB=24$ sm, $BC=36$ sm. Agar uchburchakda *BD* bissektrisa o'tkazilgan bo'lsa, hosil qilingan uchburchaklar yuzlarining nisbati topilsin.

A) $\frac{3}{4}$ yoki $\frac{4}{3}$; B) $\frac{2}{3}$ yoki $\frac{3}{2}$; S) $\frac{5}{4}$ yoki $\frac{4}{5}$; D) $\frac{2}{5}$ va $\frac{5}{2}$;

E) $\frac{3}{5}$ yoki $\frac{5}{3}$.

45. Aylanaga muntazam uchburchak va muntazam to'rtburchak ichki chizilgan. Agar to'rtburchakning tomoni 6 sm bo'lsa, uchburchakning yuzi hisoblansin.

A) $13{,}5\sqrt{3}$; B) $12{,}5\sqrt{3}$; C) $13{,}8\sqrt{3}$; D) $13{,}6\sqrt{3}$; E) $13{,}5\sqrt{2}$ sm².

46. *ABCD* parallelogrammda ichki *A* burchakning bissektrisasi *BC* tomon bilan *K* nuqtada, *CD* tomonning davomi bilan *M* nuqtada kesishadi va $BK=24$ sm, $KC=6$ sm, $\angle KMC=30°$. *MKC* uchburchakning perimetri topilsin.

A) $6(2+\sqrt{2})$; B) $4(2+\sqrt{2})$; C) $6(2+\sqrt{3})$;
D) $5(2+\sqrt{3})$; E) $3(2+\sqrt{2})$ sm².

47. Rombga ichki chizilgan doiraning yuzi $\frac{49\pi}{4}$ sm², rombning o'tmas burchagi 120° bo'lsa, uning yuzi hisoblansin.

A) $\frac{96}{\sqrt{3}}$; B) $\frac{94}{\sqrt{3}}$; C) $\frac{95}{\sqrt{3}}$; D) $\frac{98}{\sqrt{3}}$; E) $\frac{97}{\sqrt{3}}$ sm².

48. Trapetsiyaning asoslari va diagonallarining qismlari bilan hosil qilingan uchburchaklarning yuzlari *S* va *Q* ga teng. Trapetsiyaning yuzi hisoblansin.

A) S^2+Q^2; B) $S+Q$; C) $\sqrt{S}+\sqrt{Q}$; D) $(\sqrt{S}-\sqrt{Q})^2$;
E) $(\sqrt{S}+\sqrt{Q})^2$.

49. To‘g‘ri burchakli trapetsiyaning o‘rta chizig‘i 16 sm, trapetsiyaga ichki chizilgan aylananing radiusi 6 sm bo‘lsa, trapetsiyaning yon tomoni va katta asosi orasidagi burchak topilsin.

A) $\arcsin\frac{3}{5}$; B) $\arcsin\frac{2}{3}$; C) $\arcsin\frac{4}{5}$; D) $\arcsin\frac{3}{4}$;

E) $\arcsin\frac{1}{2}$.

50. Uchburchakning asosiga parallel bo‘lgan kesma yon tomonni uchidan boshlab 5:3 kabi nisbatda bo‘ladi hamda hosil qilingan qismlar yuzlarining ayirmasi 56 sm^2 ga teng. Uchburchakning yuzi hisoblansin.

A) 252; B) 256; C) 254; D) 253; E) 255 sm^2.

2-qism

STEREOMETRIYA

8-§. FAZODAGI TO'G'RI CHIZIQLAR VA TEKISLIKLAR

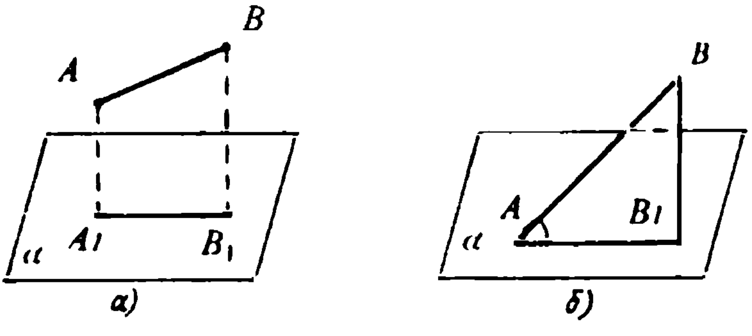
8.1. Asosiy tushunchalar va tasdiqlar

Fazoda to'g'ri chiziqlarning o'zaro vaziyati uch xil bo'lishi mumkin: a) kesishgan to'g'ri chiziqlar; b) parallel to'g'ri chiziqlar; d) ayqash to'g'ri chiziqlar.

To'g'ri chiziq va tekislikning o'zaro vaziyati ham uch xil bo'lishi mumkin: a) to'g'ri chiziq tekislikda yotadi; b) to'g'ri chiziq tekislik bilan bitta nuqtada kesishadi; v) to'g'ri chiziq va tekislik parallel bo'ladi.

Tekislikdagi har bir to'g'ri chiziqqa perpendikulyar to'g'ri chiziq shu tekislikka perpendikulyar bo'ladi.

Agar AB kesmaning uchlaridan α tekislikka (8.1-*a* chizma) AA_1 va BB_1 perpendikulyarlar o'tkazsak, A_1B_1 kesma berilgan AB kesmaning α tekislikdagi *proyeksiyasi* bo'ladi.



8.1-chizma.

Quyidagi ta'rif va tasdiqlar o'rinli

1. Tekislikda *AB* og'maning proyeksiyasiga perpendikulyar to'g'ri chiziq o'tkazilsa, bu to'g'ri chiziq og'maning o'ziga ham perpendikulyar bo'ladi va tasdiqning teskarisi ham o'rinli.

2. *AB* to'g'ri chiziq va tekislik orasidagi burchak shu to'g'ri chiziq va uning tekislikdagi proyeksiyasi orasidagi burchakka teng (8.1-*b* chizma).

3. Ikkita yarim tekislikdan va ularni chegaralab turgan umumiy to'g'ri chiziqdan tashkil topgan shakl *ikki yoqli burchak* (8.2-chizma), yarim tekisliklar ikki yoqli burchakning *yoqlari*, ularni chegaralovchi to'g'ri chiziq esa ikki yoqli burchakning *qirrasi* deyiladi.

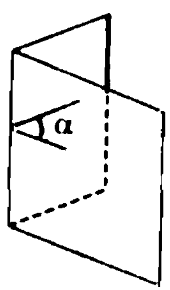
4. Ikki yoqli burchakning qirrasiga perpendikulyar tekislik uning yoqlarini ikkita yarim to'g'ri chiziq bo'yicha kesib o'tadi. Bu yarim to'g'ri chiziqlar tashkil etgan burchak ikki yoqli burchakning *chiziqli burchagi* deyiladi.

5. Ikki yoqli burchakning o'lchovi chiziqli burchakning kattaligiga tengdir.

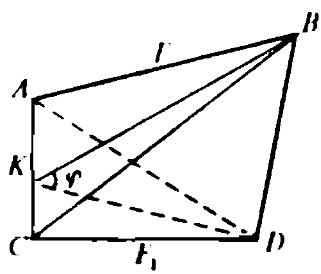
6. α tekislikda yotuvchi F shaklning β tekislikka tushirilgan proyeksiyasi F_1 shakldan iborat bo'lib, α va β tekisliklar orasidagi burchak φ bo'lsa, F_1 proyeksiyaning yuzi

$$S_{F_1} = S_F \cdot \cos\varphi \qquad (8.1)$$

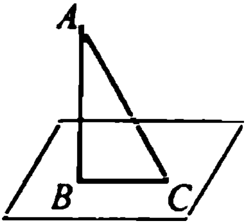
formula orqali hisoblanadi (8-3 chizma).



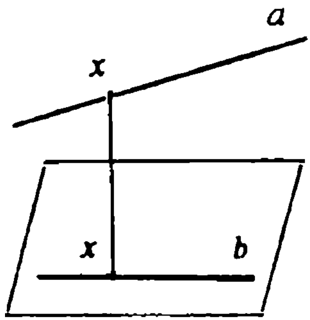
8.2-chizma.



8.3-chizma.



8.4-chizma.



8.5-chizma.

Berilgan nuqtadan berilgan tekislikka o'tkazilgan *og'ma* — bu bir uchi shu nuqtada, ikkinchi uchi tekislikda yotgan va tekislikka perpendikulyar bo'lmagan istalgan kesmadir. Kesmaning tekislikda yotgan uchi uning *asosidir.* Bitta nuqtadan o'tkazilgan perpendikulyar va og'maning asoslarini tutashtiruvchi kesma og'maning *proyeksiyasidir* (8.4-chizma).

7. (Uch perpendikulyar haqida.) Tekislikda og'maning asosidan uning proyeksiyasiga perpendikulyar qilib o'tkazilgan to'g'ri chiziq og'maning o'ziga ham perpendikulyar. Aksincha, agar tekislikdagi to'g'ri chiziq og'maga perpendikulyar bo'lsa, u og'maning proyeksiyasiga ham perpendikulyardir.

8. Ikki ayqash to'g'ri chiziqning umumiy perpendikulyari uchlari shu to'g'ri chiziqlarda bo'lib, ularning har biriga perpendikulyar kesmadir. Ikki ayqash to'g'ri chiziq bitta va faqat bitta umumiy perpendikulyarga ega. Bu perpendikulyar shu to'g'ri chiziqlar orqali o'tuvchi parallel tekisliklarning umumiy perpendikulyaridir (8.5-chizma).

8.2. Mavzuga oid masalalar

1. AB kesmaning A uchidan α tekislik o'tkazilgan, B uchidan va o'rtasidagi C nuqtadan o'zaro parallel BB_1 va CC_1 kesmalar o'tkazilgan. Bu kesmalar α tekislikni B_1 va

C_1 nuqtalarda kesib o‘tadi. Agar $BB_1=12$ sm bo‘lsa, CC_1 kesmaning uzunligi topilsin.

A) 4; B) 6; C) 5; D) 4,5; E) 7 sm.

2. AB kesmaning A uchidan tekislik o‘tkazilgan, kesmaning B uchidan va C nuqtasidan o‘zaro parallel kesmalar o‘tkazilgan va ular tekislik bilan mos ravishda B_1 va C_1 nuqtalarda kesishadi. Agar $BB_1=16$ dm va $AC:AB=3:5$ kabi bo‘lsa, CC_1 kesmaning uzunligi topilsin.

A) 9,6; B) 7,2; C) 8,4; D) 9,0; E) 7,6 dm.

3. A nuqtadan α tekislikka ikkita $AB=17$ m va $AC=10$ m og‘ma o‘tkazilgan. Ular proyeksiyalarining ayirmasi 9 m bo‘lsa, A nuqtadan α tekislikkacha bo‘lgan masofa topilsin.

A) 5; B) 6; C) 7; D) 8; E) 9 m.

4. ΔABC da $\angle B=90°$ bo‘lib, $BC=a$. Uchburchakning A uchidan uchburchak tekisligiga AD perpendikulyar shunday o‘tkazilganki, D va C nuqtalar orasidagi masofa m ga teng. D nuqtadan BC katetgacha bo‘lgan masofa topilsin.

A) $\sqrt{a^2+2m^2}$; B) $\sqrt{a^2-m^2}$; C) $\sqrt{m^2-a^2}$;

D) $\sqrt{am}$; E) $\sqrt{\frac{a^2+m^2}{2}}$.

5. Trapetsiyaning asoslaridan biri ikkinchisidan ikki marta katta. Trapetsiyaning o‘rta chizig‘i α tekislikka parallel va undan 13 sm masofada o‘tadi. Trapetsiya diagonallarining kesishish nuqtasi esa bu tekislikdan 15 sm masofada yotadi. Trapetsiyaning asoslaridan α tekislikkacha bo‘lgan masofalar topilsin.

A) 7 va 12; B) 8 va 16; C) 7 va 16; D) 8 va 11;
E) 7 sm va 19 sm.

6. 120° ga teng bo‘lgan ikki yoqli burchakning yoqlaridagi A va B nuqtalardan burchakning qirrasiga $AC=7$ sm va

$BD=8$ sm perpendikulyarlar o'tkazilgan. Agar $AB=16$ sm bo'lsa, CD kesmaning uzunligi topilsin.

A) 2; B) 3; C) 2,5; D) 4; E) 3,5.

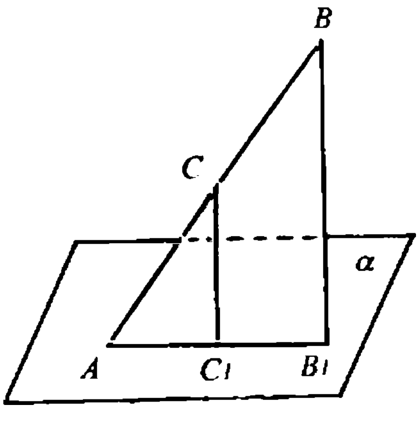
7. ΔABC da $AB=9$ m, $BC=6$ m, $AC=5$ m bo'lib, uning AC tomonidan uchburchak tekisligi bilan 45° li burchak tashkil etuvchi tekislik o'tkazilgan. ΔABC ning shu tekislikdagi proeksiyasi yuzi hisoblansin.

A) 10; B) 9; C) 8; D) 12; E) 11 sm^2.

8.3. Mavzuga oid masalalarning yechimlari

1. Berilgan. $AB \cap \alpha = A$, $BB_1 \| CC_1$, $AC=CB$, $BB_1=12$ sm.

CC_1 topilsin (8.3.1-chizma).



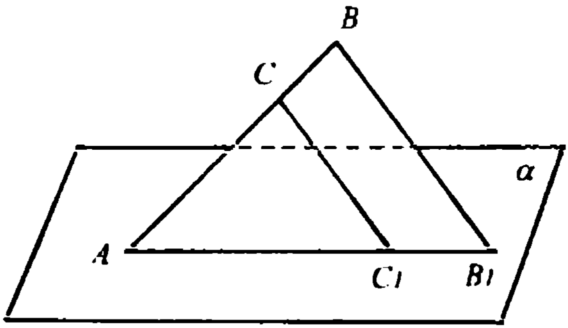
8.3.1-chizma.

Yechilishi. Berilishiga ko'ra, $BB_1 \| CC_1$ bo'lganligidan, ular bir tekislikda $BB_1 \| CC_1$ bo'lganligi-χ tekislik bilan B_1C_1 to'g'ri chiziq orqali kesishadi. C nuqta AB kesmaning o'rtasidagi nuqta va $CC_1 \| BB_1$ bo'lgani uchun, CC_1 kesma ΔABB_1 ning o'rta chizig'idir. Shuning uchun, $CC_1=\frac{1}{2}BB_1=\frac{1}{2}\cdot 12=6$ sm.

Javobi: B).

2. Berilgan: $A \in \alpha$, $B \notin \alpha$. $BB_1 \| CC_1$, $AC : AB = 3:5$, $BB_1 = 16$ dm.

CC_1 topilsin (8.3.2-chizma).



8.3.2-chizma.

Yechilishi. Berilishiga ko'ra, $BB_1 \| CC_1$ bo'lganligidan, $\Delta ACC_1 \sim \Delta ABB_1$. O'xshash uchburchaklarda mos tomonlar o'zaro proporsional bo'lganligidan,

$$\frac{AB}{BB_1} = \frac{AC}{CC_1} \text{ yoki } \frac{CC_1}{BB_1} = \frac{AC}{AB}$$

bo'ladi. U holda $CC_1 = \frac{AC}{AB} \cdot BB_1 = \frac{3}{5} \cdot 16 = \frac{48}{5} = 9{,}6$ dm.

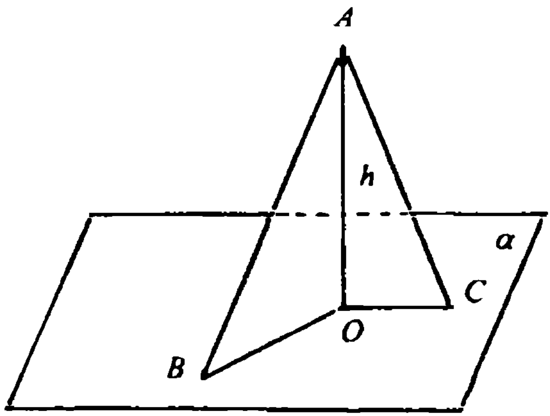
Javobi: A).

3. Berilgan. α tekislik, AB, AC — og'malar, $AB = 17$ m; $AC = 10$ m, $AO \perp \alpha$, $BO - CO = 9$ m.

AO topilsin (8.3.3-chizma).

Yechilishi. $AO = h$, $BO = x$, $CO = y$ belgilashlarni kiritamiz. ΔABO va ΔAOC larning har biri to'g'ri burchakli bo'ladi va ulardan

$$AO^2 = AB^2 - BO^2;\ AO^2 = AC^2 - OC^2;\ BO - OC = 9$$



8.3.3-chizma.

munosabatlarni olamiz. Belgilashlarimizdan foydalansak,

$$\begin{cases} h^2 = 17^2 - x^2, \\ h^2 = 10^2 - y^2, \\ x - y = 9 \end{cases} \Rightarrow \begin{cases} 17^2 - x^2 = 10^2 - y^2, \\ x - y = 9, \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x^2 - y^2 = 289 - 100. \\ x - y = 9, \end{cases} \Rightarrow \begin{cases} (x - y)(x + y) = 189, \\ x - y = 9, \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x + y = 21, \\ x - y = 9, \end{cases} \Rightarrow \begin{cases} 2x = 30, \\ x - y = 9, \end{cases} \Rightarrow \begin{cases} x = 15, \\ x - y = 9, \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x = 15, \\ 15 - y = 9, \end{cases} \Rightarrow \begin{cases} x = 15, \\ y = 6, \end{cases} \quad h^2 = 10^2 - 6^2 = 64; \ h = 8 \text{ m}.$$

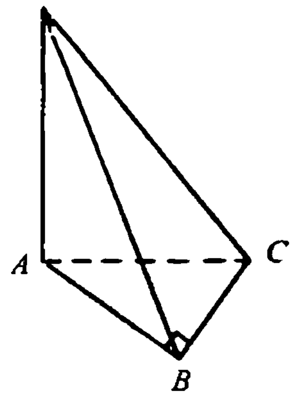
Javobi: D).

4. Berilgan. ΔABC — to'g'ri burchakli, $AD \perp (ABC)$, $BC = a$, $CD = m$.

BD topilsin (8.3.4-chizma).

Yechilishi. D nuqtadan BC katetga perpendikulyar o'tkazish kerak. Lekin berilishiga ko'ra, ΔABC — to'g'ri burchakli va $AB \perp BC$. Uch perpendikulyar haqidagi teoremaga asosan, $DB \perp BC$ bo'ladi. To'g'ri burchakli ΔDBC dan Pifagor teoremasiga asosan, $BD = \sqrt{DC^2 - BC}$ yoki $BD = \sqrt{m^2 - a^2}$.

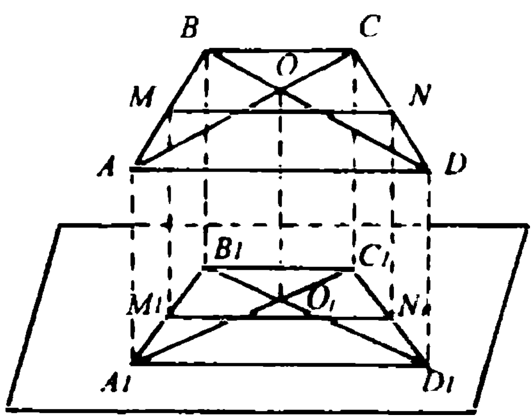
Javobi: C).



8.3.4-chizma.

5. Berilgan. $ABCD$ — trapetsiya, MN — o'rta chiziq, α tekislik, $MN \parallel \alpha$, $AD = 2 \cdot BC$, $AC \cap BD = O$, $OO_1 = 15$ sm, $MM_1 = 13$ sm.

AA_1, BB_1 topilsin (8.3.5-chizma).



8.3.5-chizma.

Y e c h i l i s h i. $MN \parallel AD$, $MN \parallel BC$ bo'lganligidan va berilishiga ko'ra, $BC \parallel \alpha$, $AD \parallel \alpha$ bo'ladi. Trapetsiyaning uchlaridan hamda M, N va O nuqtalardan α tekislikka perpendikulyar o'tkazamiz. Ular bitta tekislikka perpendikulyarlar bo'lib, o'zaro parallel bo'ladi.

$\Delta BCO \sim \Delta AOD$, chunki vertikal burchaklar bo'lganligidan, $\angle BOC = \angle AOD$, ichki almashinuvchi burchaklar bo'lgani uchun $\angle BCO = \angle OAD$. Ularning mos tomonlari o'zaro proporsionalligidan,

$$\frac{AD}{BC} = \frac{AO}{OC}, \quad \frac{AO}{OC} = \frac{2}{1} \text{ va } \frac{AO}{OC} = 2$$

$AA_1 \parallel BB_1$ bo'lgani uchun, AA_1B_1B — trapetsiya va MM_1 — uning o'rta chizig'idir. Shuning uchun,

$$MM_1 = \frac{AA_1 + BB_1}{2} \text{ yoki } 13 = \frac{1}{2}(AA_1 + BB_1), \; AA_1 + BB_1 = 26 \text{ sm.}$$

Endi AA_1C_1C trapetsiyani alohida qaraymiz. A nuqtadan $AP \parallel A_1C_1$ ni o'tkazamiz va u OO_1 bilan K nuqtada kesishgan bo'lsin.

$AA_1 = x$, $CC_1 = BB_1 = y$ deb belgilaymiz. U holda, $OK = 15 - x$, $CP = y - x$. To'g'ri burchakli ΔAOK va ΔACP larning o'xshashligidan,

$$\frac{AC}{CP} = \frac{AO}{OK}, \quad \frac{AO+OC}{AO} = \frac{CP}{OK}, \quad 1 + \frac{OC}{AO} = \frac{CP}{OK}, \quad 1 + \frac{1}{2} = \frac{y-x}{15-x},$$

$$3(15-x) = 2(y-x), \quad x + 2y = 45.$$

Natijada $\begin{cases} x + y = 26, \\ x + 2y = 45 \end{cases}$ sistemani hosil qilamiz.

Bu sistemani yechamiz:

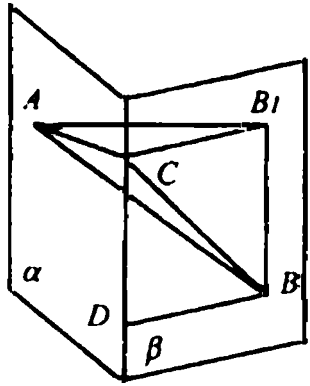
$$\begin{cases} x + y = 26, \\ 26 + y = 45 \end{cases} \Rightarrow \begin{cases} x = 26 - y, \\ y = 19 \end{cases} \Rightarrow \begin{cases} x = 7, \\ y = 19. \end{cases}$$

Javobi: E).

6. B e r i l g a n. $ACDB$ — ikki yoqli burchak, $A \in \alpha$, $B \in \beta$, $AB = 16$ sm, $\alpha \cap \beta = CD$, $AC \perp CD$, $AC = 7$ sm, $BC \perp BD$, $BD = 11$ sm.

CD t o p i l s i n (8.3.6-chizma).

Y e c h i l i s h i. β tekislikdagi C nuqtadan $CB_1 \perp CD$ o'tkazamiz va $CB_1 = 8$ sm kesma ajratamiz. Ikki yoqli burchakning chiziqli burchagi bo'lganligidan, $\angle ACB_1 = 120°$. ΔACB_1 dan kosinuslar teoremasi yordamida topamiz:



8.3.6-chizma.

$$AB_1^2 = AC^2 + B_1C^2 - 2AC \cdot B_1C \cdot \cos 120°,$$

$$AB_1^2 = 7^2 + 11^2 - 2 \cdot 7 \cdot 11\left(-\frac{1}{2}\right) = 49 + 21 + 77 = 247.$$

Ikkinchi tomondan, $B_1C = BD$, $B_1C \perp CD$, $BD \perp CD$ bo'lgani uchun, $BDCB_1$ — to'g'ri to'rtburchak va $CB_1 \perp B_1B$. U holda, uch perpendikulyar haqidagi teoremaga asosan, $AB_1 \perp BB_1$. To'g'ri burchakli ΔABB_1 dan:

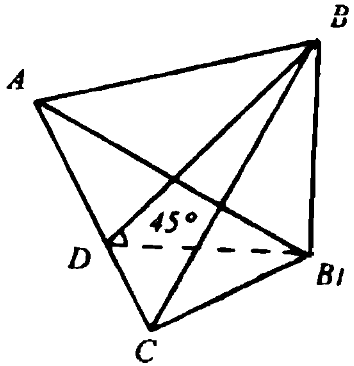
$$BB_1^2 = AB^2 - AB_1^2 = 16^2 - 247 = 9, \ BB_1 = 3 \text{ sm}.$$

Javobi: B).

7. B e r i l g a n. ΔABC, $AC = 5$ m, $AB = 9$ m, $BC = 6$ m, $AC \subset (AB_1C)$, $\angle BDB_1 = 45°$.

$S_{\Delta AB_1C}$ h i s o b l a n s i n (8.3.7-chizma).

Y e c h i l i s h i. B nuqtadan AC tomonga BD perpendikulyar, D nuqtadan AB_1C tekislikda AC ga perpendikulyar bo'lgan DB_1 to'g'ri chiziq o'tkazamiz. ΔABC ning



8.3.7-chizma.

tekislikdagi proeksiyasini yasash uchun B nuqtadan (AB_1C) tekislikka BB_1 perpendikulyar tushiramiz, natijada ΔAB_1C — izlangan proyeksiya bo'ladi.

Shakl tekislikka proyeksiyalangan bo'lib, shakl va tekislik orasidagi burchak φ bo'lsa, quyidagi formula o'rinli: $S_{pr.} = S_{sh} \cdot \cos\varphi$. Endi Geron formulasi vositasida ΔABC ning yuzini hisoblaymiz:

$$p = \frac{9+6+5}{2} = 10 \text{ sm};$$

$$S_{\Delta ABC} = \sqrt{10 \cdot 1 \cdot 4 \cdot 5} = 10\sqrt{2} \text{ sm}^2.$$

U holda

$$S_{\Delta AB_1C} = S_{pr} = S_{\Delta ABC} \cdot \cos 45° = 10 \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 10 \text{ sm}^2.$$

Javobi: A).

8.4. Mustaqil yechish uchun masalalar

1. AB kesmaning uchlaridan α tekislikka $AC=3$ va $BD=2$ m perpendikulyarlar o'tkazilgan. Agar $CD=24$ dm bo'lsa, AB kesmaning uzunligi topilsin.

A) 15; B) 20; C) 24; D) 26; E) 28 dm.

2. A nuqtadan α tekislikka ikkita: $AB=20$ sm, $AC=15$ sm og'ma o'tkazilgan. AB og'maning α tekislikdagi proyeksiyasi 16 sm bo'lsa, AC og'maning tekislikdagi proyeksiyasi topilsin.

A) 9; B) 10; C) 8; D) 12; E) 6 sm.

3. Muntazam uchburchakning tomoni 3 sm. Uchburchak tekisligiga tegishli bo'lmagan K nuqta uchburchak-

ning uchlaridan bir xil — 2 sm masofada yotadi. K nuqtadan uchburchak tekisligigacha bo'lgan masofa topilsin.

A) 9; B) 0,5; C) 0,75; D) 1; E) 1,2 sm.

4. To'g'ri burchakli ΔABC da katetlar 15 m va 20 m. To'g'ri burchakning C uchidan (ABC) tekislikka $CD=35$ m perpendikulyar o'tkazilgan. D nuqtadan AB gipotenuzagacha bo'lgan masofa topilsin.

A) 40; B) 30; C) 32; D) 29; E) 37 m.

5. P tekislikda 60° li burchak berilgan. M nuqta burchak uchidan 25 sm uzoqlikda, burchak tomonlaridan esa mos ravishda 20 sm va 7 sm uzoqlikda yotadi. M nuqtadan P tekislikkacha bo'lgan masofa topilsin.

A) 12; B) 5; C) $\sqrt{37}$; D) $\sqrt{33}$; E) 6 sm.

6. Uzunligi a bo'lgan kesma α tekislikni kesib o'tadi. Agar kesmaning uchlari α tekislikdan b va c uzoqlikda yotishi ma'lum bo'lsa, kesmaning tekislikdagi proyeksiyasi topilsin.

A) $\sqrt{a^2-(b+c)^2}$; B) $\sqrt{b^2-(a^2-ac)}$;
C) $\sqrt{c^2-(a^2+b^2)}$; D) $\sqrt{a^2b^2-c^2}$; E) $\sqrt{a^2-bc}$.

7. $ABCD$ kvadratning A uchidan AK perpendikulyar o'tkazilgan. Agar $AB=3$ dm, $BK=4$ dm bo'lsa, K nuqtadan kvadratning C uchigacha bo'lgan masofa topilsin.

A) 4; B) 6; C) 5; D) 4,5; E) $5\sqrt{2}$ dm.

8. To'g'ri burchakli ΔABC ning C to'g'ri burchagi uchidan gipotenuzaga parallel tekislik o'tkazilgan. Gipotenuza $AB=12$ sm hamda AC va BC katetlarning tekislikdagi proyeksiyalari, mos ravishda, 6 sm va 10 sm bo'lsa, gipotenuzadan tekislikkacha bo'lgan masofa topilsin.

A) 5; B) 4; C) 6; D) 1,5; E) 2 sm.

9. ΔABC ning uchlaridan α tekislikkacha bo'lgan masofalar 2, 2,5 va 4,5 dm. Uchburchak medianalarining kesishish nuqtasidan tekislikkacha bo'lgan masofa topilsin.

A) 2,5; B) 3,5; C) 2; D) 3; E) 4 dm.

10. A nuqta to'g'ri ikki yoqli burchakning yoqlaridan 3 va 4 dm uzoqlikda yotadi. A nuqtadan ikki yoqli burchakning qirrasigacha bo'lgan masofa topilsin.

A) 2; B) 4; C) 5; D) 7; E) 6 dm.

11. $AB=50$ sm kesmaning uchlaridan berilgan tekislikkacha bo'lgan masofalar $AC=30$ sm va $BD=44$ sm. AB kesmaning bu tekislikdagi proeksiyasi topilsin.

A) 36; B) 48; C) 42; D) 54; E) 39 sm.

12. $CD=26$ sm kesmaning uchlari α tekislikdan 18 sm va 8 sm uzoqlikda bo'lsa, shu kesmaning tekislikdagi proyeksiyasi topilsin.

A) 24; B) 16; C) 20; D) 21; E) 32 sm.

13. Uzunligi 15 sm bo'lgan kesmaning uchlari α tekislikdan 3 sm va 6 sm uzoqlikda yotishi ma'lum bo'lsa, kesmaning tekislikdagi proyeksiyasi topilsin.

A) 9; B) 16; C) 14; D) 10; E) 12 sm.

14. $AB=26$ sm kesmaning uchlari P tekislikdan 6 sm va 8 sm uzoqlikda joylashgan bo'lib, AB kesma tekislikni kesib o'tadi. AB kesmaning tekislikdagi proyeksiyasi topilsin.

A) 26; B) 22; C) 24; D) 20; E) 18 sm.

15. Kesma tekislikni kesib o'tadi va uning uchlari tekislikdan 3 sm va 12 sm uzoqlikda bo'lsa, kesmaning o'rta nuqtasi tekislikdan qanday uzoqlikda joylashgan?

A) 6,5; B) 6; C) 5; D) 4,5; E) 4 sm.

16. Tekislik bilan kesishmaydigan kesmaning uchlari tekislikdan 30 sm va 50 sm uzoqlikda yotadi. Shu kesmani 3:7 kabi nisbatda bo'luvchi nuqta tekislikdan qanday uzoqlikda yotadi?

A) 24 yoki 28; B) 36 yoki 44; C) 24 yoki 36;
D) 18 yoki 24; E) 18 sm yoki 28 sm.

17. *P* tekislikning *A* nuqtasidan og'ma to'g'ri chiziq o'tkazilib, unda *B* va *C* nuqtalar olingan, $AB=8$ sm va $AC=14$ sm, Agar *B* nuqtadan tekislikkacha masofa 6 sm bo'lsa, *C* nuqtadan tekislikkacha bo'lgan masofa topilsin.

A) 16; B) 13,5; C) 12,5; D) 10,5; E) 13 sm.

18. Muntazam uchburchakning uchlari *P* tekislikdan 10, 15 va 17 sm uzoqlikda joylashgan. Uchburchakning markazidan *P* tekislikkacha bo'lgan masofa topilsin.

A) 16; B) 14; C) 15; D) 12; E) 17 sm.

19. *a* uzunlikdagi *AB* kesma *P* tekislikda yotadi, har birining uzunligi *b* bo'lgan *AC* va *BD* kesmalar *P* tekislikda yotmaydi. *AC* kesma *P* tekislikka perpendikulyar, *AB* kesmaga perpendikulyar bo'lgan *BD* kesma *P* tekislik bilan 30° burchak hosil qilsa, *CD* kesma topilsin.

A) $\sqrt{2(a^2+b^2)}$; B) $\sqrt{2ab}$; C) $\sqrt{a^2+b^2}$; D) $2\sqrt{ab}$;
E) $\sqrt{a+b}$.

20. *K* nuqtadan *P* tekislikka perpendikulyar va og'ma o'tkazilgan hamda ular orasidagi burchak 45°. Perpendikulyarning uzunligi 12 sm bo'lsa, og'maning uzunligi topilsin.

A) $16\sqrt{3}$; B) 14; C) $12\sqrt{3}$; D) 12; E) $12\sqrt{2}$ sm.

21. Doiraning markazidan uning tekisligiga perpendikulyar o'tkazilgan. Agar perpendikulyarning uzunligi 8 sm, doiraning yuzi 36π sm^2 bo'lsa, perpendikulyarning ustki uchidan aylananing nuqtasigacha bo'lgan masofa topilsin.

A) 10; B) 12; C) 10; D) 14; E) 16 sm.

22. $ABCD$ kvadratning tomoni a ga teng. Kvadratning O markazidan uning tekisligiga OK perpendikulyar o'tkazilgan va $OK=b$. K nuqtadan kvadratning uchlarigacha bo'lgan masofa topilsin.

A) $\sqrt{2a^2-b^2}$; B) $\sqrt{a^2+2b^2}$; C) $\sqrt{a^2+\frac{b^2}{2}}$;
D) $\sqrt{b^2+\frac{a^2}{2}}$; E) $\frac{\sqrt{ab}}{2}$.

23. K nuqtadan P tekislikka ikkita: $KA=16$ sm va $KB=10$ sm og'ma o'tkazilgan. KA og'ma P tekislik bilan 30° li burchak tashkil etishi ma'lum bo'lsa, KB og'maning P tekislikdagi proyeksiyasi topilsin.

A) 4; B) 6; C) 5; D) 4,5; E) 5,8 sm.

24. K nuqtadan P tekislikka perpendikulyar va og'ma o'tkazilgan bo'lib, perpendikulyarning uzunligi 6 sm, og'maning uzunligi 9 sm. Perpendikulyarning og'madagi proyeksiyasi topilsin.

A) 3,5; B) 4,5; C) 4; D) 5; E) 6 sm.

25. Teng tomonli uchburchakning tomoni 6 sm. Uchburchakning har bir uchidan 4 sm uzoqlikdagi nuqta bilan uning tekisligi orasidagi masofa topilsin.

A) 1,5; B) 2,5; C) 4; D) 3; E) 2 sm.

26. Teng tomonli uchburchakning tomoni 6 sm. Uchburchakning O markazidan uchburchak tekisligiga $OK=1$

sm perpendikulyar o'tkazilgan. *K* nuqtadan uchburchakning tomonigacha bo'lgan masofa topilsin.

A) 2; B) 1; C) 2,5; D) 2; E) 1,5 sm.

27. Uchburchakning tomonlari 10, 17 va 21 sm. Shu uchburchakning katta burchagi uchidan uning tekisligiga 15 sm uzunlikdagi perpendikulyar o'tkazilgan. Uning uchlaridan uchburchakning katta tomonigacha bo'lgan masofalar topilsin.

A) 7 va 16; B) 6 va 10; C) 5 va 11; D) 8 va 17;
E) 6 sm va 12 sm.

28. ΔABC — teng yonli va $AC=6$ sm, $AB=BC=5$ sm. Uchburchakka ichki chizilgan aylananing O markazidan uchburchak tekisligiga $OK=2$ sm perpendikulyar o'tkazilgan. *K* nuqtadan uchburchakning tomonlarigacha va *B* uchigacha bo'lgan masofalar topilsin.

A) 3,5 va 4; B) 3 va $\frac{\sqrt{41}}{2}$; C) 2 va $\sqrt{39}$;
D) 1,8 va $\sqrt{41}$; E) 2,5 sm va $0,5\sqrt{41}$ sm.

29. $ABCD$ to'g'ri to'rtburchakning A uchidan uning tekisligiga AK perpendikulyar o'tkazilgan. *K* nuqtadan to'rtburchakning uchlarigacha bo'lgan masofalar $KB=6$ sm, $KC=9$ sm, $KD=7$ sm bo'lsa, KA kesmaning uzunligi topilsin.

A) 6; B) 2; C) 3; D) 4; E) 1,5 sm.

30. P tekislikka parallel AB kesmaning uchlaridan P tekislikka AC perpendikulyar va BD og'ma o'tkazilgan. Agar $AB=a$, $AC=b$ va $BD=c$ bo'lsa, CD kesmaning uzunligi topilsin.

A) $\sqrt{ab+ac+bc}$; B) $\sqrt{a^2+b^2-c}$; C) $\sqrt{a^2+c^2-b^2}$;
D) $\sqrt{b^2+c^2-a^2}$; E) $\sqrt{ab+ac}$.

31. AB va CD — o'zaro kesishgan ikki tekislikdagi parallel kesmalar; AE va DK — tekisliklarning kesishish chizig'iga o'tkazilgan perpendikulyar bo'lsin. Agar $AD=5$ sm, $EK=4$ sm bo'lsa, AB va CD to'g'ri chiziqlar orasidagi masofa topilsin.

A) 3; B) 3,5; C) 2; D) 4; E) 4,5 sm.

32. $ABCD$ trapetsiyaning AD asosi P tekislikda yotadi, BD asosi esa tekislikdan 5 sm uzoqlikdadir. Agar $AD:BC=$ $=7:3$ kabi bo'lsa, shu trapetsiya diagonallarining kesishish nuqtasi M dan P tekislikkacha bo'lgan masofa topilsin.

A) 3; B) 2,5; C) 5; D) 3,5; E) 4 sm.

33. α va β tekisliklar berilgan bo'lib, α tekislikning A va B nuqtalaridan β tekislikka $AC=37$ sm va $BD=125$ sm og'malar o'tkazilgan. Agar AC og'maning β tekislikdagi proyeksiyasi 12 sm bo'lsa, BD kesmaning proyeksiyasi topilsin.

A) 116; B) 120; C) 132; D) 96; E) 105 sm.

34. Yig'indisi c ga teng bo'lgan ikkita kesmaning uchlari ikki parallel tekislikka tiraladi, ularning proyeksiyalari, mos ravishda a va b ga teng. Shu kesmalarning uzunliklari topilsin.

A) $\frac{a+b+c}{a-b}$, $\frac{a^2+b^2-c^2}{a-b}$; B) $\frac{b^2+c^2-a^2}{2a}$, $\frac{a^2+b^2-c^2}{2a}$;

C) $\frac{a^2+b^2-c^2}{2a}$, $\frac{a^2+b^2-c^2}{2b}$; D) $\frac{2a^2+b^2-c^2}{bc}$, $\frac{2b^2+a^2-c^2}{ab}$;

E) $\frac{a^2+c^2-b^2}{2c}$, $\frac{b^2+c^2-a^2}{2c}$.

35. A nuqtadan α tekislikka $AC=6$ sm perpendikulyar va $AD=9$ sm og'ma o'tkazilgan. Perpendikulyarning og'madagi proyeksiyasi topilsin.

A) 4,5; B) 5; C) 4; D) 3; E) 6 sm.

36. P nuqtadan o'tkazilgan ikkita to'g'ri chiziq uchta parallel tekislikni A_1, A_2, A_3 va B_1, B_2, B_3 nuqtalarda kesib o'tadi. Agar $A_1A_2=4$ sm, $B_2B_3=9$ sm, $A_2A_3=B_1B_2$ bo'lsa, A_1A_3 va B_1B_3 kesmalarning uzunliklari topilsin.

A) 10 va 15; B) 9 va 16; C) 8 va 14; D) 12 va 13; E) 11 sm va 16 sm.

37. Uchburchakning tomonlari 51, 30 va 27 sm. Uchburchakning kichik burchagi uchidan uchburchak tekisligiga perpendikulyar o'tkazilgan va uning uzunligi 10 sm. Perpendikulyarning uchlaridan uchburchakning o'sha uchi qarshisidagi tomonigacha bo'lgan masofalar topilsin.

A) 24 va 26; B) 20 va 22; C) 18 va 24; D) 20 va 24; E) 32 sm va 16 sm.

38. Rombning diagonallari 60 va 80 sm. Diagonallarning kesishish nuqtasidan romb tekisligiga uzunligi 45 sm bo'lgan perpendikulyar o'tkazilgan. Perpendikulyarning uchlaridan rombning tomonigacha bo'lgan masofalar topilsin.

A) 60; B) 51; C) 48; D) 36; E) 42 sm.

39. To'g'ri burchakli uchburchakning to'g'ri burchagi uchidan uchburchak tekisligiga uzunligi 16 sm bo'lgan perpendikulyar o'tkazilgan. Uchburchakning katetlari 15 va 20 sm bo'lsa, perpendikulyarning uchlaridan gipotenuzagacha bo'lgan masofalar topilsin.

A) 12 va 21; B) 10 va 18; C) 12 va 20; D) 15 va 18; E) 16 sm va 22 sm.

40. Teng yonli trapetsiyaning asoslari 16 va 30 sm. M nuqta trapetsiyaning har bir tomonidan 11 sm uzoqlikda yotsa, M nuqtadan trapetsiya tekisligigacha bo'lgan masofa topilsin.

A) 3; B) 1,5; C) 2; D) 1; E) 4 sm.

41. K nuqtadan P tekislikka ikkita og'ma o'tkazilgan. Og'malarning har biri P tekislik bilan 45° li burchak tashkil qiladi va K nuqtadan tekislikkacha bo'lgan masofa a ga teng. Og'malarning proeksiyalari orasidagi burchak 120° bo'lsa, og'malarning uchlari orasidagi masofa topilsin.

A) $a\sqrt{2}$; B) $2a$; C) $a\sqrt{5}$; D) $\frac{a\sqrt{2}}{2}$; E) $a\sqrt{3}$.

42. To'g'ri burchakli teng yonli uchburchakning kateti orqali tekislik o'tkazilgan. Uchburchakning ikkinchi kateti shu tekislikka 45° li burchak ostida og'madan iborat bo'lsa, uchburchakning bissektrisasi va tekislik orasidagi burchak topilsin.

A) 30°; B) 45°; C) 60°; D) 15°; E) 75°.

43. 60° li ikki yoqli burchakning bitta tomonida M nuqta olingan va M nuqtadan ikki yoqli burchakning ikkinchi tomonigacha bo'lgan masofa c ga teng. M nuqtadan ikki yoqli burchakning qirrasigacha bo'lgan masofa topilsin.

A) $\frac{c\sqrt{3}}{2}$; B) $\frac{2c\sqrt{3}}{3}$; C) $2c\sqrt{3}$; D) $\frac{2c\sqrt{2}}{3}$; E) $\frac{c\sqrt{2}}{2}$.

44. 120° li ikki yoqli burchakning ichki qismida M . uqta olingan bo'lib, M nuqtadan ikki yoqli burchakning har bir tomonigacha bo'lgan masofalar p ga teng. M nuqtadan ikki yoqli burchakning qirrasigacha bo'lgan masofa topilsin.

A) $\frac{p^2\sqrt{15}}{4}$; B) $\frac{2p\sqrt{2}}{3}$; C) $\frac{2p\sqrt{3}}{3}$; D) $\frac{p^2\sqrt{2}}{4}$; E) $\frac{2p^2}{3}$.

45. Ikkita teng yonli uchburchak umumiy asosga ega bo'lib, ularning tekisliklari orasidagi burchak 60°. Umumiy asosning uzunligi 12 sm, bitta uchburchakning yon tomoni 10 sm, ikkinchi uchburchakning yon tomonlari o'zaro

perpendikulyardir. Uchburchaklarning uchlari orasidagi masofa topilsin.

A) $5\sqrt{12}$; B) $4\sqrt{3}$; C) 12; D) $2\sqrt{13}$; E) $2\sqrt{15}$ sm.

46. AB kesmaning uchlari o'zaro perpendikulyar bo'lgan ikkita tekislikda joylashgan. A va B nuqtalardan tekisliklarning kesishish chiziqlariga $AD=a$ va $CB=b$ perpendikulyarlar o'tkazilgan. Agar $BD=c$ bo'lsa, AC kesma va uning proyeksiyalari uzunliklari topilsin.

A) $\sqrt{2(a^2+c^2-b^2)}, \sqrt{a^2}-c^2, \sqrt{d^2-c^2}$;

B) $\sqrt{2(a^2+b^2)-bc}, a\sqrt{2}, bc\sqrt{2}$;

C) $\sqrt{(a^2+b^2-c^2)}, \sqrt{a^2-c^2}, \sqrt{b^2-c^2}$;

D) $\sqrt{abc}, \sqrt{a^2+c^2}, \sqrt{b^2+c^2}$;

E) $\sqrt{(a^2+b^2+c^2)}, \sqrt{a^2+c^2}, \sqrt{b^2+c}$.

47. ΔABC ning tomonlari 13, 14 va 15 sm. Uchburchakning bitta tomoni orqali uchburchak tekisligi bilan 30° li burchak hosil qiluvchi tekislik o'tkazilgan. Uchburchakning shu tekislikdagi proyeksiyasi yuzi hisoblansin.

A) $42\sqrt{3}$; B) 42; C) $48\sqrt{3}$; D) 48; E) $44\sqrt{2}$ sm^2.

48. Yassi ko'pburchak proeksiyasining yuzi 20 sm^2, ko'pburchak va proyeksiyaning tekisliklari orasidagi burchak 45° bo'lsa, ko'pburchakning yuzi hisoblansin.

A) 20; B) $20\sqrt{2}$; C) $16\sqrt{2}$; D) $24\sqrt{2}$; E) 24 sm^2.

49. ΔABC — to'g'ri burchakli bo'lib, uning gipotenuzasi 12 sm. Fazodagi P nuqta ΔABC ning uchlaridan bir xil — 10 sm uzoqlikda yotsa, P nuqtadan (ABC) tekislikkacha bo'lgan masofa topilsin.

A) 7; B) 6; C) 8; D) $8\sqrt{2}$; E) 9 sm.

50. Teng yonli ΔABC ning AC asosi α tekislikda yotadi, B uchi esa α tekislikdan 3 sm uzoqlikda joylashgan. Agar uchburchakning asosi $AC=18$ sm, uchburchak tekisligi va α tekislik orasidagi burchak 45° bo'lsa, ΔABC ning yuzi hisoblansin.

A) 64; B) 48; C) 36; D) 54; E) 60 sm^2.

51. Uchlari ikkita parallel tekislikda joylashgan kesmalar uzunliklarining nisbati 2:3 kabi. Kesmalarning tekisliklar bilan tashkil qilgan burchaklari o'lchovlarining nisbati 2:1 kabi bo'lsa, shu burchaklar topilsin.

A) $\frac{\pi}{4}$ va 2 arcsin $\frac{2}{3}$; B) arctg 3 va 2α; C) arccos $\frac{5}{6}$ va $\frac{3\alpha}{2}$;
D) 2 arcsin $\frac{4}{5}$ va 3α; E) $\alpha=2\arccos\frac{3}{4}$ va 2α.

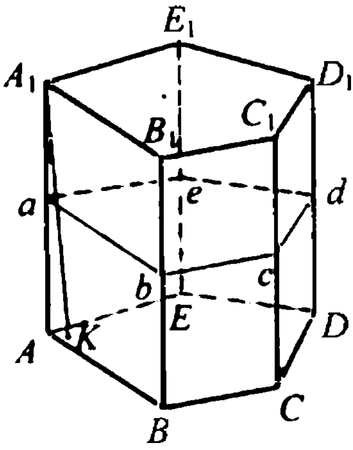
9-§. PRIZMA

9.1. Asosiy tushunchalar va tasdiqlar

Ko'pyoq chekli sondagi tekisliklar bilan chegaralangan jism bo'lib, ko'pyoqning chegarasi uning sirtidan iborat. Ko'pyoq o'zini chegaralovchi tekisliklarning har biridan bir tomonda yotsa, u *qavariqdir*. Ko'pyoqning sirti bilan uni chegaralab turgan tekislikning umumiy qismi uning yog'i, ko'pyoq *yoqlarining tomonlari* — *qirralari*, ko'pyoq yoqlarining uchlari — ko'pyoqning *uchlaridir*. Masalan, kub qavariq ko'pyoqdir, uning sirti oltita kvadratdan — yoqlardan tashkil topgan. Bu kvadratlarning tomonlari kubning qirralari, uchlari esa kubning uchlaridir. Kubda oltita yoq, o'n ikkita qirra va sakkizta uch bor.

Prizma ikkita parallel tekislik orasiga joylashgan barcha parallel to'g'ri chiziqlar kesmalaridan tuzilgan ko'pyoq bo'lib, bu kesmalar shu tekisliklardan birida yotgan yassi ko'pbuchakni kesib o'tadi. Prizmaning parallel tekisliklarda yotgan yoqlari — ko'pburchaklar uning *asoslari*, qolganlari uning *yon yoqlari* bo'ladi. Demak, prizmaning yon yoqlari parallelogrammlardir.

To'g'ri prizma yon qirralari asosiga perpendikulyar bo'lgan prizmadir. Aks holda prizma og'ma prizmadan iborat bo'ladi.



9.1-chizma.

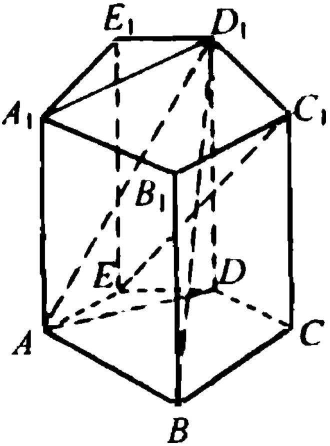
9.1-chizmada $ABCDEA_1B_1C_1D_1E_1$ prizma keltirilgan.

Prizmaning AA_1 qirrasida ixtiyoriy a nuqtani olamiz va bu nuqtadan AA_1 qirraga perpendikulyar tekislik o'tkazsak, tekislik prizmaning sirtini $abcde$ ko'pburchak bo'ylab kesadi. Bu kesim prizmaning *perpendikulyar kesimi* va uning *ab, bc, cd, de, ea* tomonlari prizmaning yon qirralariga perpendikulyar bo'ladi.

Prizmaning asoslari *yuqori* (ustki) va *quyi* (pastki) *asoslar* deyilib, yuqori asosning ixtiyoriy nuqtasidan pastki asosga o'tkazilgan A_1K perpendikulyar (9.1-chizma) prizmaning *balandligidir*.

Muntazam prizma asoslari muntazam ko'pburchaklar bo'lgan to'g'ri prizmadir. To'g'ri va muntazam prizmalarning balandliklari ularning yon qirralariga tengdir.

Prizmaning diagonali uning bitta yog'iga tegishli bo'lmagan ikkita uchini birlashtiruvchi kesmadir (9.2-chizmada AD, AD_1, BD_1, ...). Quyi va yuqori asoslarning mos AD va A_1D_1 diagonallarini o'tkazamiz (9.2-chizma).



9.2-chizma.

Ulardan o'tuvchi kesim prizmaning diagonal kesimidir (9.2-chizma ADD_1A_1, AA_1C_1C, ...). To'g'ri va muntazam prizmalarning diagonal kesimlari to'g'ri to'rtburchaklar, og'ma prizmada esa parallelogrammlar bo'ladi.

Prizma *yon sirtining yuzi* deb uning yon yoqlari yuzlari yig'indisiga, *to'la sirtining* yuzi deb prizma yon sirtining yuzi bilan uning asoslari yuzlarining yig'indisiga aytiladi.

Quyidagi tasdiqlar o'rinli.

1. Prizma yon sirtining yuzi uning perpendikulyar kesimi bilan yon qirrasining ko'paytmasiga teng:

$$S_{yon} = P_{perp.kes} \cdot l, \text{ bunda } l = AA_1. \qquad (9.1)$$

To'g'ri prizmaning yon sirti asosining perimetri P_{ac} bilan yon qirrasi uzunligi l ning ko'paytmasiga teng:

$$S_{yon} = P_{ac} \cdot l.$$

2. Prizma to'la sirtining yuzi

$$S_t = S_{yon} + 2S_{asos} \qquad (9.2)$$

formula orqali hisoblanadi.

3. Prizmaning hajmi uning asosi yuzi bilan balandligi ko'paytmasiga teng:

$$V = S_{asos} \cdot h, \text{ bunda } h = A_1K. \qquad (9.3)$$

4. Og'ma prizma shunday prizmaga tengdoshki, uning asosi og'ma prizmaning perpendikulyar kesimiga, balandligi esa og'ma prizmaning yon qirrasiga tengdir.

9.2. Mavzuga oid masalalar

1. To'rtburchakli muntazam prizma asosining diagonali 8 sm, yon yog'ining diagonali 7 sm bo'lsa, uning diagonali topilsin.

A) 6; B) 11; C) 9; D) 8; E) 10 sm.

2. Uchburchakli muntazam prizma asosining tomoni a, yon qirrasi b ga teng. Prizma quyi asosining tomoni va qarama-qarshi yon qirrasining o'rtasidan o'tkazilgan kesimning yuzi topilsin.

A) $\frac{a}{2}\sqrt{3a^2+b^2}$; B) $\frac{a}{4}\sqrt{3a^2+b^2}$; C) $a\sqrt{a^2+3b^2}$;

D) $\frac{a}{2}\sqrt{a^2+2b^2}$; E) $\frac{a}{3}\sqrt{3a^2+b^2}$.

3. To'rtburchakli muntazam prizmaning diagonali yon yoq bilan 30° li burchak tashkil qiladi. Shu diagonal va asos tekisligi orasidagi burchak topilsin.

A) 55°; B) 60°; C) 35°; D) 70°; E) 45°.

4. Uchburchakli og'ma prizmaning yon qirralari 8 sm dan, prizma perpendikulyar kesimining tomonlari 9:10:17 kabi nisbatda bo'lib, uning yuzi 144 sm^2 ga teng bo'lsa, prizma yon sirtining yuzi topilsin.

A) 456; B) 544; C) 525; D) 576; E) 624 sm^2.

5. Prizmaning asosi kvadratdan iborat bo'lib, yuqori asosning bitta uchi pastki asosning uchlaridan bir xil masofada joylashgan. Agar prizma asosining tomoni a, yon qirrasi b ga teng bo'lsa, prizma to'la sirtining yuzi topilsin.

A) $2a\sqrt{4b^2-a^2}+2a^2$; B) $a\sqrt{2b^2-a^2}+2a^2$;

C) $a\sqrt{4b^2-a^2}+a^2$; D) $2a\sqrt{2b^2-a^2}$.

6. Uchburchakli og'ma prizmaning yon qirralari 15 sm, ular orasidagi masofalar 26, 25, 17 sm bo'lsa, prizmaning hajmi hisoblansin.

A) 3060; B) 3025; C) 3225; D) 3100; E) 3200 sm^2.

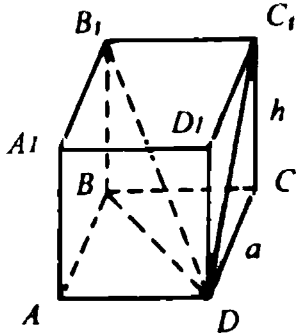
7. Oltiburchakli muntazam prizma eng katta diagonal kesimining yuzi Q, prizma qarama-qarshi yon yoqlari orasidagi masofa b bo'lsa, prizmaning hajmi hisoblansin.

A) $\frac{2bQ}{3}$; B) $\frac{3bQ}{2}$; C) $\frac{3bQ}{4}$; D) $\frac{4bQ}{3}$; E) $\frac{bQ}{2}$.

9.3. Mavzuga oid masalalarning yechimlari

1. Berilgan. $ABCDA_1B_1C_1D_1$ muntazam to'rtburchakli prizma, $BD=8$ sm, $DC_1=7$ sm, $ABCD$ kvadrat.

B_1D topilsin (9.3.1-chizma).



9.3.1-chizma.

Yechilishi. Asosdagi $ABCD$ kvadratning tomonini a bilan, prizmaning yon qirrasini $AA_1=h$ deb belgilaymiz. So'ngra ABD, DCC_1, BB_1D to'g'ri burchakli uchburchaklardan Pifagor teoremasiga (2-§) asosan quyidagilarni topamiz:

ΔABD dan: $BD^2=a^2+a^2$; $2a^2=8^2$; $a^2=32$;

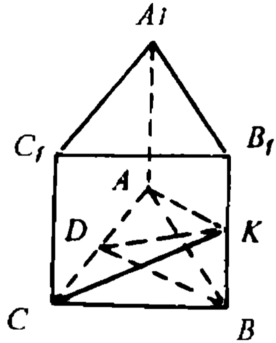
ΔDCC_1 dan: $C_1D^2=h^2+a^2$; $h^2=7^2-32=17$;

ΔBB_1D dan: $B_1D^2=h^2+BD^2=17+64=81$, $B_1D=9$ sm.

Javobi: C).

2. B e r i l g a n. $ABCA_1B_1C_1$ muntazam prizma, $AB=a$; $AA_1=b$, $BK=KB_1$.

S_{AKC} h i s o b l a n s i n (9.3.2-chizma).



9.3.2-chizma.

Y e c h i l i s h i. Prizmaning muntazamligidan $AK=KC$ bo'ladi, ya'ni ΔAKC — teng yonli, uning KD medianasi balandlik ham bo'ladi, natijada kesimning yuzi

$$S_{AKC}=\frac{1}{2}AC\cdot KD$$

formuladan topiladi. Uch perpendikulyar haqidagi teoremaga asosan (8-§), $BD\perp AC$. ΔABC dan

$$BD=BC\cdot\sin 60^{\circ}=\frac{a\sqrt{3}}{2}.$$

Berilganiga muvofiq, $BK=\frac{1}{2}BB_1=\frac{b}{2}$. U holda ΔBDK dan $DK^2=BD^2+BK^2$ ifodani olamiz, ya'ni

$$DK^2=\left(\frac{a\sqrt{3}}{2}\right)^2+\left(\frac{b}{2}\right)^2=\frac{3a^2}{4}+\frac{b^2}{4}=\frac{1}{4}(3a^2+b^2),$$

bu yerdan

$$DK=\frac{1}{2}\sqrt{3a^2+b^2}.$$

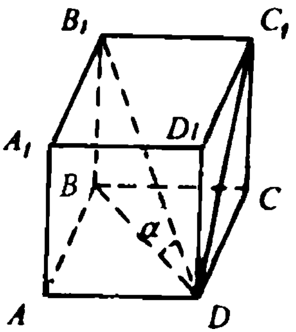
Demak, kesim yuzi:

$$S_{AKC}=\frac{1}{2}a\frac{1}{2}\sqrt{3a^2+b^2}=\frac{a}{4}\sqrt{3a^2+b^2}.$$

Javobi: B).

3. Berilgan. $ABCDA_1B_1C_1D_1$ — muntazam prizma, $ABCD$ — kvadrat, $\angle B_1D_1C_1=30°$.

$\angle B_1DB$ topilsin (9.3.3-chizma).



9.3.3-chizma.

Yechilishi. To'g'ri chiziq va tekislik orasidagi burchakni yasash uchun B_1 nuqtadan yon yoqqa va asosga perpendikulyarlar o'tkazish kerak. Prizma muntazam bo'lganligidan, $B_1C_1\perp C_1D_1$ va $B_1B\perp(ABCD)$. To'g'ri chiziq va tekislikning perpendikulyarlik alomatiga ko'ra (8-§), $B_1B\perp BD$ bo'ladi. Shu sababli, ta'rifga ko'ra, $\angle B_1DC_1=30°$ diagonal va yon yoq orasidagi burchakdan, $\angle B_1DB$ esa diagonal va asos tekisligi orasidagi burchakdan iborat bo'ladi.

Faraz qilaylik, $AB=a$ bo'lsin. U holda to'g'ri burchakli ΔDB_1C_1 dan: $B_1D=\dfrac{B_1C_1}{\sin 30°}=2a$ va ΔABD dan $BD=\sqrt{a^2+a^2}=a\sqrt{2}$.

Endi to'g'ri burchakli ΔBB_1D dan

$\cos\alpha=\dfrac{BD}{B_1D}=\dfrac{a\sqrt{2}}{2a}=\dfrac{\sqrt{2}}{2}$; demak, $\alpha=45°$.

Javobi: E).

4. Berilgan. $ABCA_1B_1C_1$ og'ma prizma, (abc) perpend. kesim, $AA_1=8$ sm, $ab:bc:ac=9:10:17$, $S_{pk}=144$ sm^2.

$S_{yon\,s}$ hisoblansin (9.3.4-chizma).

Yechilishi. Perpendikulyar kesim tomonlarining nisbati ma'lum bo'lganligidan, ularni quyidagicha yozib olamiz: $ab=9x$, $bc=10x$, $ac=17x$. Geron formulasi (2-§) yordamida perpendikulyar kesim — abc ning yuzini x orqali ifodalaymiz:

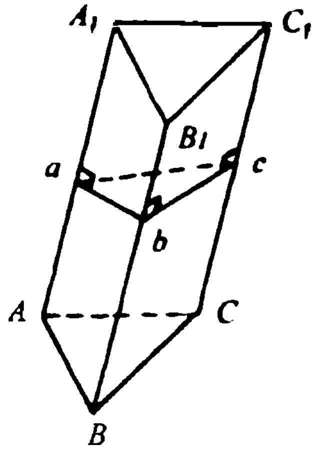
$$p=\frac{9x+10x+17x}{2}=18x,$$

$$S_{pk}=S_{abc}=\sqrt{18x\cdot 9x\cdot 8x\cdot x}=36x^2$$

Berilganlarni hisobga olsak, $36x^2=144$ sm^2, $x^2=4$, $x=2$ sm.

Demak, $ab=18$, $bc=20$, $ac=34$ sm va prizmaning yon sirti

$$S_{yon}=(18+20+34)\cdot 8=576 \text{ sm}^2.$$



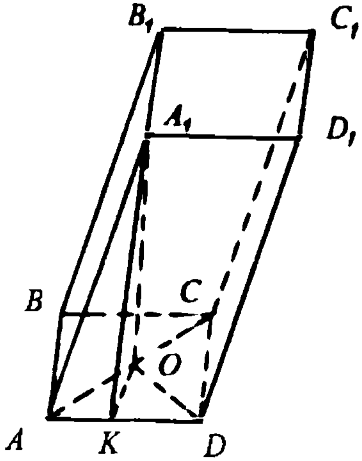
9.3.4-chizma.

Javobi: D).

5. B e r i l g a n. $ABCDA_1B_1C_1D_1$ prizma, $ABCD$ kvadrat, $A_1A=A_1B=A_1C=A_1D$, $AB=a$, $AA_1=b$.

$S_{t.s}$ h i s o b l a n s i n (9.3.5-chizma).

Y e c h i l i s h i. A_1 nuqta kvadratning uchlaridan bir xil masofada bo'lganligidan AC diagonalning o'rtasidagi O nuqta kvadratga tashqi chizilgan aylananing markazi yoki kvadrat diagonallarining kesishish nuqtasi bo'ladi. Demak, $ABCD$ kvadratning diagonali



9.3.5-chizma.

$$AC = \sqrt{a^2 + a^2} = a\sqrt{2} \text{ va } AO = \frac{1}{2} AC = \frac{a\sqrt{2}}{2}.$$

Endi A_1 nuqtadan AD tomonga A_1K perpendikulyar o'tkazamiz. O nuqta kvadratning markazi bo'lganligidan, $OK = \frac{a}{2}$ va uch perpendikulyar haqidagi teoremaga asosan (8-§) $OK \perp AD$ va $AK = \frac{a}{2}$. To'g'ri burchakli ΔAA_1K dan Pifagor teoremasiga ko'ra

$$A_1K = \sqrt{AA_1^2 - AK^2} = \sqrt{b^2 - \frac{a^2}{4}} = \frac{1}{2}\sqrt{4b^2 - a^2}$$

bo'ladi. U holda,

$$S_{\text{yon.s.}} = R_{\text{asos}} \cdot AK = 4a\frac{1}{2}\sqrt{4b^2 - a^2} = 2a\sqrt{4b^2 - a^2}$$

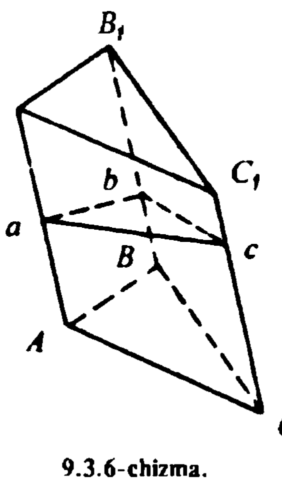
Prizma asosining yuzi $S_{\text{asos}} = a^2$. Prizmaning to'la sirtini hisoblaymiz:

$$S_{\text{t.s}} = S_{\text{yon s}} + 2S_{\text{asos}} = 2a\sqrt{4b^2 - a^2} + 2a^2.$$

Javobi: A).

6. Berilgan. $ABCA_1B_1C_1$ og'ma prizma, $AA_1 = 15$ sm, $AA_1 \perp (ABC)$, $ab = 25$ sm, $ac = 26$ sm, $bc = 17$ sm.

V_{prizma} hisoblansin (9.3.6-chizma).



9.3.6-chizma.

Yechilishi. 4-tasdiqqa ko'ra $V_{\text{prizma}} = S_{abc} \cdot AA_1$ bo'lishi kerak, bu yerda S_{abc} — perpendikulyar kesimning yuzi. Perpendikulyar kesimning yuzini Geron formulasidan foydalanib topamiz:

$$p = \frac{ab+ac+bc}{2} = \frac{25+26+17}{2} = 34 \text{ sm},$$

demak,

$$S_{abc} = \sqrt{34 \cdot 9 \cdot 8 \cdot 17} = 17 \cdot 4 \cdot 3 = 204 \text{ sm}^2.$$

Endi prizmaning hajmi $V_{prizma} = 204 \cdot 15 = 3060$ sm².

Javobi: A)

7. B e r i l g a n. $ABCDEFA_1B_1D_1E_1F_1$ oltiburchakli muntazam prizma, $S_{AA_1D_1D} = Q$, $A_1C_1 = b$.

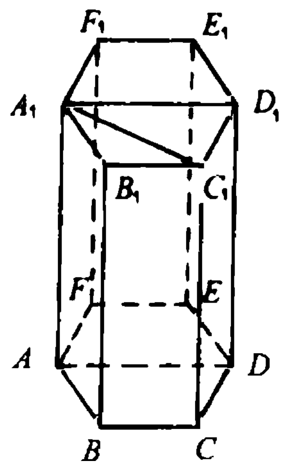
V_{prizma} h i s o b l a n s i n (9.3.7-chizma).

Y e c h i l i s h i. Eng katta diagonal kesim muntazam oltiburchakning markazidan oʻtadigan kesimdir. Qarama-qarshi yon yoqlar oʻzaro parallel boʻlganligidan, ular orasidagi masofa $AC=b$ diagonalning uzunligiga teng. Prizmaning hajmi

$$V = S_{asos} \cdot H, \; H = AA_1$$

formula boʻyicha hisoblanadi. Agar muntazam oltiburchakning tomoni $AB=a$ boʻlsa, uning yuzi

$$S_{asos} = 6 \cdot S_{\Delta AOB} = 6\frac{a^2\sqrt{3}}{4} = \frac{3a^2\sqrt{3}}{2},$$



9.3.7-chizma.

katta diagonal kesimning yuzi

$$S_{kesim} = AD \cdot AA_1 = 2aH, \; AD = 2a.$$

U holda prizmaning hajmi

$$V = \frac{3a^2\sqrt{3}}{2} \cdot H$$

bo'ladi. Masalaning berilishiga ko'ra, $Q=2a\cdot H$ tenglikdan foydalansak, $V=\frac{3a\sqrt{3}}{4}Q$.

Endi $AB=a$ ning qiymatini topish uchun ΔABC ga kosinuslar teoremasini (2-§) qo'llaymiz:

$$AC^2=AB^2+BC^2-2AB\cdot BC\cdot\cos120^\circ$$

(chunki muntazam oltiburchakning ichki burchagi 120° ga teng, $\angle ABC=120^\circ$), ya'ni

$$b^2=a^2+a^2-2a^2\cos120^\circ=2a^2(1-\cos120^\circ)=4a^2\sin^2 60^\circ=3a^2;$$

$$a^2=\frac{1}{3}b^2; a=\frac{b}{\sqrt{3}}.$$

Demak, prizmaning hajmi

$$V=\frac{b}{\sqrt{3}}\cdot\frac{3\sqrt{3}}{4}Q=\frac{3bQ}{4}$$

Javobi: C).

9.4. Mustaqil yechish uchun masalalar

1. Uchburchakli to'g'ri prizmaning yon qirrasi 15 sm, asosining tomonlari 25, 39 va 40 sm. Yon qirra va asosning o'rta balandligi orqali o'tkazilgan kesimning yuzi hisoblansin.

A) 325; B) 360; C) 380; D) 350; E) 240 sm^2.

2. Oltiburchakli muntazam prizmaning diagonallari 15 sm va 17 sm. Uning diagonali kesimlarining yuzlari hisoblansin.

A) $16\sqrt{33}$; $24\sqrt{11}$; B) $18\sqrt{29}$; $24\sqrt{7}$; C) $16\sqrt{23}$; $18\sqrt{7}$; D) $17\sqrt{35}$; $13\sqrt{29}$; E) $16\sqrt{11}$; $24\sqrt{33}$.

3. To'rtburchakli muntazam prizma diagonali kesimining yuzi S bo'lsa, prizma yon sirtining yuzi hisoblansin.

A) $2\sqrt{3}S$; B) $3\sqrt{2}S$; C) $2\sqrt{2}S$; D) $4\sqrt{3}S$; E) $5\sqrt{2}S$.

4. To'g'ri prizmaning asosi rombdan iborat bo'lib, prizmaning diagonallari 8 sm va 5 sm, balandligi 2 sm. Prizma asosining tomoni uzunligi hisoblansin.

A) 4; B) 5,5; C) 4,8; D) 5; E) 4,5 sm.

5. Uchburchakli og'ma prizmaning yon qirralari orasidagi masofalar mos ravishda 37, 13 va 40 sm. Prizmaning katta yon yog'i bilan uning qarshisidagi yon qirra orasidagi masofa topilsin.

A) 11; B) 12; C) 13; D) 14; E) 10 sm.

6. To'rtburchakli muntazam prizmaning diagonali 14 sm, yon yog'ining diagonali 10 sm bo'lsa, prizmaning yon sirti topilsin.

A) $32\sqrt{3}$; B) $36\sqrt{2}$; C) $36\sqrt{3}$; D) $23\sqrt{6}$; E) $32\sqrt{2}$ sm².

7. Uchburchakli muntazam $ABCA_1B_1C_1$ prizmaning balandligi 6 dm ga, uning asosi va A_1BC orasidagi burchak 45° ga teng bo'lsa, prizmaning to'la sirti topilsin.

A) $96\sqrt{3}$; B) 72; C) 96; D) $84\sqrt{2}$; E) 80 dm².

8. To'g'ri prizmaning asosi trapetsiyadan iborat bo'lib, uning perimetri 58 sm ga teng. Prizmaning parallel yon yoqlarining yuzlari 96 sm² va 264 sm², boshqa yon yoqlarining yuzlari 156 sm² va 180 sm² bo'lsa, prizmaning hajmi hisoblansin.

A) 2100; B) 1840; C) 2240; D) 2160; E) 1960 sm³.

9. Prizmaning asosi ΔABC da $AC=2$ dm, $AB=BC=$ $=3$ dm. Prizmaning yon qirrasi 6 dm ga teng va asos tekisligi bilan 30° li burchak tashkil qilsa, prizmaning hajmi hisoblansin.

A) 7,6; B) 6,5; C) $7\sqrt{2}$ D) 8; E) $6\sqrt{2}$ dm³.

10. To'g'ri prizmaning yon sirti S ga teng bo'lib, asosi teng yonli uchburchakdan iborat. Uchburchakning teng tomonlari a ga, ular orasidagi burchak α ga teng bo'lsa, prizmaning hajmi hisoblansin.

A) $\frac{1}{2}a^3 \operatorname{tg}\frac{\pi-\alpha}{8}$; B) $\frac{aS}{2}\sin\frac{\alpha}{2}\operatorname{tg}\frac{\pi-\alpha}{4}$; C) $a^2\sqrt{S}\cos\alpha\operatorname{tg}\frac{\alpha}{2}$;
D) $\frac{1}{6}a^2\sqrt{S}\sin\frac{\alpha}{2}\operatorname{tg}\alpha$; E) $\frac{1}{3}aS\frac{\alpha}{2}\operatorname{ctg}\frac{\alpha}{4}$.

11. Oltiburchakli muntazam prizma asosining tomoni a, yon yoqlari esa kvadratlardan iborat bo'lsa, prizmaning diagonallari topilsin.

A) $a\sqrt{6}$ va $a\sqrt{2}$; B) $2a$ va $a\sqrt{5}$; C) $a\sqrt{3}$ va $a\sqrt{5}$;
D) $3a$ va $a\sqrt{3}$; E) $2a$ va $a\sqrt{2}$.

12. Oltiburchakli muntazam prizmaning asosiga tashqi chizilgan aylananing radiusi R, yon yoqlari esa kvadratlardan iborat bo'lsa, diagonal kesimlarning yuzlari hisoblansin.

A) $R^2\sqrt{7}$ va $R^2\sqrt{3}$; B) $R^2\sqrt{6}$ va $2R^2$; C) $R^2\sqrt{3}$ va $2R^2$;
D) $R^2\sqrt{2}$ va $R^2\sqrt{3}$; E) R^2 va $2R^2$.

13. Uchburchakli muntazam prizmaning hamma qirralari o'zaro teng va ularning uzunligi a bo'lib, pastki asos tomonidan va prizma o'qining o'rtasidan tekislik o'tkazilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) $\frac{4a^2\sqrt{3}}{9}$; B) $\frac{a^2\sqrt{3}}{9}$; C) $\frac{9a^2}{4}$; D) $\frac{3a^2}{4}$; E) $\frac{4a^2\sqrt{3}}{9}$.

14. Uchburchakli to'g'ri prizma asosining bir tomoni orqali qarshidagi yon qirrani kesuvchi va asos tekisligiga 45° og'ma bo'lgan tekislik o'tkazilgan. Prizma asosining yuzi P bo'lsa, kesimning yuzi hisoblansin.

A) $3P$; B) $R\sqrt{6}$; C) $2P$; D) $P\sqrt{2}$; E) $P\sqrt{3}$.

15. Uchburchakli to'g'ri prizma asosining tomonlari 10 sm, 17 sm va 21 sm, prizmaning balandligi 18 sm.

Prizmaning yon qirrasi va asosning kichik balandligi orqali tekislik o'tkazilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) 156; B) 172; C) 144; D) 168; E) 162 sm^2.

16. To'g'ri prizmaning asosi — romb, diagonallari 8 sm va 5 sm, balandligi 2 sm bo'lsa, prizma asosining perimetri topilsin.

A) 15; B) 18; C) 16; D) 24; E) 20 sm.

17. Uchburchakli og'ma prizmaning yon qirralari orasidagi masofalar, mos ravishda 37, 13 va 40 sm. Prizmaning katta yon yog'i bilan uning qarshisidagi yon qirrasi orasidagi masofa topilsin.

A) 12; B) 14; C) 10; D) 13; E) 15 sm.

18. Og'ma prizmaning yon qirrasi l ga teng va asos tekisligi bilan α burchak tashkil qiladi. Prizmaning balandligi topilsin.

A) $\frac{l}{\cos\alpha}$; B) $\sqrt{l^2 + \mathrm{tg}^2\alpha}$; C) $l\,\mathrm{tg}\alpha$; D) $\sqrt{l\sin\alpha}$;

E) $l\sin\alpha$.

19. Oltiburchakli muntazam prizma nechta diagonal kesimga ega?

A) 5 ta; B) 3 ta; C) 9 ta; D) 8 ta; E) 6 ta.

20. n-burchakli muntazam prizma nechta diagonal kesimga ega?

A) $\frac{1}{3}n(n+1)$; B) $\frac{1}{2}n(n-3)$; C) $\frac{1}{3}n(n-2)$;

D) $\frac{1}{2}n(n-1)$; E) $\frac{1}{2}n(n+1)$.

21. Uchburchakli muntazam prizma asosining tomoni a, yon qirrasi b ga teng. Asosning tomoni va unga qarama-

qarshi yon qirraning o'rtasidan o'tkazilgan kesimning yuzi hisoblansin.

A) $\frac{a\sqrt{3a^2+5b^2}}{8}$; B) $\frac{b\sqrt{a^2+b^2}}{4}$; C) $\frac{a\sqrt{3a^2+b^2}}{4}$;
D) $\frac{a\sqrt{3a^2+b^2}}{2}$; E) $\frac{b\sqrt{3a^2+b^2}}{2}$.

22. Uchburchakli muntazam prizmaning har bir qirrasi a ga teng. Uning quyi asosi tomoni va yuqori asosining o'rta chizig'idan tekislik o'tkazilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) $\frac{2a^2\sqrt{3}}{9}$; B) $\frac{a^2\sqrt{2}}{4}$; C) $\frac{3a^2\sqrt{2}}{4}$; D) $\frac{a^2\sqrt{15}}{4}$; E) $\frac{3a^2\sqrt{19}}{16}$.

23. To'rtburchakli muntazam prizma asosining diagonali d ga teng, asosning diagonali va ikkinchi asosning uchidan kesim o'tkazilgan bo'lib, u asos tekisligi bilan α o'tkir burchak tashkil qiladi. Kesimning yuzi hisoblansin.

A) $\frac{d^2}{4\cos\alpha}$; V) $\frac{d^2}{4\sin\alpha}$; C) $\frac{d^2}{2\cos\alpha}$; D) $\frac{1}{2}d^2\sin\alpha$;
E) $\frac{1}{4}d^2\cos\alpha$.

24. To'g'ri prizmaning asosi teng yonli trapetsiya bo'lib, trapetsiyaning asoslari 25 va 9 sm, balandligi esa 8 sm. Prizmaning yon qirralaridagi ikkiyoqli burchaklarning kattaliklari topilsin.

A) 60° va 120°; B) 45° va 135°; C) 30° va 150°;
D) 90° va 90°; E) 80° va 110°.

25. To'rtburchakli muntazam prizmaning diagonali yon yoq tekisligi bilan 30° li burchak tashkil qiladi. Ushbu diagonal va asos tekisligi orasidagi burchak topilsin.

A) 60°; B) 75°; C) 90°; D) 45°; E) 30°.

26. To'rtburchakli muntazam prizma asosining diagonali orqali prizmaning diagonaliga parallel tekislik o'tka-

zilgan. Agar prizma asosining tomoni 2 sm, prizmaning balandligi 4 sm bo'lsa, hosil qilingan kesimning yuzi hisoblansin.

A) $4\sqrt{2}$; B) $3\sqrt{2}$; C) $2\sqrt{3}$; D) $4\sqrt{3}$; E) $6\sqrt{3}$ sm².

27. $ABCA_1B_1C_1$ og'ma prizmaning asosi teng yonli uchburchak bo'lib, uning tomonlari $AC=AB=13$ sm, $BC=10$ sm, prizmaning yon qirrasi esa asos tekisligi bilan 45° li burchak tashkil qiladi. Prizma yuqori asosining A_1 uchi pastki asosning markaziga proyeksiyalanadi. CC_1B_1B yoqning yuzi hisoblansin.

A) 80; V) $80\sqrt{2}$; C) $80\sqrt{3}$; D) $40\sqrt{2}$; E) $60\sqrt{2}$ sm.

28. $ABCA_1B_1C_1$ to'g'ri prizmaning asosi to'g'ri burchakli uchburchak bo'lib, $\angle B=90°$. Prizmaning BB_1 qirrasidan AA_1C_1C tekislikka perpendikulyar tekislik o'tkazilgan. Agar $AA_1=10$ sm, $AD=27$ sm, $DC=12$ sm bo'lsa, hosil qilingan kesimning yuzi hisoblansin.

A) 225; B) 196; C) 240; D) 180; E) 214 sm².

29. Asosning tomoni a, yon qirrasi b ga teng bo'lgan uchburchakli muntazam prizma to'la sirtining yuzi hisoblansin.

A) $\frac{ab}{2}+b^2\sqrt{3}$; B) $ab+\frac{b\sqrt{5}}{4}$; C) $3ab+\frac{a^2\sqrt{3}}{2}$;

D) $2ab+\frac{b^2\sqrt{3}}{2}$; E) $4ab+\frac{ab}{\sqrt{2}}$.

30. Asosning tomoni a, yon qirrasi b ga teng bo'lgan to'rtburchakli muntazam prizma to'la sirtining yuzi hisoblansin.

A) $4ab+2a^2$; B) $2ab+4a^2$; C) $3ab+5a^2$; D) $5ab+2a^2$;
E) $6\,ab$.

31. Asosning tomoni a, yon qirrasi b ga teng bo'lgan oltiburchakli muntazam prizma to'la sirtining yuzi hisoblansin.

A) $4ab+2a^2\sqrt{3}$; B) $6ab+3a^2\sqrt{3}$; C) $4ab+3a^2\sqrt{2}$;
D) $4ab+3a^2\sqrt{2}$; E) $6ab+3a^2\sqrt{5}$.

32. Uchburchakli muntazam prizma asosining bir tomoni va uning qarshisidagi qirraning o'rtasidan o'tgan tekislik asos bilan 45° li burchak tashkil qiladi. Prizma asosining tomoni l ga teng bo'lsa, prizma yon sirtining yuzi hisoblansin.

A) $l^2\sqrt{21}$; B) $l^2\sqrt{15}$; C) $2l^2\sqrt{2}$; D) $2l^2\sqrt{3}$; E) $3l^2\sqrt{3}$.

33. Uchburchakli to'g'ri prizma asosining tomonlari 25, 29 va 36 dm ga teng. Agar prizma to'la sirtining yuzi 1620 dm² bo'lsa, prizma yon sirtining yuzi va balandligi topilsin.

A) 25 va 4; B) 16 va 2; C) 9 va 1; D) 12 m² va 4 m.

34. Uchburchakli to'g'ri prizma asosi tomonlarining nisbati 17:10:9 kabi, yon qirrasi 16 sm, to'la sirtining yuzi 1440 sm² bo'lsa, prizma asosining tomonlari topilsin.

A) 17, 10, 9; B) 34, 20, 18; C) 51, 30, 27;
D) 48, 50, 26; E) 39 sm, 26 sm, 24 sm.

35. To'g'ri prizmaning asosi $ABCD$ teng yonli trapetsiya bo'lib, uning tomonlari $AB=CD=13$ sm, $BC=11$ sm, $AD=21$ sm, diagonal kesimning yuzi 180 sm² bo'lsa, prizma to'la sirtining yuzi hisoblansin.

A) 932; B) 880; C) 1024; D) 906; E) 864 cm².

36. Uchburchakli og'ma prizmaning yon qirralari orasidagi masofalar 37 sm, 15 sm va 26 sm ga teng, yon sirti esa perpendikulyar kesimga tengdosh bo'lsa, prizmaning yon qirrasi topilsin.

A) 7; B) 2,5; C) 3; D) 4; E) 2 sm.

37. Uchburchakli og'ma prizmaning yon qirralari 8 sm dan, perpendikulyar kesimning tomonlari 9:10:17 kabi nisbatda va uning yuzi 144 sm^2 bo'lsa, prizma yon sirtining yuzi hisoblansin.

A) 576; B) 676; C) 625; D) 584; E) 600 sm^2.

38. Uchburchakli og'ma prizmaning ikkita yon yog'i o'zaro perpendikulyar bo'lib, ularning umumiy qirrasi 24 sm va qolgan ikki yon qirradan 12 sm va 35 sm uzoqlikda turadi. Prizma yon sirtining yuzi hisoblansin.

A) 2048; B) 2016; C) 1896; D) 1924; E) 3200 sm^2.

39. Og'ma prizmaning asosi ABC teng yonli uchburchakdan iborat va $AB=AC=10$ sm, $BC=12$ sm. Prizmaning A_1 uchi A va C uchlardan bir xil uzoqlikda va $AA_1=13$ sm bo'lsa, uning to'la sirti yuzi hisoblansin.

A) 468; B) 366; C) 492; D) 429; E) 524 sm^2.

40. To'rtburchakli muntazam prizma diagonal kesimining yuzi 6 bo'lsa, prizma yon sirtining yuzi hisoblansin.

A) $13\sqrt{3}$; B) $13\sqrt{2}$; C) $12\sqrt{3}$; D) 12; E) $12\sqrt{2}$.

41. To'g'ri prizmaning asosi uchburchakdan iborat bo'lib, uning ikkita tomoni 3,5 sm va ular orasidagi burchak 120°. Agar prizma eng katta yon yog'ining yuzi 35 sm^2 bo'lsa, prizma yon sirtining yuzi hisoblansin.

A) 58; B) 96; C) 72; D) 75; E) 64 sm^2.

42. Oltiburchakli muntazam prizma quyi asosining tomoni va yuqori asosining unga qarama-qarshi tomonidan tekislik o'tkazilgan. Agar prizmaning har bir qirrasi a bo'lsa, hosil qilingan kesimning yuzi hisoblansin.

A) $3a^2$; B) $4a^2$; C) $2\sqrt{3}a^2$; D) $3\sqrt{2}a^2$; E) $6a^2$.

43. To'rtburchakli muntazam prizma asosining tomoni a ga teng bo'lib, asosning diagonali orqali asos tekisligi bilan α burchak tashkil qiluvchi tekislik o'tkazilgan. Bu tekislikning yon qirrani kesib o'tishidan hosil qilingan kesimning yuzi hisoblansin.

A) $a^2\sin\alpha$; B) $\frac{a^2}{2\cos\alpha}$; C) $a^2\cos\alpha$; D) $2a^2\cos\alpha$; E) $a^2\text{tg}\alpha$.

44. Uchburchakli muntazam prizma asosining tomoni 10 sm, balandligi 15 sm bo'lsa, prizma to'la sirtining yuzi hisoblansin.

A) $60(90+\sqrt{3})$; B) $50(9+\sqrt{2})$; C) $50(9+\sqrt{3})$;

D) $50(6+\sqrt{3})$; E) $(450+29\sqrt{3})$ sm².

45. Oltiburchakli muntazam prizma asosining tomoni uzunligi 8 dm, balandligi 5 dm ga teng bo'lsa, prizma to'la sirtining yuzi hisoblansin.

A) $(180+96\sqrt{3})$; B) $(225+96\sqrt{3})$; C) $(220+192\sqrt{2})$;

D) $(240+192\sqrt{3})$; E) $(196+37\sqrt{5})$ dm².

46. To'g'ri prizmaning asosi to'g'ri burchakli uchburchakdan iborat bo'lib, uning bitta kateti c, unga yopishgan burchagi α ga teng. Berilgan katet va yuqori asosning qarama-qarshi uchidan tekislik o'tkazilgan. Hosil qilingan kesim asos tekisligi bilan β burchak tashkil qilsa, prizma yon sirtining yuzi hisoblansin.

A) $\frac{c^2\sqrt{2}\text{tg}\sin 2\beta}{\sin(\alpha-\beta)}$; B) $\frac{\sqrt{3}c^2\sin 2\alpha\text{tg}\beta}{\cos\left(45^\circ-\frac{\alpha}{2}\right)}$; C) $\frac{c^2\text{tg}\alpha\text{ctg}\beta}{\sin(\alpha+\beta)}$;

D) $\frac{\sqrt{3}c^2\sin\alpha\sin\beta}{\sin(2\alpha+\beta)}$; E) $\frac{\sqrt{2}c^2\text{tg}\alpha\text{tg}\beta\cos\frac{\alpha}{2}}{\sin\left(45^\circ-\frac{\alpha}{2}\right)}$.

47. To'rtburchakli muntazam prizmaning diagonali a va yon yoq tekisligi bilan 30° li burchak tashkil qilsa, prizmaning hajmi hisoblansin.

A) $\frac{a^3\sqrt{2}}{8}$; B) $\frac{a^3\sqrt{3}}{6}$; C) $\frac{a^3\sqrt{5}}{8}$; D) $\frac{a^3\sqrt{3}}{4}$; E) $\frac{a^3\sqrt{2}}{16}$.

48. Uchburchakli to'g'ri prizmaning asosi ABC teng yonli uchburchakda $AB=BC=m$ va $\angle ABC=\varphi$ bo'lib, prizmaning yon qirrasi asosning BD balandligiga teng bo'lsa, prizmaning hajmi topilsin.

A) $2m^3\sin\varphi$; B) $\frac{1}{2}m^3\sin\varphi\cos\frac{\varphi}{2}$; C) $m^3\cos\varphi\sin\frac{\varphi}{2}$;
D) $\frac{1}{6}m^3\sin2\varphi\,\mathrm{tg}\frac{\varphi}{2}$; E) $\frac{1}{3}m^3\sin\frac{\varphi}{2}\cos\frac{3\varphi}{2}$.

49. Oltiburchakli muntazam prizma katta diagonalining uzunligi 8 sm bo'lib, u yon qirra bilan 30° li burchak tashkil qilsa, prizmaning hajmi topilsin.

A) 75; B) 68; C) 72; D) 66; E) 64 sm^3.

50. To'g'ri prizmaning asosi ABC teng yonli uchburchakda $AB=BC=a$, $\angle ABC=\alpha$ bo'lib, AB tomon va C uchidan tekislik o'tkazilgan hamda hosil qilingan kesim asos tekisligi bilan φ burchak tashkil etadi. Prizmaning hajmi topilsin.

A) $a^3\sin2\alpha\,\mathrm{tg}\frac{\varphi}{2}$; B) $\frac{1}{8}a^3\sin\frac{3\alpha}{2}\cos\varphi$; C) $\frac{1}{3}a^3\sin\frac{\alpha}{2}\mathrm{tg}2\varphi$;
D) $\frac{1}{2}a^3\sin^2\alpha\,\mathrm{tg}\varphi$; E) $a^3\sin2\alpha\,\mathrm{tg}\varphi$.

51. Og'ma prizmaning asosi to'g'ri burchakli uchburchakdan iborat bo'lib, uning bitta o'tkir burchagi 30° ga, gipotenuzasi esa c ga teng. Prizmaning yon qirrasi b va asos tekisligi bilan 60° li burchak tashkil qiladi. Prizmaning hajmi hisoblansin.

A) $\frac{3cb^2}{40}$; B) $\frac{b^2c}{24}$; C) $\frac{3bc^2}{8}$; D) $3bc^2$; E) $\frac{3bc^2}{16}$.

52. Og‘ma prizmaning asosi tomoni a bo‘lgan muntazam uchburchakdan iborat bo‘lib, prizma yon yoqlaridan biri asosga perpendikulyar va rombdan iborat. Ushbu rombning kichik diagonali c bo‘lsa, prizmaning hajmi hisoblansin.

A) $\frac{1}{8}ac\sqrt{12a^2-3c^2}$, $2a>c$; B) $\frac{1}{16}a^2\sqrt{4a^2-3c^2}$;

C) $\frac{1}{4}ac\sqrt{16a^2-4c^2}$; D) $\frac{1}{2}ac\sqrt{4a^2-c^2}$;

E) $\frac{1}{4}c^2\sqrt{4c^2-3a^2}$.

53. Og‘ma prizmaning asosi — tomonlari 10, 10 va 12 sm bo‘lgan uchburchakdan iborat bo‘lib, prizmaning yon qirrasi 8 sm va asos tekisligiga 60° li burchak ostida og‘madir. Prizmaning hajmi topilsin.

A) 192; B) $192\sqrt{3}$; C) 196; D) $192\sqrt{2}$; E) $200\sqrt{3}$ sm^3.

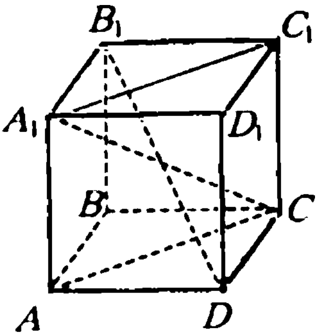
54. Uchburchakli og‘ma prizma yon qirralari orasidagi masofalar mos ravishda 37, 13 va 30 sm, prizma yon sirtining yuzi 480 sm^2 bo‘lsa, uning hajmi topilsin.

A) 960; B) 1024; C) 1080; D) 988; E) 1054 sm^3.

10-§. PARALLELEPIPED

10.1. Asosiy tushunchalar va tasdiqlar

Parallelepiped asoslari parallelogrammlar bo‘lgan prizmadir. Agar prizmaning yon yoqlari ham parallelogrammlardan iborat bo‘lsa, u *og‘ma parallelepiped*, yon yoqlari asoslarga perpendikulyar bo‘lsa, parallelepiped to‘g‘ri bo‘ladi, hamma yoqlari to‘g‘ri to‘rtburchaklardan iborat parallelepiped *to‘g‘ri burchaklidir*. Parallelepipedning *o‘lchovlari* to‘g‘ri burchakli parallelepipedning bitta uchidan chiqqan uchta qirrasining uzunliklaridir. O'lchovlari o'zaro teng bo'lgan parallelepiped kubdir.



10.1-chizma.

Parallelepipedning bitta yog'iga tegishli bo'lmagan ixtiyoriy ikkita qarama-qarshi uchini tutashtiruvchi kesma parallelepipedning *diagonalidir* (10.1-chizmada AC_1, B_1D, A_1C, BD_1). Parallelepipedning *diagonal kesimi* — parallelepiped asoslarining mos diagonallaridan o'tuvchi tekislik bilan parallelepipedning kesishishidan hosil qilingan to'rtburchaklardir (AA_1C_1C; BB_1D_1D).

Quyidagi tasdiqlar o'rinli:

1. Parallelepipedning qarama-qarshi tomonlari teng va parallel.

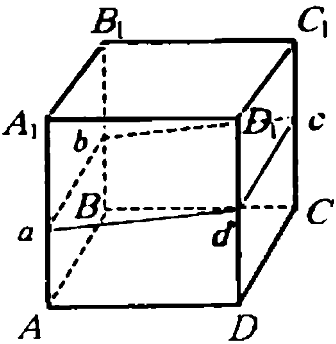
2. Parallelepipedning diagonallari bitta nuqtada kesishadi va kesishish nuqtasida teng ikkiga bo'linadi.

3. Parallelepiped diagonallari kvadratlarining yig'indisi uning hamma qirralari kvadratlarining yig'indisiga teng.

4. To'g'ri burchakli parallelepiped istalgan diagonalining kvadrati uning uchta o'lchovi kvadratlari yig'indisiga teng.

Parallelepipedning *perpendikulyar kesimi* uning yon qirrasiga perpendikulyar o'tkazilgan tekislik va parallelepipedning kesishishidan hosil bo'lgan kesimdir.

5. Og'ma parallelepipedning *yon sirti* perpendikulyar kesimning perimetri bilan yon qirrasining ko'paytmasiga teng, ya'ni agar *abcd* — perpendikulyar kesim, $AA_1 = l$ — yon qirra (10.2-chizma) bo'lsa, u holda



10.2-chizma.

$$S_{yon} = P_{perp.kes.} \cdot l.$$

6. To'g'ri parallelepipedning yon sirti uning asosi perimetri bilan yon qirrasining ko'paytmasiga teng, ya'ni agar $ABCD$ — asos, $AA_1 = H$ — yon qirrasi bo'lsa, u holda

$$S_{yon} = P_{asos} \cdot H.$$

Parallelepipedning hajmi — uning asosi yuzi va balandligining ko'paytmasiga teng:

$$V_{par\cdot d} = S_{asos} \cdot H.$$

10.2. Mavzuga doir masalalar

1. To'g'ri parallelepiped asosining tomonlari 3 sm va 5 sm, asos diagonallaridan biri 4 sm. Agar parallelepipedning kichik diagonali asos tekisligi bilan 60° li burchak tashkil qilsa, uning diagonallari topilsin.

A) 8 va 12; B) 8 va 10; C) 7 va 11; D) 12 va 6;
E) 9 va 11 sm.

2. Asosi $ABCD$ bo'lgan to'g'ri parallelepipedda $AB=29$ sm, $AD=36$ sm, $BD=25$ sm va uning yon qirrasi 48 sm bo'lsa, AB_1C_1D kesimning yuzi hisoblansin.

A) 1900; B) 2000; C) 1560; D) 1680; E) 1872 sm².

3. To'g'ri parallelepipedning asosi rombdan iborat. Parallelepiped pastki asosining bir tomoni va yuqori asosining qarama-qarshi tomoni orqali kesim o'tkazilgan. Bu

kesim parallelepiped asosi bilan 45° li burchak tashkil qiladi va kesimning yuzi Q. Parallelepiped yon sirtining yuzi topilsin.

A) $2Q$, B) $\sqrt{2}\,Q$, C) $2\sqrt{2}\,Q$; D) $2Q\sqrt{3}$; E) $3Q$.

4. To'g'ri burchakli parallelepiped qo'shni yon yoqlarining diagonallari asos tekisligi bilan mos ravishda, α va β burchaklar tashkil qiladi. Ushbu diagonallar orasidagi burchak topilsin.

A) arccos (sin2α); B) arccos (sin$\alpha \cdot$ sinβ);
C) arccos (cos$\alpha \cdot$ cosβ); D) arcsin (cos$\alpha \cdot$ cosβ);
E) arctg (sin$\alpha \cdot$ cosβ).

5. To'g'ri parallelepipedning asosi rombdan iborat bo'lib, parallelepiped diagonal kesimlarining yuzlari S_1 va S_2 bo'lsa, parallelepiped yon sirtining yuzi topilsin.

A) $\frac{1}{2}\sqrt{S_1^2+S_2^2}$; B) $\sqrt{S_1^2+S_2^2}$; C) $S_1^2+S_2^2$;
D) $\sqrt{S_1S_2}$; E) $2\sqrt{S_1^2+S_2^2}$.

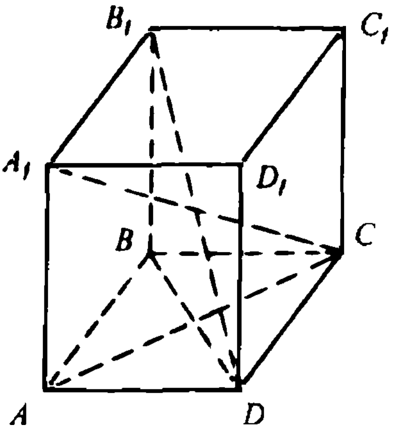
10.3. Mavzuga doir masalalarning yechimlari

1. Berilgan. $ABCDA_1B_1C_1D_1$ — to'g'ri parallelepiped, $AB=3$ sm; $AD=5$ sm, $BD=4$ sm, $\angle BDB_1=60°$.

A_1C va B_1D topilsin (10.3.1-chizma).

Yechilishi. Parallelepiped asosi $ABCD$ parallelogrammning ikkinchi diagonalini topamiz. Ma'lumki, parallelogramm diagonallari kvadratlarining yig'indisi uning tomonlari kvadratlarining yig'indisiga teng, ya'ni

$$AC^2+BD^2=2(AB^2+AD^2),\ 4^2+AC^2=2(3^2+5^2),$$



10.3.1-chizma.

$AC^2=2\cdot 34-16=52;$

$AC=\sqrt{52}$ sm;

Demak, AC — parallelepiped asosining katta diagonalidir. Berilishiga ko'ra, ΔBB_1D — to'g'ri burchakli, shu sababli,

$$\operatorname{tg}60^\circ=\frac{BB_1}{BD}, \quad BB_1=BD\cdot\operatorname{tg}60^\circ=4\sqrt{3} \text{ sm}.$$

Yana ΔBB_1D dan: $B_1D^2=BB_1^2+BD^2=(4\sqrt{3})^2+4^2=48+16=64$, $B_1D=8$ sm.

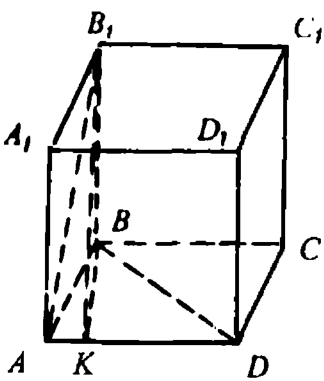
ΔAA_1C ham to'g'ri burchakli bo'lganligidan,

$A_1C^2=AA_1^2+AC^2, A_1C^2=(4\sqrt{3})^2+(\sqrt{52})^2=48+52=100$, $A_1C=10$ sm.

Javobi: B).

2. Berilgan. $ABCDA_1B_1C_1D_1$ — to'g'ri parallelepiped, $AD=36$ sm, $BD=25$ sm, $AA_1=48$ sm, $AB=29$ sm.

$S_{AB_1C_1D}$ hisoblansin (10.3.2-chizma).



10.3.2-chizma.

Yechilishi. To'g'ri parallelepipedning asosi parallelogramm bo'lganligidan, AB_1C_1D kesim ham parallelogrammdir. Uning yuzini hisoblash uchun B_1 uchidan AD tomoniga balandlik tushiramiz; $B_1K\perp AD$.

Uch perpendikulyar haqidagi teoremaga asosan (8-§) B_1K perpendikulyarning parallelepiped asosidagi BK proyeksiyasi ham AD ga perpendikulyar bo'ladi; $BK \perp AD$. Berilishiga ko'ra, ΔABD ning hamma tomonlari ma'lum, uning yuzini Geron formulasi orqali topish mumkin:

$$p = \frac{36+29+25}{2} = 45;$$

$$S_{\Delta ABD} = \sqrt{45(45-36)(45-29)(45-25)} =$$

$$= \sqrt{45 \cdot 9 \cdot 16 \cdot 20} = \sqrt{9 \cdot 5 \cdot 9 \cdot 16 \cdot 4 \cdot 5} = 5 \cdot 9 \cdot 8 = 360 \text{ sm}^2.$$

Ikkinchi tomondan, $S_{\Delta ABD} = \frac{1}{2} AD \cdot BK \Rightarrow 360 = \frac{1}{2} 36 \cdot BK \Rightarrow$ $\Rightarrow BK = 20$ sm.

Endi to'g'ri burchakli ΔBB_1K dan Pifagor teoremasi (2-§) orqali B_1K gipotenuzani topamiz:

$$B_1K^2 = BB_1^2 + BK^2 \Rightarrow B_1K^2 = 48^2 + 20^2 = 2704; \; B_1K = 52 \text{ sm}.$$

U holda AB_1C_1D kesimning yuzi:

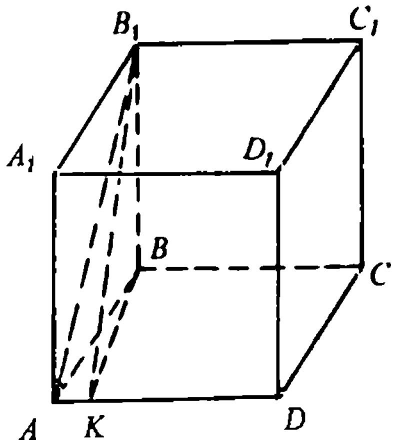
$$S = AD \cdot B_1K = 36 \cdot 52 = 1872 \text{ sm}^2.$$

Javobi: E).

3. Berilgan. $ABCDA_1B_1C_1D_1$ — to'g'ri parallelepiped, $ABCD$ — romb, $\angle B_1KB = 45°$, $S_{AB_1C_1D} = Q$.

$S_{yon\,s.}$ hisoblansin (10.3.3-chizma).

Yechilishi. Avvalo kerakli yasashlarni bajaramiz. A va B_1, D va C_1 nuqtalar mos ravishda AA_1B_1B va DD_1C_1C tekisliklarda yotganini hisobga olib, AB_1 va C_1D kesmalarni o'tkazamiz. Hosil qilingan kesim AB_1C_1D rombdan iborat bo'ladi, rombning B_1 uchidan AD tomonga B_1K perpendikulyar o'tkazamiz: $B_1K \perp AD$. Uch perpendikulyar haqidagi teoremaga asosan, B_1K ning BK proyeksiyasi ham AD tomonga perpendikulyardir: $BK \perp AD$. U holda $\angle B_1KB$ —



10.3.3-chizma.

ikki yoqli burchakning chiziqli burchagi bo'ladi va berilishiga ko'ra, $\angle B_1KB=$ $=45°$.

Ikkinchi tomondan, $BB_1 \perp BK$, demak, $\Delta BB_1K=$ $=90°-45°=45°$.

Uchburchakning ikkita burchagi o'zaro teng bo'lganligidan, u — teng-yonli, ya'ni $BK=BB_1$. Endi $BB_1=BK=x$ deb olsak, Pifagor teoremasiga (2-§) asosan, $B_1K^2=x^2+x^2=2x^2$ va $B_1K=x\sqrt{2}$. AB_1C_1D rombning yuzi $S_{AB_1C_1D}=ADB_1K$ formuladan topiladi. $AD=a$ deb olsak, $Q=a\cdot x\sqrt{2}$, $ax=\frac{Q}{\sqrt{2}}$.

Nihoyat, to'g'ri parallelepipedning yon sirti

$$S_{yon}=R_{asos}B_1B=4ax \text{ yoki } S_{yon}=4\frac{Q}{\sqrt{2}}=2\sqrt{2}\,Q.$$

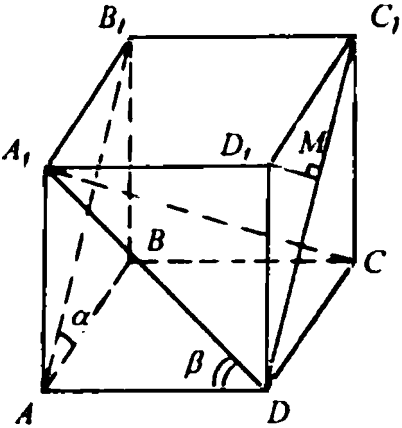
Javobi: S).

4. Berilgan. $ABCDA_1B_1C_1D_1$ — to'g'ri burchakli parallelepiped, $\angle B_1AB=\alpha$, $\angle A_1DA=\beta$

$(A_1D\,\widehat{\ }\,AB_1)$ topilsin (10.3.4-chizma).

Yechilishi. Avvalo 3-masaladagiga o'xshash kerakli yasashlarni bajaramiz. To'g'ri chiziq va tekislik orasidagi burchakning ta'rifiga ko'ra, $\angle A_1DA=\beta$ va $\angle B_1AB=\alpha$ hamda AB_1 va A_1D lar parallelepiped qo'shni yoqlarining o'zaro kesishmaydigan diagonallaridir. Parallelepipedda AB_1 ga parallel bo'lgan DC_1 kesmani o'tkazamiz. U holda $\angle A_1DC_1$ yoqlarning A_1D va AB_1 diagonallari orasidagi burchakka teng bo'lgan burchakdir, uni $\angle A_1DC_1=x$ deb belgilaymiz.

Parallelepipedning DD_1C_1C yon yog'ida $D_1P \perp DC_1$ kesma o'tkazamiz. Uch perpendikulyar haqidagi teoremaga asosan, $A_1P \perp DC_1$, demak, ΔA_1DP — to'g'ri burchakli bo'ladi va $\angle A_1DC=x$ ni topish uchun burchakning ikkita tomonini bitta o'lchov orqali ifodalash kerak. Buning uchun $A_1D=l$ belgilash kiritamiz. U holda ΔA_1AD dan $DP=$ $=l \cdot \cos x$. To'g'ri burchakli ΔA_1D_1D dan $\angle DA_1D_1=\angle A_1DA=\beta$, $DD_1=l \cdot \sin\beta$. Berilishiga ko'ra, $\angle C_1DC=\alpha$ va $D_1P \perp CD_1$, $DD_1 \perp DC$ bo'lgani uchun, $\angle DD_1P=\angle C_1DC=\alpha$ va $DM=DD_1\sin\alpha$, $DP=l \sin\beta \sin\alpha$. Demak, bir tomondan, $DM=l \cdot \cos x$, ikkinchi tomondan, $DP=l \cdot \sin\beta \cdot \sin\alpha$, ularni tenglashtirsak,



10.3.4-chizma.

$$\cos x=\sin\alpha \cdot \sin\beta$$

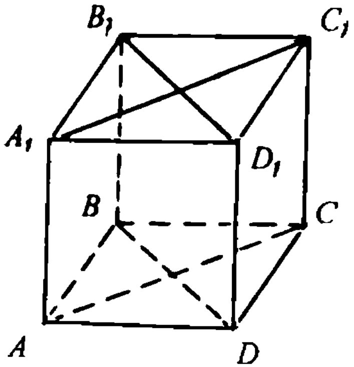
tenglikni olamiz, bu yerdan $x=\arccos(\sin\alpha \cdot \sin\beta)$.

Javobi: B).

5. Berilgan $ABCDA_1B_1C_1D_1$ — to'g'ri parallelepiped, $ABCD$ — romb, $S_{AA_1C_1C} = S_1$; $S_{BB_1D_1D} = S_2$.

S_{yon} hisoblansin (10.3.5-chizma).

Yechilishi. Ma'lumki, parallelepipedning yon sirti $S_{yon}=P_{asos} \cdot H$ formula orqali hisoblanadi. $AB=a$, $AA_1=H$ belgilashlar kiritsak, parallelepipedning asosi romb bo'lganligidan, $P_{asos}=4a$ va $S_{yon}=4a \cdot H$ bo'ladi. Asosning



10.3.5-chizma.

diagonallari $AC=d_1$, $BD=d_2$ bo‘lsa, parallelepiped diagonal kesimlarining yuzlari

$$S_1=d_1H,\ S_2=d_2H \text{ va } d_1=\frac{S_1}{H};\quad d_2=\frac{S_2}{H}.$$

Parallelogramm diagonallari kvadratlarining yig‘indisi uning hamma tomonlari kvadratlarining yig‘indisiga teng bo‘lganligidan, romb uchun

$$d_1^2+d_2^2=4a^2$$

ifodani olamiz. Demak,

$$4a^2=\frac{S_1^2}{H^2}+\frac{S_2^2}{H^2},\ 4a^2H^2=S_1^2+S_2^2.$$

Bu yerdan, $2a\cdot H=\sqrt{S_1^2+S_2^2}$. U holda parallelepipedning yon sirti $S_{yon}=2\cdot 2aH=2\sqrt{S_1^2+S_2^2}$ bo‘ladi.

Javobi: E).

10.4. Mustaqil yechish uchun masalalar

1. To‘g‘ri burchakli parallelepipedning uchta o‘lchovi berilgan bo‘lsa, uning diagonali topilsin: 1) 2, 1, 2; 2) 7, 6, 6; 3) 12, 21, 16.

1) A) 4; B) 3; C) 2; D) 3,5; E) 2,8.
2) A) 12; B) 8; C) 9; D) 10; E) 11.
3) A) 25; B) 27; C) 29; D) 26; E) 28.

2. To‘g‘ri parallelepiped asosining tomonlari 3 dm va 4 dm bo‘lib, o‘zaro 60° li burchak tashkil qiladi. Parallelepipedning yon qirrasi asosning tomonlari orasida o‘rta proporsional bo‘lsa, uning katta diagonali topilsin.

A) 6; B) 4,5; C) 8; D) 7; E) 10 dm.

3. To‘g‘ri parallelepipedning yon qirrasi 1 m, asosining tomonlari 23 dm va 11 dm bo‘lib, asos diagonallarining nisbati 2:3 kabi. Parallelepiped diagonal kesimlarining yuzlari hisoblansin.

A) 2 va 3; B) 3 va 4; C) 1 va 6; D) 2,5 va 4,5;
E) 12 m² va 10 m².

4. To‘g‘ri burchakli parallelepipedning uchta yog‘ining yuzlari mos ravishda 42 sm², 72 sm² va 84 sm² bo‘lsa, uning diagonali topilsin.

A) 15; B) 16; C) $\sqrt{180}$; D) $\sqrt{240}$; E) $\sqrt{229}$ sm.

5. To‘g‘ri burchakli parallelepipedning diagonali 13 dm, balandligi 12 dm, asosining bitta tomoni 4 dm bo‘lsa, parallelepiped to‘la sirtining yuzi hisoblansin.

A) 180; B) 196; C) 192; D) 200; E) 156 dm².

6. To‘g‘ri parallelepipedning asosi rombdan iborat bo‘lib, rombning kichik diagonali d, o‘tkir burchagi α ga teng. Agar parallelepipedning balandligi $\frac{d}{2}$ bo‘lsa, parallelepiped to‘la sirtining yuzi hisoblansin.

A) $\frac{1}{2}d^2\sin\alpha$; B) $d^2\operatorname{ctg}\frac{\alpha}{4}$; C) $\frac{1}{4}d^2\cos^2\frac{\alpha}{2}$;
D) $\frac{1}{8}d^2\sin\frac{\alpha}{2}$; E) $\frac{1}{2}d^2\operatorname{tg}\frac{\alpha}{4}$.

7. To‘g‘ri burchakli parallelepipedning diagonali d ga teng va bitta yoq bilan 30° li, ikkinchi yoq bilan 45° li

burchak tashkil qiladi. Parallelepiped yon sirtining yuzi hisoblansin.

A) $\frac{1}{4}d^2\sqrt{2}$; B) $\frac{1}{8}d^2$; C) $\frac{1}{4}d^2$; D) $\frac{d^2(\sqrt{2}+1)}{2}$;

E) $\frac{1}{6}d^2\sqrt{2}$.

8. To'g'ri burchakli parallelepipedning diagonali asos tekisligi bilan α burchak tashkil qiladi, uning diagonal kesimi va yon yog'i tashkil etgan ikki yoqli burchak β ga teng. Agar parallelepiped asosining diagonali d bo'lsa, uning hajmi hisoblansin.

A) $\frac{1}{4}d^3\sin\beta\,\mathrm{tg}\alpha$; B) $\frac{1}{2}d^3\cos2\beta\,\mathrm{tg}\alpha$; C) $\frac{1}{4}d^3\cos\beta\,\mathrm{tg}\alpha$;

D) $\frac{1}{2}d^3\sin^2\beta\,\mathrm{tg}\alpha$; E) $\frac{1}{2}d^3\sin2\beta\,\mathrm{tg}\alpha$.

9. To'g'ri parallelepiped asosining tomonlari a va b bo'lib, ular orasidagi burchak 60° ga teng. Parallelepipedning kichik diagonali asosning katta diagonaliga teng bo'lsa, uning hajmi hisoblansin.

A) $ab\sqrt{6ab}$; B) $\frac{1}{2}ab\sqrt{5ab}$; C) $\frac{1}{2}ab\sqrt{6ab}$;

D) $\frac{1}{2}ab\sqrt{3ab}$; E) $ab\sqrt{5ab}$.

10. To'g'ri parallelepipedning asosi o'tkir burchagi α va kichik diagonali d bo'lgan rombdan iborat va parallelepipedning balandligi asosning tomonidan ikki marta kichik bo'lsa, uning hajmi hisoblansin.

A) $\frac{d^3\mathrm{ctg}\frac{\alpha}{2}}{8\sin\frac{\alpha}{2}}$; B) $\frac{d^3}{8}\mathrm{tg}\frac{\alpha}{2}$; C) $\frac{d^3\sin\frac{\alpha}{2}}{4\mathrm{ctg}\frac{\alpha}{2}}$;

D) $\frac{d^3\mathrm{ctg}\frac{\alpha}{2}}{8}$; E) $\frac{d^3\mathrm{tg}\frac{\alpha}{2}}{8}$.

11. To'g'ri burchakli parallelepipedning bitta uchidan chiqqan qirralari uzunliklari 6, 6 va 8 m bo'lib, ularning

o'rta nuqtalaridan kesim o'tkazilgan. Shu kesimning yuzi hisoblansin.

A) $\frac{5\sqrt{14}}{2}$; B) $\frac{3\sqrt{14}}{2}$; C) $\frac{4\sqrt{14}}{3}$; D) $5\sqrt{14}$; E) $4\sqrt{14}$ m^2.

12. To'g'ri burchakli parallelepiped uchta yon yoqlarining yuzlari, mos ravishda, 2, 3 va 4 m^2 bo'lsa, parallelepiped to'la sirtining yuzi hisoblansin.

A) $\sqrt{24}$; B) 16; C) 24; D) 9; E) 18 m^2.

13. To'g'ri burchakli parallelepiped yon yoqlarining yuzlari S_1, S_2 va S_3 bo'lsa, parallelepipedning hajmi hisoblansin.

A) $\sqrt{S_1S_2}+\sqrt{S_1S_3}$; B) $\sqrt{S_1S_2S_3}$; C) $\sqrt{S_1+S_2+S_3}$;
D) $S_1\sqrt{S_2S_3}$; E) $S_2\sqrt{S_1S_3}$.

14. To'g'ri parallelepipedning diagonallari 9 va $\sqrt{33}$ sm, asosining perimetri 18 sm va yon qirrasi 4 sm bo'lsa, parallelepiped to'la sirtining yuzi hisoblansin.

A) 98; B) 92; C) 96; D) 104; E) 108 sm^2.

15. To'g'ri parallelepiped asosining tomonlari a va b bo'lib, ular orasidagi burchak α ga teng. Parallelepipedning kichik diagonali asosning katta diagonaliga teng bo'lsa, uning hajmi hisoblansin.

A) $2\sqrt{(ab)^3\cos\alpha}$; B) $2\cos\alpha\sqrt{(ab)^2\sin\alpha}$;

C) $\sin\alpha\sqrt{ab\cos\alpha}$; D) $4\cos\alpha\sqrt{\sin\alpha}$;

E) $2\sin\alpha\sqrt{(ab)^3\cos\alpha}$.

16. To'g'ri parallelepipedning asosi rombdan iborat. Parallelepiped pastki asosining bir tomoni va yuqori asosining qarama-qarshi tomoni orqali tekislik o'tkazilgan. Hosil qilingan kesimning yuzi Q bo'lib, u asos tekisligi bilan β

burchak tashkil qiladi. Parallelepiped yon sirtining yuzi hisoblansin.

A) $4\,Q\sin\beta$; B) $2Q\,\mathrm{tg}\beta$; C) $Q\,\mathrm{ctg}\beta$; D) $\frac{1}{2}\,Q\sin\beta$;
E) $3\,Q\sin\beta$.

17. To'g'ri burchakli parallelepipedning asosi to'g'ri to'rtburchakdan iborat bo'lib, uning kichik tomoni a, diagonallari orasidagi burchak $60°$. Agar parallelepiped asosining katta tomoni uning yon qirrasiga teng bo'lsa, parallelepipedning hajmi hisoblansin.

A) $6\,a^3$; B) $4\,a^3$; C) $3\,a^3$; D) $\frac{1}{8}\,a^3$; E) $2\,a^3$.

18. To'g'ri burchakli parallelepipedning diagonali 13 sm, yon yoqlarining diagonallari $4\sqrt{10}$ va $3\sqrt{17}$ sm bo'lsa, parallelepipedning hajmi hisoblansin.

A) 148; B) 156; C) 128; D) 144; E) 120 sm^2.

19. To'g'ri burchakli parallelepipedning diagonali l va asos tekisligi bilan α burchak tashkil qiladi. Agar parallelepiped asosining yuzi S bo'lsa, uning hajmi hisoblansin.

A) $S\cdot l\sin\alpha$; B) $2lS\,\mathrm{tg}\alpha$; C) $4l^3\cos\alpha$; D) $(S+l^2)l\sin^2\alpha$;
E) $5l\,S\,\mathrm{tg}\alpha$.

20. To'g'ri parallelepipedning balandligi h, uning diagonallari asos tekisligi bilan α va β burchaklarni tashkil qilsa, parallelepiped yon sirtining yuzi hisoblansin.

A) $\sqrt{h^2\mathrm{ctg}\alpha\,\mathrm{ctg}\beta}$; B) $2h^2\sqrt{\mathrm{ctg}^2\alpha+\mathrm{ctg}^2\beta}$;

C) $h^2\sqrt{\mathrm{ctg}^2\alpha+\mathrm{ctg}^2\beta}$; D) $\frac{h^2}{4}\sqrt{\mathrm{ctg}^2\alpha+\mathrm{ctg}^2\beta}$;

E) $h^2\sqrt{\mathrm{tg}^2\alpha+\mathrm{tg}^2\beta}$.

21. To'g'ri parallelepipedning asosi parallelogramm bo'lib, uning tomonlari 1 va 4 sm, ular orasidagi burchak 60°. Parallelepipedning katta diagonali 5 sm bo'lsa, uning yon sirtining yuzi hisoblansin.

A) 28; B) 16; C) 24; D) 18; E) 20 sm^2.

22. Kubda diagonal va kubning u bilan kesishmaydigan qirrasi orasidagi masofa d bo'lsa, kub to'la sirtining yuzi hisoblansin.

A) 15 d^2; B) 18 d^2; C) 12 d^2; D) 14 d^2; E) 16 d^2.

23. To'g'ri burchakli parallelepipedning diagonali l bo'lib, yon yoqlari bilan mos ravishda 30° li va 45° li burchaklar tashkil qiladi. Parallelepipedning hajmi hisoblansin.

A) $\frac{l^3}{4}$; B) $\frac{l^3\sqrt{2}}{8}$; C) $\frac{l^3}{8}$; D) $\frac{l^3\sqrt{2}}{4}$; E) $\frac{l^3\sqrt{3}}{8}$.

24. To'g'ri burchakli parallelepipedning diagonali d, o'lchovlari nisbati $m:n:p$ kabi bo'lsa, uning hajmi hisoblansin.

A) $\frac{d^3}{(m+n+p)^{3/2}}$; B) $\frac{(m+n+p)d^3}{\sqrt{m^2+n^2+p^2}}$; C) $\frac{mnpd}{\sqrt{m^2+n^2+p^2}}$;
D) $\frac{mnpd^3}{(m^2+n^2+p^2)^{3/2}}$; E) $\frac{(mn+np+mp)d}{(m^2+n^2+p^2)^{3/2}}$.

25. Kub to'la sirtining yuzi 36 sm^2 bo'lsa, uning ikkita ayqash qirralari orasidagi masofaning kvadrati topilsin.

A) 8; B) 3; C) 6; D) 4; E) 5 sm.

26. To'g'ri parallelepipedning asosi parallelogramm, uning o'tkir burchagi 60°, tomonlari esa 1 va 4 m. Parallelepipedning katta diagonali 5 sm bo'lsa, uning hajmi hisoblansin.

A) $4\sqrt{3}$; B) $4\sqrt{2}$; C) $6\sqrt{2}$; D) $6\sqrt{2}$; E) 12 m^3.

27. Kubning diagonali va unga ayqash bo'lgan yon qirra orasidagi masofa d bo'lsa, kubning hajmi hisoblansin.

A) $d^3\sqrt{2}$; B) $d^3\sqrt{3}$; C) $2d^3$; D) $d^3\sqrt{2}$; E) $2d^3\sqrt{2}$.

28. To'g'ri parallelepipedning asosi romb bo'lib, uning yuzi S. Parallelepiped diagonal kesimlarining yuzlari S_1 va S_2 bo'lsa, uning hajmi hisoblansin.

A) $\sqrt{S_1}\frac{S_2+S_3}{2}$; B) $\frac{S_1+S_2+S_3}{2}$; C) $\sqrt{\frac{S_1S_2S}{2}}$;
D) $\frac{\sqrt{S_1S_2S_3}}{2}$; E) $\frac{\sqrt{S_1+S_2+S_3}}{2}$.

29. To'g'ri parallelepiped asosining tomonlari a va b, ular orasidagi burchak 30°. Parallelepiped yon sirtining yuzi S ga teng bo'lsa, uning hajmi hisoblansin.

A) $\frac{abS}{8(a+b)}$; B) $\frac{abS}{4(a+b)}$; C) $\frac{(a+b)S}{4ab}$; D) $\frac{abS}{a+b}$; E) $\frac{S}{a+b}$.

30. Kub to'la sirtining yuzi 36 m² bo'lsa, uning ikkita ayqash qirrasi o'rta nuqtalari orasidagi masofa topilsin.

A) 3,5; B) 5; C) 4; D) 3; E) 6 sm.

31. To'g'ri burchakli $ABCDA_1B_1C_1D_1$ parallelepipedning A, C va D_1 uchlaridan tekislik o'tkazilgan. Shu tekislik va asos tekisligi orasidagi burchak 60°, asosning tomonlari 4 va 3 sm bo'lsa, parallelepipedning hajmi hisoblansin.

A) $\frac{144\sqrt{3}}{5}$; B) $\frac{128\sqrt{3}}{5}$; C) $\frac{156\sqrt{3}}{5}$; D) $\frac{108\sqrt{2}}{5}$; E) $\frac{148\sqrt{3}}{7}$.

32. To'g'ri parallelepipedning asosi parallelogramm va uning o'tkir burchagi 30°. Parallelepiped asosining yuzi 4 dm², yon yoqlarining yuzlari 6 dm² va 12 dm² bo'lsa, uning hajmi hisoblansin.

A) 10; B) 15; C) 8; D) 16; E) 12 dm³.

33. Og'ma parallelepipedning asosi — romb va uning tomoni a, o'tkir burchagi 60°. Parallelepipedning yon qirrasi AA_1 ham a ga teng hamda AB va AD qirralar bilan 45° li burchak tashkil qilsa, uning hajmi hisoblansin.

A) $\frac{1}{8}a^3$; B) $\frac{1}{5}a^3$; C) $\frac{1}{2}a^3$; D) $\frac{1}{4}a^3$; E) $\frac{1}{3}a^3$.

34. To'g'ri burchakli parallelepiped yon yoqlarining diagonallari a, b va c. Parallelepiped to'la sirtining yuzi hisoblansin.

A) $ab+bc+ac$; B) $\frac{a^2+b^2+c^2}{2}$; C) $a^2+b^2+c^2$;

D) $\sqrt{a^4-(b^4-c^2)^2}+\sqrt{b^4-(c^2-a^2)^2}+\sqrt{c^4-(a^2-b^2)^2}$;

E) $\sqrt{a^2-(b-c)^2}+\sqrt{b^2-(c-a)^2}+\sqrt{c^2-(a-b)^2}$.

35. To'g'ri burchakli parallelepipedning diagonali l va yon qirra bilan α burchak tashkil etadi. Parallelepiped asosining perimetri p bo'lsa, uning hajmi hisoblansin.

A) $(p^2-4l^2\sin^2\alpha)l\cos\alpha$; B) $\frac{1}{8}(p^2-4l^2\sin^2\alpha)l\cos\alpha$;

C) $\frac{1}{2}(p^2-l^2\sin^2\alpha)l\,\mathrm{tg}\alpha$; D) $\frac{1}{4}(p^2-8l^2)\,l\cos\alpha$;

E) $(p^2-l^2)\,\mathrm{tg}\alpha$.

36. To'g'ri parallelepipedning asosi — romb va uning o'tkir burchagi α va kichik diagonali d. Parallelepipedning balandligi asosining tomonidan ikki marta kichik bo'lsa, parallelepiped to'la sirtining yuzi hisoblansin.

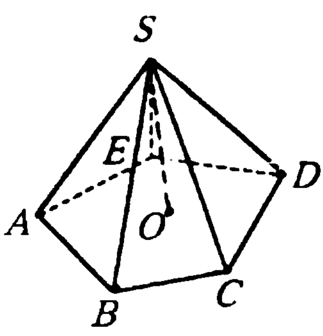
A) $\frac{d^2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\sin^2\frac{\alpha}{2}}$; B) $d^2\mathrm{ctg}^2\frac{\alpha}{2}$; C) $d^2\mathrm{tg}^2\frac{\alpha}{4}$;

D) $\frac{d^2\sin\alpha}{8}$; E) $\frac{d^2\cos^2\frac{\alpha}{2}}{\sin^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}$.

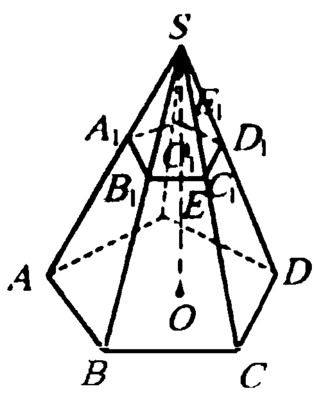
11-§. PIRAMIDA

11.1. Asosiy tushunchalar va tasdiqlar

Piramida — berilgan nuqtani yassi ko‘pburchakning nuqtalari bilan tutashtiradigan barcha kesmalardan tashkil topgan ko‘pyoqdan iborat. Shu berilgan nuqta piramidaning *uchi*, ko‘pburchak esa piramidaning *asosidir*. Piramidaning sirti uning asosi va yon yoqlaridan iborat, yon yoqlari uchburchaklardir. *Yon qirra* piramidaning uchini asosi uchi bilan tutashtiradigan yoki ikki yon yog‘ining kesishishidan hosil bo‘ladigan kesmadir. Piramidaning *balandligi* uning uchidan asos tekisligiga tushirilgan perpendikulyardir. 11.1-chizmada: S — piramidaning uchi; $ABCDE$ — piramidaning asosi; $\Delta SAB, \Delta SBC, \Delta SCD, \Delta SDE, \Delta SEA$ — piramidaning yon yoqlari; SA, SB, SC, SD, SE — piramidaning yon qirralari; SO — piramidaning balandligidir. *Muntazam piramida* — asosi muntazam ko‘pburchak bo‘lib, balandligi asosning markazidan o‘tadigan piramidadir. Muntazam piramidaning *o‘qi* uning balandligi yotgan to‘g‘ri chiziqdan iborat. *Apofema* — muntazam piramida yon yog‘ining uchidan o‘tkazilgan balandlikdir.



11.1-chizma.



11.2-chizma.

Quyidagi xossalar va tasdiqlar o'rinli.

1. Piramidaning asosiga parallel va uni kesib o'tadigan tekislik o'tkazilgan bo'lsa: a) shu piramidaga o'xshash piramida ajratadi (11.2-chizma); b) piramidaning yon qirralari va balandligi proporsional kesmalarga ajraladi:

$$\frac{AS}{A_1S} = \frac{BS}{B_1S} = \ldots = \frac{SO}{SO_1};$$

d) kesimdagi ko'pburchak piramidaning asosiga o'xshash bo'ladi:

$$ABCDE \sim A_1B_1C_1D_1E_1;$$

e) piramidaning asosi va kesim yuzlarining nisbati piramida uchidan asoslargacha bo'lgan mos masofalar kvadratlarining nisbatiga teng:

$$\frac{S_{ac}}{S_{kec}} = \frac{H^2}{h^2}.$$

2. Muntazam piramidaning yon sirti asosining perimetri va apofemasi ko'paytmasining yarmiga teng:

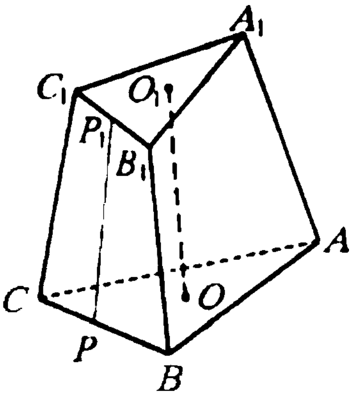
$S_{yon} = \frac{1}{2} P_{as} \cdot l$ (l — apofema, P_{as} — asosning perimetri).

3. Piramidaning hajmi asosining yuzi bilan balandligi ko'paytmasining uchdan biriga teng:

$V_{pir} = \frac{1}{3} S_{ac} \cdot H$ (S_{ac} — asosi yuzi, H — balandligi).

Piramidaning asosi tekisligiga parallel va piramidani kesib o'tuvchi tekislik va asosi bilan chegaralangan qismi *kesik piramidadir* (11.3-chizma).

4. Muntazam kesik piramidaning yon sirti — uning asoslari perimetrlari yig'idisining yarmi bilan apofemasining ko'paytmasiga teng:



11.3-chizma.

$$S_{yon} = \frac{P_1 + P_2}{2} \cdot l,$$

(P_1 — quyi asos perimetri, P_2 — yuqori asos perimetri, l — apofema).

5. Muntazam bo'lmagan kesik piramidaning yon sirti uning yon yoqlari yuzlarining yig'indisiga teng.

6. Agar kesik piramida asoslarining yuzlari, mos ravishda, S_1 va S_2, balandligi H bo'lsa, kesik piramidaning hajmi

$$V = \frac{1}{3} H(S_1 + S_2 + \sqrt{S_1 S_2})$$

formula orqali hisoblanadi.

11.2. Mavzuga oid masalalar

1. Yon qirrasi b, asosining tomoni a ga ko'ra:

a) uchburchakli; b) to'rtburchakli va d) oltiburchakli muntazam piramidaning balandligi topilsin.

a) A) $\sqrt{8b^2 - 3a^2}$; B) $\frac{1}{3}\sqrt{9b^2 - 3a^2}$; C) $\frac{1}{2}\sqrt{4b^2 - a^2}$;

D) $\frac{1}{3}\sqrt{4b^2 + a^2}$; E) $\frac{1}{2}\sqrt{6b^2 - 2a^2}$;

b) A) $\frac{1}{2}\sqrt{2(2b^2 - a^2)}$; B) $2\sqrt{2b^2 - a^2}$; C) $\frac{1}{2}\sqrt{a^2 + b^2}$;

D) $\frac{1}{4}\sqrt{a^2 - b^2}$; E) $\frac{1}{2}\sqrt{2b^2 - a^2}$.

d) A) $\frac{1}{2}\sqrt{a^2 - b^2}$; B) $\sqrt{a^2 - b^2}$; C) $\sqrt{b^2 - a^2}$;

D) $\sqrt{a^2 - b^2}$; E) $\sqrt{ab}$;

2. Piramidaning asosi asosi 12 sm, yon tomoni 10 sm bo'lgan teng yonli uchburchak bo'lib, piramidaning yon qirralari o'zaro teng va har biri 13 sm. Piramidaning balandligi topilsin.

A) 1,6; B) $\frac{\sqrt{10}}{2}$; C) $\frac{\sqrt{48}}{5}$; D) $\frac{\sqrt{51}}{4}$; E) 1,5 sm.

3. Piramidaning asosi asosi 12 sm, yon tomoni 10 sm bo'lgan teng yonli uchburchak, yon yoqlari asos tekisligi bilan o'zaro teng va har biri 45° dan iborat burchak hosil qiladi. Piramidaning hajmi hisoblansin.

A) 62; B) 49; C) 54; D) 45; E) 48 sm^3.

4. Piramidaning balandligi 16 sm, asosining yuzi 512 m^2. Yuzi 50 m^2 bo'lgan parallel kesim asosdan qanday masofada joylashgan?

A) 20; B) 11; C) 12; D) 16; E) 18 m.

5. Uchburchakli piramidaning yon qirralari o'zaro perpendikulyar va uzunliklari, mos ravishda, $\sqrt{70}$, $\sqrt{99}$ va $\sqrt{126}$ ga teng. Piramidaning hajmi hisoblansin.

A) $21\sqrt{55}$; B) $2\sqrt{110}$; C) $4\sqrt{68}$; D) $16\sqrt{33}$;
E) $29\sqrt{22}$.

6. Piramidaning asosi — o'tkir burchagi 45° bo'lgan romb. Rombga ichki chizilgan aylananing radiusi 3 sm bo'lib, piramidaning balandligi shu aylana markazidan o'tadi va 4 sm. Piramida yon sirtining yuzi hisoblansin.

A) 60; B) $60\sqrt{3}$; C) $60\sqrt{2}$; D) 80; E) 108 sm^2.

7. Piramidaning asosi — yuzi 1 m^2 bo'lgan to'g'ri to'rtburchak, ikki yon yog'i asosiga perpendikulyar bo'lib,

qolgan ikkitasi esa asosi bilan 30° va 60° li burchaklar tashkil etadi. Piramidaning hajmi hisoblansin.

A) $\frac{2}{3}$; B) $\frac{3}{4}$; C) $\frac{1}{2}$; D) $\frac{1}{3}$; E) $\frac{2}{5}$ m².

8. Uchburchakli muntazam piramidaning yon sirti va asosi yuzlarining nisbati k ga teng. Piramidaning yon qirrasi va balandligi orasidagi burchak topilsin.

A) 45°; B) $\arcsin\frac{\sqrt{k-1}}{2}$, $k\geq 1$; C) $\operatorname{arctg}\frac{\sqrt{k^2-1}}{2}$, $k>1$;
D) $\arccos\frac{\sqrt{k-1}}{2}$, $k>1$; E) $\operatorname{arcctg}\frac{\sqrt{k^2-1}}{2}$, $k>1$.

9. Uchburchakli muntazam kesik piramida asoslarining tomonlari 6 dm va 12 dm, balandligi 1 dm. Piramida yon sirtining yuzi hisoblansin.

A) 52; B) 54; C) 56; D) 60; E) 63 dm².

10. To'rtburchakli muntazam kesik piramidaning diagonali 9 sm, asoslarining tomonlari 7 va 5 sm. Piramidaning hajmi hisoblansin.

A) 109; B) 104; C) 96; D) 98; E) 105 sm³.

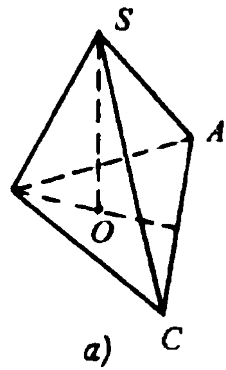
11.3. Mavzuga oid masalalarning yechimlari

1. a) B e r i l g a n. $SABC$ — muntazam uchburchakli piramida, $AB=a$, $AS=BS=CS=b$, $SO\perp(ABC)$.

SO t o p i l s i n (11.3.1 a)-chizma.).

Y e c h i l i s h i. Muntazam piramidaning yon qirralari o'zaro teng va ularning proyeksiyalari ham o'zaro teng: $AO=BO=CO$. U holda O nuqta — piramida asosiga tashqi chizilgan aylananing markazi bo'lib, $AO=R$. Muntazam uchburchakning har bir burchagi 60° bo'lganligidan, sinuslar teoremasiga ko'ra, $\frac{a}{\sin 60°} = 2R$ va u holda

$R = \frac{a}{2\cdot\frac{\sqrt{3}}{2}} = \frac{a}{\sqrt{3}}$. $SO \perp (ABC)$ bo'lganligidan, SO — tekislikdagi kesishish nuqtasi O dan o'tuvchi ixtiyoriy to'g'ri chiziqqa perpendikulyardir. Shu sababli, $SO \perp OB$ va ΔSOB — to'g'ri burchakli. Pifagor teoremasiga asosan, $SB^2 = SO^2 + BO^2$ va



11.3.1 a)- chizma.

$SO^2 = SB^2 - BO^2$; $SO^2 = b^2 - (\frac{a}{\sqrt{3}})^2 = b^2 - \frac{3a^2}{9} = \frac{9b^2 - 3a^2}{9}$;

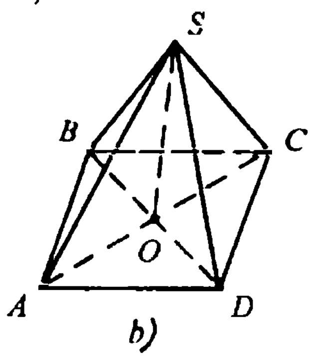
$SO = \frac{1}{3}\sqrt{9b^2 - 3a^2}$.

Javobi: B).

b) Berilgan. $SABCD$ — to'rtburchakli muntazam piramida, $ABCD$ — kvadrat, $AB = a$, $SA = SB = SC = SD = b$.

SO topilsin (11.3.1 b)-chizma).

Yechilishi. SO balandlik va SC yon qirra to'g'ri burchakli ΔSOC ning tomonlaridir. Agar biz OC tomon uzunligini topsak, SO ni hisoblashimiz oson bo'ladi. Ikkinchi tomondan, OC kesma — $ABCD$ kvadrat diagonalining yarmiga teng: $OC = \frac{1}{2}AC$. AC tomonni to'g'ri burchakli ΔACD dan topamiz:



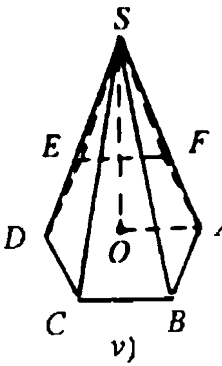
11.3.1 b)- chizma.

$AC^2=AD^2+DC^2$, $AC=\sqrt{AD^2+DC^2}=a\sqrt{2}$ va $OS=\frac{a\sqrt{2}}{2}$.
Demak, piramidaning balandligi

$$SO=\sqrt{SC^2-OC^2}=\sqrt{b^2-\frac{2a^2}{4}}=\frac{1}{2}\sqrt{2(2b^2-a^2)}.$$

Javobi: A).

d) Berilgan. $SABCDEF$ — oltiburchakli muntazam piramida; $AB=a$, $AC=b$.



11.3.1 v-chizma.

SO topilsin (11.3.1 v-chizma).

Yechilishi. Piramidaning asosi muntazam oltiburchak bo'lganligidan, unga tashqi chizilgan aylananing radiusi o'sha oltiburchakning tomoniga teng: $OA=a$. U holda piramidaning balandligini to'g'ri burchakli ΔOSA dan topiladi:

$$SO=\sqrt{AS^2-OA^2}=\sqrt{b^2-a^2}.$$

2. Berilgan. $SABC$ uchburchakli piramida, ΔABC — teng yonli, $BD\perp AC$, $AC=12$ sm, $AB=BC=10$ sm, $AS=BS=CS=13$ sm.

SO topilsin (11.3.2-chizma).

Yechilishi. Yon qirralari teng bo'lgani uchun, ularning proyeksiyalari ham teng: $AO=BO=CO$. Demak, O nuqta ΔABC ga tashqi chizilgan aylananing markazi va $AO=R$ ushbu aylananing radiusi va uni quyidagi formula yordamida topamiz: $R=\frac{abc}{4S}$.

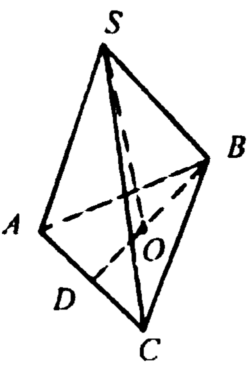
Uchburchakning yuzini Geron formulasi yordamida topamiz:

$$p = \frac{10+10+12}{2} = 16,$$

$$S_{\Delta} = \sqrt{16(16-10)^2(16-12)} =$$

$$= 4 \cdot 6 \cdot 2 = 48 \text{ sm}^2.$$

U holda $R = \frac{10^2 \cdot 12}{4 \cdot 48} = \frac{100}{4 \cdot 4} = \frac{25}{4}$ sm.



11.3.2-chizma.

To'g'ri burchakli ΔAOS dan Pifagor teoremasi yordamida topamiz:

$$SO^2 = AS^2 - AO^2 = 13^2 - \left(\frac{25}{4}\right)^2 = \frac{4 \cdot 169 - 625}{4^2},$$

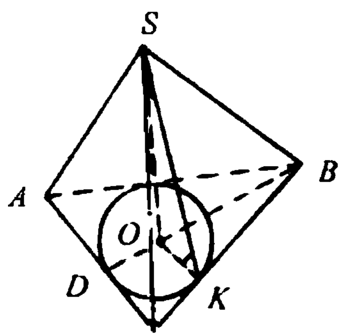
$$SO = \sqrt{\frac{(26-25)(26+25)}{4^2}} = \frac{\sqrt{51}}{4} \text{ sm}.$$

Javobi: D).

3. Berilgan. $SABC$ — uchburchakli piramida, $AB = BC = 10$ sm, $AC = 12$ sm. $\angle SKO = 45^\circ$.

V_{pir} hisoblansin (11.3.3-chizma).

Yechilishi. Avvalo yasashlar bajaramiz. Yon yoq va asos tekisligi orasidagi ikki yoqli burchakning chiziqli burchagini yasash uchun S nuqtadan asos tekisligiga SO va asosning BC tomoniga SK perpendikulyarlarni o'tkazamiz. Uch perpendikulyar haqidagi teoremaga asosan (8-§) $OK \perp BC$ bo'ladi. De-



11.3.3-chizma.

mak, chiziqli burchak $\angle OKS=45°$. Qolgan chiziqli burchaklarni ham shunga o'xshash yasaymiz. Hosil qilingan to'g'ri burchakli uchburchaklar o'zaro teng bo'lganligidan, $OK=OD=OF$. Ikkinchi tomondan, O nuqta uchburchakning tomonlaridan bir xil uzoqlikda yotganligidan, u uchburchakka ichki chizilgan aylananing markazi bo'ladi, $OK=r$ — ichki chizilgan aylananing radiusidir.

Endi Geron formulasi yordamida ΔABC ning yuzini hisoblaymiz:

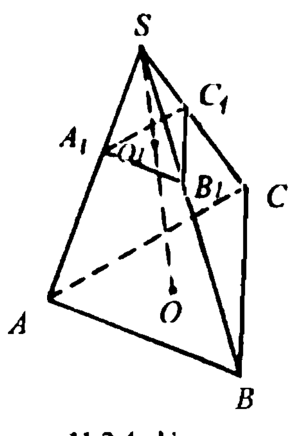
$$p=\frac{1}{2}(12+10+10)=16,\ S_{a}=\sqrt{16(16-12)(16-10)^2}=$$

$= 4\cdot 2\cdot 6=48$ sm².

Ichki chizilgan aylananing radiusi $r=\frac{S_{ac}}{p}=\frac{48}{16}=3$ sm.

To'g'ri burchakli ΔSOK ning bitta o'tkir burchagi 45°, demak, ikkinchi o'tkir burchagi ham 45° va ΔSOK — teng yonli, ya'ni $OS=OK=3$ sm. Piramidaning hajmini hisoblaymiz: $V=\frac{1}{3}S_{ac}\cdot H=\frac{1}{3}\cdot 48\cdot 3=48$ sm³.

Javobi: E).



11.3.4-chizma.

4. B e r i l g a n. $SABC$ — piramida, $S_{ac}=512$ m²; $OS=H=16$ m, $(A_1B_1C_1)\|(ABC)$, $S_{kes}=50$ m².

OO_1 t o p i l s i n (11.3.4-chizma).

Y e c h i l i s h i. Agar piramida asosiga parallel kesim o'tkazilsa, $\frac{S_{ac}}{S_{kes.}}=\frac{H^2}{h^2}$ munosabat bajariladi, bu yerda $H=SO$, $h=SO_1$. Demak, $\frac{512}{50}=\frac{16^2}{h^2}$, $h^2=\frac{256\cdot 50}{512}=25$;

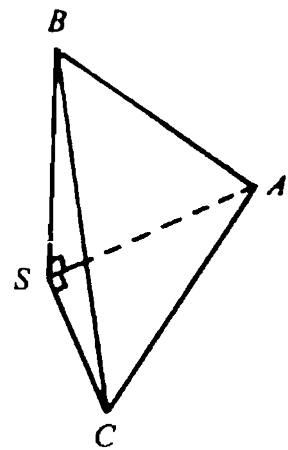
$h=5$ m. Natijada tekisliklar orasidagi masofa $OO_1=16-5=11$ m bo'ladi.

Javobi: B).

5. Berilgan. $SABC$ — uchburchakli piramida, $SA\perp SB$, $SB\perp SC$, $SA\perp SC$, $SA=\sqrt{70}$, $SB=\sqrt{99}$, $SC=\sqrt{126}$.

V_{pir} hisoblansin (11.3.5-chizma).

Yechilishi. Agar piramidaning asosi sifatida uning yon yoqlaridan birini qabul qilsak, masala juda oson yechiladi. Piramidaning yon qirralari o'zaro perpendikulyar bo'lganligidan, asos sifatida tanlab olingan uchburchak — to'g'ri burchakli uchburchak bo'ladi, piramidaning balandligi esa SB qirraga tengdir. U holda,



11.3.5-chizma.

$$S_{\Delta ASC}=\frac{1}{2}AS\cdot CS=\frac{1}{2}\sqrt{70}\cdot\sqrt{126}.$$

Piramidaning hajmini hisoblaymiz:

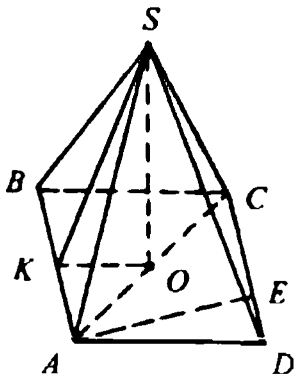
$$V=\frac{1}{3}S_{\Delta ASC}\cdot SB=\frac{1}{6}\sqrt{70}\cdot\sqrt{99}\cdot\sqrt{126}=\frac{1}{6}\sqrt{7\cdot2\cdot5\cdot2\cdot9\cdot7\cdot9\cdot11}=$$

$$=\frac{1}{6}\cdot7\cdot2\cdot9\sqrt{55}=21\sqrt{55}.$$

Javobi: A).

6. Berilgan. $SABCD$ — piramida, $ABCD$ — romb, $\angle ADC=45°$, $OK=r=3$ sm, $SO\perp(ABCD)$, $SO=4$ sm.

S_{yon} hisoblansin (11.3.6-chizma).



11.3.6-chizma.

Yechilishi. Aylana bilan romb tomonining urinish nuqtasini K deb belgilaymiz. Urinish nuqtasiga o'tkazilgan $OK=r$ radius AB urinmaga perpendikulyar bo'ladi, $OK \perp AB$. U holda uch perpendikulyar haqidagi teoremaga asosan (8-§), $SK \perp AB$ bo'ladi. Demak, SK kesma ΔABS ning balandligidir. ΔKSO dan $SK=\sqrt{SO^2+OK^2}=\sqrt{4^2+3^2}=$ $=\sqrt{25}=5$ sm. Ikkinchi tomondan, $S_{\Delta ASB}=\frac{1}{2}AB\cdot SK$. $ABCD$ rombning A uchidan $AE\perp CD$ balandlik o'tkazamiz. $AE\perp AB$, $OK\perp AB$ bo'lganligidan, ular o'zaro parallel va $AE=2\cdot OK=2\cdot 3=6$ sm. Endi rombning tomoni uzunligini to'g'ri burchakli ΔAED dan topamiz. $AD=\frac{AE}{\sin 45^\circ}=6\sqrt{2}$ sm. Demak, $S_{\Delta ASB}=\frac{1}{2}6\sqrt{2}\cdot 5=15\sqrt{2}$ sm^2 va piramidaning yon sirti:

$$S_{yon.}=4\cdot S_{\Delta ASB}=4\cdot 15\sqrt{2}=60\sqrt{2}\ \text{sm}^2.$$

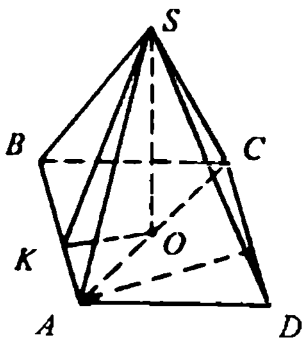
Javobi: C).

7. Berilgan. $SABCD$ — piramida, $ABCD$ — to'g'ri to'rtburchak, $S_{a.}=1$ m^2, $\angle SCB=30°$; $\angle SAB=60°$, $(SAB)\perp(ABCD)$; $(SBC)\perp(ABCD)$

V_{pir} hisoblansin (11.3.7-chizma).

Yechilishi. (SAB) va (SBC) tekisliklar piramidaning asosiga perpendikulyar bo'lganligidan, ularning kesishish chizig'i SB asosga perpendikulyar bo'ladi, demak, u piramidaning balandligidir.

Piramidaning asosi to'g'ri to'rtburchak bo'lganligidan, $BC\perp CD$, $AB\perp AD$. Uch perpendikulyar haqidagi teoremaga muvofiq (8-§), $SA\perp AD$, $BC\perp CD$ va chiziqli burchaklar mos ravishda, $\angle SCB=30°$, $\angle SAB=60°$ bo'ladi.



11.3.7-chizma.

$SB=H$ bo'lsin. ΔSBC dan: $BC=H\cdot \mathrm{ctg}30°=H\sqrt{3}$, ΔASB dan $AB=H\cdot \mathrm{ctg}60°=H\cdot \frac{\sqrt{3}}{3}$ U holda asos — $ABCD$ to'g'ri to'rtburchakning yuzi $S=AB\cdot BC$ bo'ladi yoki $H\sqrt{3}\cdot H\frac{1}{\sqrt{3}}=1\text{m}^2$; $H^2=1$; $H=1$ m.

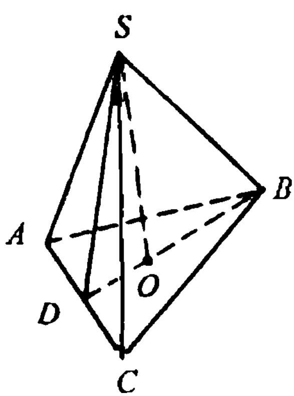
Demak, piramidaning hajmi $V=\frac{1}{3}S_{as}\cdot H=\frac{1}{3}\cdot 1\cdot 1=$ $=\frac{1}{3}$ m³.

Javobi: D).

8. Berilgan. $SABC$ — muntazam piramida, $\frac{S_{yon}}{S_{as}}=k$.

$\angle BSO$ topilsin (11.3.8-chizma).

Yechilishi. Uchburchakli muntazam piramidaning SO balandligi ΔABC medianalarining kesishish nuqtasidan o'tadi va $OB=R$ — tashqi chizilgan aylananing radiusi, $OD=r$ — ichki chizilgan aylananing radiusi. $AB=BC=AC=a$ bo'lsin, u holda



11.3.8-chizma.

$$R = \frac{a}{2\sin 60^\circ} = \frac{a}{\sqrt{3}}; \quad r = \frac{R}{2} = \frac{a}{2\sqrt{3}}.$$

$\angle BSO=\alpha$ deb olamiz, u vaqtda to'g'ri burchakli ΔSOB dan:

$SO = OB \cdot \text{ctg}\alpha = \frac{a\,\text{ctg}\alpha}{\sqrt{3}}$. ΔSOD dan SD apofemani topamiz:

$$SD^2 = SO^2 + OS^2 = \frac{a^2}{12} + \frac{a^2\text{ctg}^2\alpha}{3} = \frac{a^2}{12}(1+4\,\text{ctg}^2\alpha),$$

$$CD = \frac{a}{6}\sqrt{3(1+4\text{ctg}^2\alpha)}.$$

U holda piramidaning yon sirti $S_{yon} = \frac{1}{2} \cdot 3a \cdot \frac{a}{6}\sqrt{3(1+4\text{ctg}^2\alpha)}$.

Asosning yuzi $S_{ac} = \frac{a^2\sqrt{3}}{4}$. Berilganlardan foydalansak, α ga nisbatan

$$\frac{a^2\sqrt{3}}{4}\sqrt{3(1+4\text{ctg}^2\alpha)} = k\frac{a^2\sqrt{3}}{4}, \quad 1+4\text{ctg}^2\alpha = k^2,$$

tenglamani hosil qilamiz. Bu yerdan, $\text{ctg}^2\alpha = \frac{k^2-1}{4}$; $\text{ctg}\alpha = \frac{\sqrt{k^2-1}}{2}$; $k>1$. Natijada $\alpha = \text{arcctg}\frac{\sqrt{k^2-1}}{2}$; $k>1$ bo'ladi.

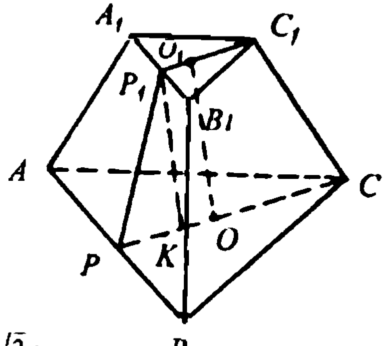
Javobi: E).

9. Berilgan. $ABCA_1B_1C_1$ — muntazam kesik piramida, $AB=12$ dm, $A_1B_1=6$ dm, $N=1$ dm.

S_{yon} hisoblansin (11.3.9-chizma).

Yechilishi. Piramida muntazam bo'lganligidan, (11.3.9-chizma) ΔABC va $\Delta A_1B_1C_1$ teng tomonli uchburchaklardir. O va O_1 lar bu uchburchaklar medianalarining kesishish nuqtalari bo'lib, medianalarning xossasiga asosan, $OR = \frac{1}{3}CP$, $O_1P_1 = \frac{1}{3}C_1P_1$, bu yerda CP va C_1P_1 —

asoslarning medianalari. P_1 nuqtadan quyi asosga P_1K perpendikulyar o‘tkazamiz, $P_1K=OO_1=1$ dm.



11.3.9-chizma.

$CP\perp AB$, $\angle ABC=60°$ bo‘lganligidan, to‘g‘ri burchakli $\triangle CBP$ da:

$CP=CB\cdot\sin60°=12\frac{\sqrt{3}}{2}=6\sqrt{3}$;

to‘g‘ri burchakli $\triangle C_1B_1P_1$ da:

$C_1P_1=C_1B_1\sin60°=6\cdot\frac{\sqrt{3}}{2}=3\sqrt{3}$.

U holda, $OR=\frac{1}{3}CP=\frac{1}{3}\cdot6\sqrt{3}=2\sqrt{3}$; $O_1P_1=\frac{1}{3}C_1P_1=\sqrt{3}$ dm.

$$PK=OP-O_1P_1=2\sqrt{3}-\sqrt{3}=\sqrt{3}\text{ dm.}$$

To‘g‘ri burchakli $\triangle PP_1K$ dan PP_1 apofemani topamiz:

$$PP_1=\sqrt{P_1K^2+PK^2}=\sqrt{1+3}=2\text{ dm.}$$

Asoslarning perimetrlari 36, 18 dm ligi ravshan.

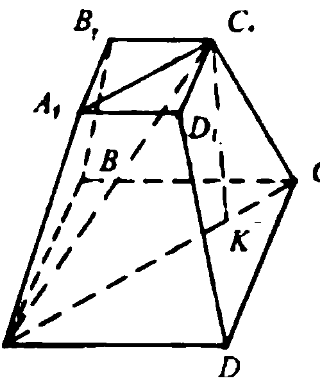
Natijada kesik piramidaning yon sirti $S_{yon}=\frac{36+18}{2}\cdot2=$ $=54$ dm².

Javobi: B).

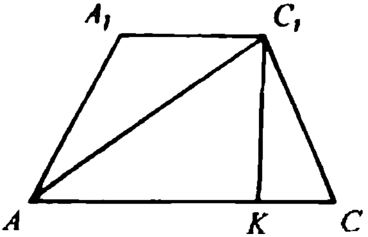
10. B e r i l g a n. $ABCDA_1B_1C_1D_1$ — muntazam kesik piramida, $ABCD$ — kvadrat, $A_1B_1C_1D_1$ — kvadrat, $AB=7$ sm; $A_1B_1=5$ sm, $AC_1=9$ sm.

V_{kp} h i s o b l a n s i n (11.3.10-chizma).

Y e c h i l i s h i. Ma’lumki, kesik piramidaning hajmi $V_{k.p.}=\frac{1}{3}H(S_1+S_2+\sqrt{S_1S_2})$ formuladan topiladi. Kesik



11.3.10-chizma.



11.3.11-chizma.

piramidaning asoslari kvadratlardan iborat bo'lganligidan, asoslarning yuzlarini S_1, S_2 deb belgilasak, $S_1=7^2=49$ sm²; $S_2=5^2=25$ sm² bo'ladi. Kesik piramidaning diagonal kesimi AA_1C_1C teng yonli trapetsiyadan iborat (13.3.11-chizma).

ΔACD dan: $AC=\sqrt{AD^2+DC^2}=7\sqrt{2}$ sm,

$\Delta A_1C_1D_1$ dan: $A_1C_1=\sqrt{A_1D_1^2+D_1C_1^2}=5\sqrt{2}$ sm.

$KC=\frac{1}{2}(7\sqrt{2}-5\sqrt{2})=\sqrt{2}$ sm.

ΔAC_1K ni qaraymiz: unda $\angle AKC=90°$
va $C_1K^2=AC_1^2-AK^2=9^2-(7\sqrt{2}-\sqrt{2})^2=81-72=$
$=9$ sm²; $C_1K=H=3$ sm.

Demak, $V=\frac{1}{3}\cdot 3(49+25+7\cdot 5)=109$ sm³.

Javobi: A).

11.4. Mustaqil yechish uchun masalalar

1. To'rtburchakli muntazam piramida yon qirrasining uzunligi b va asos tekisligi bilan α burchak tashkil qilsa, piramida diagonal kesimining yuzi hisoblansin.

A) $\frac{1}{2}b^2\cos2\alpha$; B) $\frac{1}{2}b^2\sin2\alpha$; C) $b^2\sin2\alpha$; D) $\frac{1}{4}b^2\sin2\alpha$;
E) $\frac{1}{8}b^2\sin2\alpha$.

2. Uchburchakli muntazam piramida yon qirrasining uzunligi l va asos tekisligi bilan α burchak tashkil qiladi. Piramidaning hajmi hisoblansin.

A) $l^3\sin2\alpha$; B) $\frac{\sqrt{2}}{4}l^3\sin\alpha\cos2\alpha$; C) $\frac{\sqrt{2}}{9}l^3\sin2\alpha\cos\alpha$;
D) $\frac{\sqrt{3}}{8}l^3\sin2\alpha\cos\alpha$; E) $l^3\sin2\alpha$.

3. Uchburchakli muntazam piramidaning yon qirrasi b ga teng va asos tekisligi bilan 60° li burchak tashkil etadi. Piramida asosining bir tomonidan qarama-qarshi yon qirraga perpendikulyar tekislik o'tkazilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) $\frac{9b^2}{32}$; B) $\frac{9b^2}{8}$; C) $\frac{3b^2}{4}$; D) $\frac{5b^2}{9}$; E) $\frac{7b^2}{16}$.

4. To'rtburchakli muntazam piramidaning balandligi h, asosidagi ikki yoqli burchagi 60° ga teng bo'lsa, piramidaning yon sirti hisoblansin.

A) $2h^2$; B) $\frac{4}{3}h^2$; C) $\frac{8h^2}{3}$; D) $3h^2$; E) $\frac{5}{6}h^2$.

5. Piramidaning asosi teng yonli uchburchakdan iborat bo'lib, uning teng tomonlari 6 sm dan, uchinchi tomoni esa 8 sm. Piramidaning yon qirralari o'zaro

teng va har biri 9 sm bo'lsa, piramidaning hajmi hisoblansin.

A) 42; B) 32; C) 56; D) 64; E) 48 sm^3.

6. Muntazam tetraedrning balandligi h bo'lsa, uning to'la sirti hisoblansin.

A) $\frac{2}{5}h^2$; B) $2h^2\sqrt{3}$; C) $\frac{4\sqrt{2}h^2}{3}$; D) $\frac{3\sqrt{3}h^2}{2}$; E) $\frac{3\sqrt{2}h^2}{2}$.

7. Piramidaning asosi teng yonli uchburchakdan iborat. Uchburchakning yon tomoni a, uchidagi burchagi α. Piramidaning barcha yon qirralari asos tekisligi bilan o'zaro teng β burchak tashkil etishi ma'lum bo'lsa, uning hajmi hisoblansin.

A) $\frac{2}{3}a^3\cos\alpha\cdot\mathrm{tg}\frac{\beta}{2}$; B) $\frac{1}{6}a^3\sin\frac{\alpha}{2}\cdot\mathrm{tg}\beta$;

C) $\frac{1}{2}a^3\sin\alpha\cdot\mathrm{tg}\frac{\beta}{2}$; D) $\frac{1}{12}a^3\sin\frac{\alpha}{2}\cdot\mathrm{tg}\frac{\beta}{2}$;

E) $\frac{1}{6}a^3\sin\alpha\cdot\mathrm{tg}\beta$.

8. To'rtburchakli muntazam piramida asosining tomoni a, asosidagi ikki yoqli burchagi α bo'lsa, uning hajmi hisoblansin.

A) $\frac{1}{6}a^3\mathrm{tg}\alpha$; B) $\frac{1}{2}a^3\cos\alpha$; C) $\frac{1}{6}a^3\sin\alpha$; D) $\frac{1}{12}a^3\mathrm{ctg}\alpha$;

E) $\frac{1}{4}a^3\mathrm{tg}\alpha$.

9. Oltiburchakli muntazam piramidaning apofemasi m, asosidagi ikki yoqli burchagi α ga teng bo'lsa, piramidaning to'la sirtini hisoblang.

A) $4m^2\cos\alpha\cdot\sin\frac{\alpha}{2}$; B) $\sqrt{3}\,m^2\sin\alpha\cdot\mathrm{tg}^2\frac{\alpha}{2}$;

C) $2\sqrt{3}\,m^2\sin\alpha\cdot\cos^2\frac{\alpha}{2}$; D) $3m^2\cos\alpha\cdot\sin^2\frac{\alpha}{2}$;

E) $4\sqrt{3}\,m^2\cdot\cos\alpha\cdot\cos^2\frac{\alpha}{2}$.

10. Piramidaning asosi — yon tomoni a, o'tkir burchagi α bo'lgan teng yonli trapetsiyadan iborat. Piramidaning yon yoqlari asos tekisligi bilan bir xil β burchak tashkil qilsa, piramidaning hajmi hisoblansin.

A) $\frac{1}{2}a^3\cos2\alpha\cdot\mathrm{tg}\beta$; B) $\frac{1}{3}a^3\sin2\alpha\cdot\mathrm{tg}\beta$;

C) $\frac{1}{6}a^3\sin2\alpha\cdot\mathrm{tg}\beta$; D) $\frac{2}{3}a^3\sin2\alpha\cdot\mathrm{tg}2\beta$;

E) $\frac{1}{6}a^3\sin2\alpha\cdot\cos2\beta$.

11. Uchburchakli muntazam kesik piramida asoslarining tomonlari a va $b (a>b)$ bo'lib, yon qirrasi asos tekisligi bilan $60°$ li burchak tashkil etadi. Kesik piramidaning balandligi topilsin.

A) $a-b$; B) $\sqrt{ab}$; C) $a+b$; D) $\sqrt{a^2+b^2}$; E) $\sqrt{a^2-b^2}$.

12. To'rtburchakli muntazam kesik piramida asoslarining tomonlari 10 va 2 m, piramidaning balandligi 4 m. Kesik piramida to'la sirtining yuzi hisoblansin.

A) 216; B) 256; C) 248; D) 242; E) 238 m^2.

13. To'rtburchakli kesik piramida asoslarining tomonlari a va b $(a>b)$, katta asosidagi ikki yoqli burchagi α teng bo'lsa, kesik piramidaning hajmi hisoblansin.

A) $\frac{1}{12}(a^3-b^3)\mathrm{tg}\alpha$; B) $\frac{1}{6}(a^3-b^3)\mathrm{tg}\alpha$; C) $\frac{1}{2}(a^3+b^3)\mathrm{ctg}\alpha$;

D) $\frac{a}{3}(a^3-b^3)\mathrm{tg}\alpha$; E) $\frac{2}{3}(a^3-b^3)\mathrm{ctg}\alpha$.

14. Oltiburchakli muntazam piramidaning yon qirrasi b va asos tekisligi bilan α burchak tashkil qiladi. Piramida eng katta diagonal kesimining yuzi hisoblansin.

A) $\frac{1}{2}b^2\cos2\alpha$; B) $2b^2\cos2\alpha$; C) $\frac{1}{3}b^2\cos\alpha$; D) $\frac{1}{2}b^2\sin2\alpha$;

E) $b^2\sin\alpha$.

15. Uchburchakli muntazam piramidaning apofemasi m va asos tekisligi bilan α burchak tashkil qiladi. Piramida yon sirtining yuzi hisoblansin.

A) $\frac{1}{6}m^2\cos\alpha$; B) $m^2\sin 2\alpha$; C) $3\sqrt{3}\,m^2\cos\alpha$;

D) $\sqrt{3}\,m^2\cos\alpha$; E) $3m^2\cos\alpha$.

16. To'rtburchakli muntazam piramida asosining tomoni 10 m, balandligi 12 m bo'lsa, piramida to'la sirtining yuzi hisoblansin.

A) 360; B) 480; C) 345; D) 540; E) 420 m².

17. Piramidaning asosi kvadratdan iborat bo'lib, balandligi h va asosining uchidan o'tadi. Kvadratning tomoni a bo'lsa, piramida yon sirtining yuzi hisoblansin.

A) $4a(h+\sqrt{a^2+h^2})$; B) $2h(h+\sqrt{ah})$;

C) $h(a+\sqrt{a^2+h^2})$; D) $2a(h+\sqrt{ah})$; E) $a(h+\sqrt{h^2a^2})$.

18. Piramidaning asosi muntazam uchburchakdan iborat bo'lib, yon yoqlaridan biri asosga perpendikulyar, boshqa ikkitasi asos tekisligi bilan 60° li burchaklar tashkil qilsa, piramidaning katta yon qirrasi asos tekisligiga qanday burchak ostida og'ma bo'ladi?

A) $\arcsin\frac{3}{5}$; B) $\mathrm{arctg}\frac{\sqrt{3}}{2}$; C) $\arcsin\frac{2}{3}$; D) arctg2;

E) $\arccos\frac{1}{4}$.

19. Piramidaning asosi — o'tkir burchagi 45° bo'lgan rombdan iborat va rombga ichki chizilgan aylananing radiusi 3 *sm*. Piramidaning balandligi 4 sm va rombga ichki chizilgan aylananing markazidan o'tadi. Piramida yon sirtining yuzi hisoblansin.

A) $64\sqrt{3}$; B) $62\sqrt{2}$; C) 60; D) $60\sqrt{2}$; E) $60\sqrt{3}$ sm².

20. Oltiburchakli muntazam piramida asosining tomoni va piramidaning balandligi 8 dm bo'lsa, kichik diagonal kesimning yuzi hisoblansin.

A) $16\sqrt{15}$; B) $16\sqrt{3}$; C) $16\sqrt{2}$;
D) $16\sqrt{11}$; E) $8\sqrt{19}$ dm.

21. To'rtburchakli muntazam piramidaning balandligi h, asosining diagonali d bo'lsa, uning hajmi hisoblansin.

A) $\frac{d^2h}{12}$; B) $\frac{dh^2}{4}$; C) $\frac{d^2h}{6}$; D) $\frac{dh^2}{6}$; E) $\frac{d^2+h^2}{3}$.

22. Uchburchakli muntazam piramida asosining tomoni a, balandligi h. Asosining bir tomonidan qarama-qarshi qirraga perpendikulyar tekislik o'tkazilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) $\frac{1}{2}(a^2+h^2)$; B) $\frac{4}{5}ah\sqrt{2}$; C) $\frac{2}{5}ah\sqrt{3}$; D) $\frac{4}{5}ah\sqrt{2}$;
E) $\frac{a^2h}{2\sqrt{3h^2+a^2}}$.

23. Piramidaning asosi teng yonli uchburchak va uning tomonlari 10, 10 va 12 sm. Piramidaning yon yoqlari asos tekisligi bilan 45° li burchak tashkil qilsa, uning balandligi topilsin.

A) 7; B) 3; C) 4; D) 5; E) 2 sm.

24. Piramidaning asosi — asosi 6 sm, balandligi 9 sm bo'lgan teng yonli uchburchakdan iborat. Piramidaning yon qirralari o'zaro teng va 13 sm bo'lsa, piramidaning balandligi topilsin.

A) 16; B) 8; C) 14; D) 12; E) 10 sm.

25. Piramidaning balandligi H ga teng. Uning asosiga parallel tekislik o'tkazilgan. Agar kesimning yuzi asos yuzining $\frac{1}{5}$ qismini tashkil qilsa, piramida uchidan kesimgacha bo'lgan masofa topilsin.

A) $\frac{H}{\sqrt{5}}$; B) $\frac{H}{\sqrt{3}}$; C) $\frac{H}{\sqrt{2}}$; D) $H\sqrt{3}$; E) $H\sqrt{5}$.

26. Muntazam to'rtburchakli piramidaning yon yoqlari asos tekisligi bilan bir xil burchak tashkil qiladi. Agar piramida to'la sirti yuzining asosi yuziga nisbati k bo'lsa, yon yoqlarining asos tekisligi bilan tashkil qilgan burchagini toping. Masala k ning qanday qiymatlarida ma'noga ega?

A) $\operatorname{arctg}\frac{k+1}{2}$, $k>1$; B) $\arccos\frac{k}{k-1}$, $k>2$;

C) $\arccos\frac{1}{k-1}$, $k>2$; D) $\operatorname{arctg}\frac{2-k}{k+1}$, $k>1$;

E) $\arcsin\frac{k-1}{2}$, $k>2$.

27. Piramida asosidagi rombning kichik diagonali d, o'tkir burchagi α. Piramida asosidagi hamma ikki yoqli burchaklar β bo'lsa, uning hajmi hisoblansin.

A) $\frac{1}{2}d^3\operatorname{tg}^2\alpha\cdot\sin 2\beta$; B) $\frac{1}{3}d^3\operatorname{tg}\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}$;

C) $\frac{1}{6}d^3\sin 2\alpha\cdot\operatorname{tg}\beta$; D) $\frac{1}{12}d^3\operatorname{ctg}\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}\operatorname{tg}\beta$;

E) $d^3\operatorname{tg}\frac{\alpha}{2}\operatorname{tg}\beta$.

28. Uchburchakli muntazam piramida uchidagi yassi burchagi 90°. Piramida yon sirtining yuzi va asosi yuzining nisbati topilsin.

A) 2:3; B) $\sqrt{3}$; C) $\sqrt{2}:\sqrt{3}$; D) $\sqrt{2}$; E) 5:7.

29. Muntazam piramida asosidagi ko'pburchak ichki burchaklarining yig'indisi 720°. Agar piramidaning yon qirrasi l bo'lib, uning balandligi bilan 30° li burchak tashkil qilsa, piramidaning hajmi hisoblansin.

A) $2l^3$; B) $\frac{2l^3}{15}$; C) $\frac{3l^3}{19}$; D) $\frac{3l^3}{8}$; E) $\frac{3l^3}{16}$.

30. To'rtburchakli muntazam piramida asosining tomoni uning yon qirrasiga teng bo'lib, a ga teng. Piramida to'la sirtining yuzi hisoblansin.

A) $\frac{a^2(1+\sqrt{7})}{2}$; B) $\frac{a^2(1+\sqrt{3})}{2}$; C) $\frac{a^2(\sqrt{2}+1)}{4}$;
D) $\frac{a^2(\sqrt{5}+3)}{2}$; E) $\frac{a^2(1+\sqrt{3})}{4}$.

31. Uchburchakli muntazam piramida asosining tomoni a, asosidagi ikki yoqli burchagi 60° bo'lsa, piramida to'la sirtining yuzi hisoblansin.

A) $\frac{3a^2\sqrt{2}}{4}$; B) $\frac{3a^2}{4}$; C) $\frac{3a^2\sqrt{3}}{4}$; D) $\frac{a^2\sqrt{3}}{2}$; E) $\frac{2a^2\sqrt{5}}{4}$.

32. Oltiburchakli muntazam piramidaning apofemasi l, asosidagi ikki yoqli burchagi 60° bo'lsa, piramida to'la sirtining yuzi hisoblansin.

A) $\frac{3}{5}l^2\sqrt{5}$; B) $\frac{4}{5}l^2\sqrt{2}$; C) $\frac{l^2\sqrt{3}}{2}$; D) $\frac{3l^2\sqrt{3}}{2}$; E) $\frac{l^2\sqrt{5}}{2}$.

33. Piramida asosidagi uchburchakning tomonlari a, a va b bo'lib, piramida yon qirralarining har biri asos tekisligi bilan 60° li burchak tashkil qiladi. Piramidaning hajmi hisoblansin.

A) $\frac{(a^2+b^2)\sqrt{2}}{4}$; B) $\frac{a^2b\sqrt{3}}{12}$; C) $\frac{ab^2\sqrt{3}}{12}$; D) $\frac{ab^2\sqrt{3}}{12}$;
E) $\frac{ab^2\sqrt{2}}{6}$.

34. Uchburchakli muntazam piramida asosining tomoni 1 sm, yon sirtining yuzi 3 sm² bo'lsa, piramidaning hajmi hisoblansin.

A) $\frac{\sqrt{29}}{2}$; B) $\frac{\sqrt{61}}{7}$; C) $\frac{\sqrt{45}}{14}$; D) $\frac{\sqrt{35}}{12}$; E) $\frac{\sqrt{47}}{24}$ sm³.

35. Piramidaning asosi — diagonali c ga teng va diagonallari orasidagi burchak 60° bo'lgan to'g'ri to'rtburchakdan iborat. Piramidaning yon qirralaridan har biri asos tekisligi bilan 45° li burchak tashkil qiladi. Piramidaning hajmi hisoblansin.

A) $\frac{c^3\sqrt{3}}{24}$; B) $\frac{c^3\sqrt{2}}{12}$; C) $\frac{c^3}{6}$; D) $\frac{c^3\sqrt{3}}{14}$; E) $\frac{c^3\sqrt{2}}{6}$.

36. Uchburchakli piramidaning yon qirralari juft-juft perpendikulyar va mos ravishda, $\sqrt{70}$, $\sqrt{99}$ va $\sqrt{126}$ sm ga teng. Piramidaning hajmi hisoblansin.

A) $42\sqrt{3}$; B) $35\sqrt{7}$; C) $21\sqrt{55}$; D) $21\sqrt{33}$;
E) $42\sqrt{11}$ sm^3.

37. Uchburchakli muntazam piramida asosining tomoni a va yon yoqlari asosiga 45° li burchak ostida og'ma bo'lsa, piramida to'la sirtining yuzi topilsin.

A) $\frac{a^2\sqrt{3}}{4}$; B) $\frac{a^2\sqrt{3}(\sqrt{2}+1)}{2}$; C) $\frac{a^2\sqrt{2}(\sqrt{2}+1)}{8}$;
D) $\frac{a^2\sqrt{3}}{4}(\sqrt{2}+1)$; E) $\frac{a^2(\sqrt{2}+1)}{4}$.

38. To'rtburchakli piramidaning asosi — o'tkir burchagi 30° bo'lgan rombdan iborat. Piramida yon yoqlarining har biri asos tekisligi bilan 60° li burchak tashkil etadi. Agar rombga ichki chizilgan aylananing radiusi r bo'lsa, piramida to'la sirtining yuzi hisoblansin.

A) $16r^2$; B) $24r^2$; C) $18r^2$; D) $22r^2$; E) $32r^2$.

39. Uchburchakli muntazam piramidaning yon qirrasi l, balandligi h. Uning asosidagi ikki yoqli burchak topilsin.

A) $\arccos\frac{2h}{h+1}$; B) $\arcsin\frac{h+l}{l}$; C) $2\text{arctg}\frac{h}{l}$;
D) $\arcsin\frac{2h^2}{h^2+l^2}$; E) $\text{arctg}\frac{2h}{\sqrt{l^2-h^2}}$.

40. n burchakli muntazam piramida asosining yuzi Q bo'lib, piramidaning har bir yon yog'i balandlik bilan φ burchak tashkil qiladi. Piramida to'la sirtining yuzi hisoblansin.

A) $\frac{Q(1+\text{tg}\varphi)}{\sin\varphi}$; B) $\frac{Q(1-\text{tg}\varphi)}{\cos\varphi}$; C) $\frac{Q\sin\varphi}{\sqrt{3}}$; D) $\frac{Q(1+\sin\varphi)}{\sin\varphi}$,
E) $\frac{Q(1+\cos\varphi)}{\sin\varphi}$.

41. Uchburchakli muntazam kesik piramida asoslarining tomonlari a va $b(a>b)$, yon qirralari asos tekisligi bilan α burchak tashkil qiladi. Piramidaning hajmi hisoblansin.

A) $\frac{3}{5}b^3\sin2\alpha$; B) $\frac{4}{5}a^3\cos2\alpha$; C) $\frac{1}{12}(a^3-b^3)\text{tg}\alpha$;

D) $\frac{2}{3}(a^3-b^3)\cos\alpha$; E) $\frac{1}{6}(a^3-b^3)\text{ctg}\alpha$.

42. Piramidaning asosi gipotenuzasi c, o'tkir burchagi α bo'lgan to'g'ri burchakli uchburchakdir. Piramidaning barcha yon qirralari asos tekisligiga bir xil β burchak ostida og'ma bo'lsa, piramidaning hajmi hisoblansin.

A) $\frac{c^3}{8}\cos2\alpha\cdot\text{tg}\beta$; B) $\frac{c^3}{24}\sin2\alpha\cdot\text{tg}\beta$; C) $\frac{c^3}{16}\sin2\alpha\cdot\text{ctg}\beta$;

D) $\frac{c^3}{6}\sin^2\alpha\cdot\text{tg}\beta$; E) $\frac{c^3}{3}\cos^2\alpha\cdot\text{tg}\beta$.

43. Piramidaning asosi — gipotenuzasi c, o'tkir burchagi α bo'lgan to'g'ri burchakli uchburchakdir. Piramidaning hamma yon qirralari asos tekisligi bilan bir xil, β burchak tashkil qiladi. Piramidaning uchidan gipotenuzaga qarama-qarshi yassi burchak topilsin.

A) $180°-2\beta$; B) $90°-\beta$; C) $90°+2\beta$; D) $\frac{3\beta}{2}$;
E) $180°-4\beta$.

44. To'rtburchakli muntazam kesik piramidaning balandligi H bo'lib, u piramidaning yon qirrasi va diagonali asos tekisligi bilan, mos ravishda α va β burchaklar tashkil qiladi. Piramida yon sirtining yuzi hisoblansin.

A) $\frac{1}{3}H^2\cos2\beta\cdot\text{tg}^2\frac{\beta}{2}$; B) $\frac{1}{6}H^2\text{tg}\beta\cdot\sin^2\frac{\beta}{2}$;

C) $\frac{1}{2}H^2\sqrt{3+\cos^2\beta}$; D) $2H^2\text{ctg}\beta\sqrt{2+\text{ctg}^2\alpha}$;

E) $2H\cdot\text{ctg}\beta\sqrt{1+\sin^2\alpha}$.

45. To‘rtburchakli muntazam kesik piramidaning balandligi H, diagonali d va asosidagi ikki yoqli burchagi α bo‘lsa, uning hajmi hisoblansin.

A) $2(d^2-H^2)+H^2\sin 2\alpha$; B) $\frac{H^2}{6}(2(d^2-H^2)\sin 2\alpha)$;

C) $\frac{H^2}{6}((H^2-d^2)\mathrm{ctg}^2\alpha)$; D) $\frac{H^2}{2}(2(d^2-H^2)H^2\mathrm{tg}^2\alpha)$;

E) $\frac{H^2}{6}(3(d^2-H^2)+2H^2\mathrm{ctg}^2\alpha)$.

46. To‘rtburchakli muntazam kesik piramidaning asoslari tomonlari a va $\sqrt{3}$ bo‘lib, yon yog‘i asos tekisligi bilan γ burchak tashkil qiladi. Kesik piramida to‘la sirtining yuzi hisoblansin.

A) $\frac{a^3(3+4\sin\gamma)}{\cos^2\gamma}$; B) $\frac{2a^2(1+2\cos\gamma)}{\cos\gamma}$; C) $\frac{a^2(1+\cos\gamma)}{2\cos\gamma}$;

D) $\frac{2a^2(1+2\sin^2\gamma)}{\cos 2\gamma}$; E) $\frac{a^2(2-\cos 2\gamma)}{2\sin\gamma}$.

47. To‘rtburchakli muntazam piramida asosining tomoni a, asosidagi ikki yoqli burchak α. Piramida asosining tomoni orqali asos tekisligi bilan β burchak tashkil qiluvchi tekislik o‘tkazilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) $\frac{a^2\cos^2(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$; B) $\frac{a^2\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$; C) $\frac{a^2\sin^2\alpha\cdot\cos\beta}{\sin^2(\alpha+\beta)}$;

D) $\frac{a^2\cos^2\alpha\cdot\sin\beta}{\sin^2(\alpha+\beta)}$; E) $\frac{a^2\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$.

48. To‘rtburchakli muntazam piramidaning yon qirrasi l, qo‘shni yon yoqlar orasidagi ikki yoqli burchak β bo‘lsa, uning hajmi hisoblansin.

A) $\frac{2}{5}l^3\cos^2 2\beta\cdot\mathrm{tg}\frac{\beta}{2}$; B) $\frac{1}{6}l^3\sin^2\frac{\beta}{2}\cdot\mathrm{ctg}\beta$;

C) $\frac{1}{12}l^3\sin\beta\cdot\mathrm{tg}\frac{3\beta}{2}$; D) $\frac{2}{3}l^3\frac{\mathrm{ctg}\frac{\beta}{2}\cdot\cos\beta}{\sin^2\frac{\beta}{2}}$; E) $\frac{2}{3}l^3\sin 2\beta$.

49. Piramidaning asosi yon tomonlari va kichik asosi o‘zaro teng bo‘lib, katta asosi a va o‘tmas burchagi α bo‘lgan trapetsiyadan iborat. Piramidaning barcha yon qirralari asos tekisligi bilan bir xil, β burchak tashkil qiladi. Piramidaning hajmi hisoblansin.

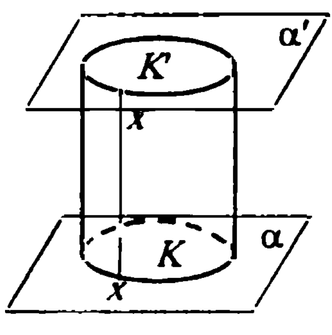
A) $\dfrac{a^3\sin^3\alpha\cdot\operatorname{tg}\beta}{12\cos^3(180^\circ-\frac{3\alpha}{2})}$; B) $\dfrac{a^3\sin\beta\cdot\operatorname{tg}^2\alpha}{4\cos^2(\alpha+\varphi)}$; C) $\dfrac{a^3\cos\alpha\cdot\operatorname{tg}\beta}{4(1+\sin\alpha)}$;

D) $\dfrac{a^3\sin^3\alpha}{4\operatorname{tg}\beta}$; E) $\dfrac{a^3\sin^2\alpha\cdot\operatorname{tg}\beta}{12\cos\frac{\alpha}{2}}$.

12-§. SILINDR

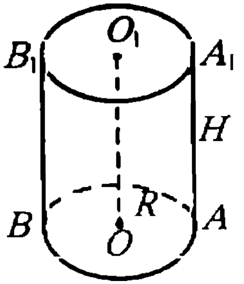
12.1. Asosiy tushunchalar va tasdiqlar

Silindr ikkita parallel tekislik orasida joylashgan va bu tekisliklardan biridagi doirani kesib o‘tadigan hamma parallel to‘g‘ri chiziqlar kesmalaridan tashkil topgan jismdir. Silindrning *yasovchilari* uchlari shu doiraning aylanasida yotgan kesmalardir. Silindrning *sirti* silindr asoslaridan — parallel tekisliklarda yotgan ikkita doiradan va yon sirtidan iborat. Yasovchilari asos tekisliklariga perpendikulyar silindr *to‘g‘ri silindr* bo‘ladi (12.1-chizma). Chizmada: α va α' — parallel tekisliklar, XX' kesmalar — yasovchilar, K, K' doiralar — silindrning asoslaridir.

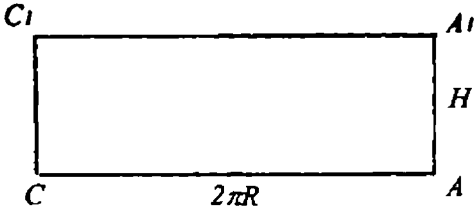


12.1-chizma.

Silindrning *radiusi* uning asosining radiusi, *balandligi* silindr asoslari tekisliklari orasidagi masofadan iborat. Silindrning *o‘qi* uning asoslari-



12.2-chizma.



ning markazlaridan o‘tuvchi to‘g‘ri chiziqdir. Silindrning *o‘q kesimi* uning o‘qi orqali o‘tuvchi kesimdan iborat.

1. Agar silindr asosining radiusi $OA=R$, yasovchisi $AA_1=H$ bo‘lsa (12.2-chizma), silindr yon sirtining yuzi

$$S_{yon}=2\pi R\cdot H \qquad (12.1)$$

(asos aylanasi uzunligi bilan balandligining ko‘paytmasiga teng).

2. Silindr to‘la sirtining yuzi uning yon sirti va asoslari yuzlarining yig‘indisiga teng:

$$S_t=S_{yon}+2S_{asos}=2\pi R\cdot H+2\pi R^2=2\pi R(H+R). \qquad (12.2)$$

3. Silindrning hajmi asosining yuzi bilan balandligining ko‘paytmasiga teng:

$$V_t=S_{asos}\cdot H=\pi R^2 H. \qquad (12.3)$$

12.2. Mavzuga oid masalalar

1. Silindrning o‘q kesimi kvadratdan iborat va uning yuzi Q bo‘lsa, silindr asosining yuzi hisoblansin.

A) $\frac{\pi Q}{6}$; B) $2\pi Q$; C) $\frac{\pi Q}{4}$; D) $\frac{\pi Q}{3}$; E) $\frac{\pi Q}{12}$.

2. Silindrning balandligi 8 dm, asosining radiusi 5 dm. Silindrning o'qiga parallel tekislik shunday o'tkazilganki, kesimda kvadrat hosil bo'lgan. Bu kesimdan silindrning o'qigacha bo'lgan masofa topilsin.

A) 3; B) 4; C) 2,5; D) 5; E) 3,5 dm.

3. Silindr asosi yuzining o'q kesimi yuziga nisbati π:4 kabi. O'q kesimning diagonallari orasidagi burchak topilsin.

A) $\frac{\pi}{6}$; B) $\frac{\pi}{2}$; C) $\frac{\pi}{12}$; D) $\frac{\pi}{4}$; E) $\frac{\pi}{3}$.

4. Silindr to'la sirtining yuzi 62 sm^2, yon sirtining yuzi 30 sm^2 bo'lsa, silindrning balandligi topilsin.

A) $\frac{8}{\sqrt{\pi}}$; B) $\frac{16}{\sqrt{\pi}}$; C) $\frac{24}{\sqrt{\pi}}$; D) $\frac{15}{4\sqrt{\pi}}$; E) $\frac{18}{\sqrt{\pi}}$.

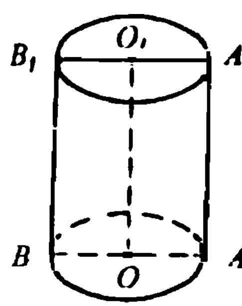
5. Silindrning o'qiga parallel qilib, o'qdan a uzoqlikda kesim o'tkazilgan. Kesim silindrning asosidagi aylanadan α radianga teng bo'lgan yoyni ajratadi. Agar kesimning yuzi S bo'lsa, silindrning hajmi topilsin.

A) $\frac{\pi S\sqrt{a}}{\sin\alpha}$; B) $\frac{\pi S^2}{a\cos\alpha}$; C) $\pi a S \operatorname{tg}\alpha$; D) $\frac{\pi Sa}{\cos\alpha}$; E) $\frac{\pi a S}{\sin\alpha}$.

12.3. Mavzuga oid masalalarning yechimlari

1. B e r i l g a n. AB_1 — silindr, AA_1B_1B — kvadrat, $S_{AA_1B_1B} = Q$.

S_{acos} h i s o b l a n s i n (12.3.1-chizma).



12.3.1-chizma.

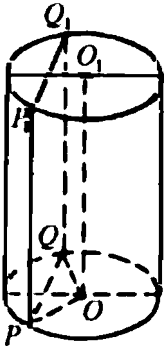
Y e c h i l i s h i. Agar silindr asosining radiusi $OA=R$ bo'lsa, asosining yuzi $S_{asos}=\pi R^2$ bo'ladi. AA_1B_1B o'q kesim kvadrat bo'lganligidan, $AA_1=AB$ yoki $N=2R$. U holda, o'q kesimning yuzi $Q=2R\cdot H=4R^2$ bo'ladi va $R^2=\frac{Q}{4}$.

Demak, silindr asosining yuzi $S=\frac{\pi Q}{4}$ bo'ladi.

Javobi: C).

2. Berilgan. AB_1 — silindr, $OO_1 \parallel (PP_1QQ_1)$, PP_1QQ_1 — kvadrat, $OO_1=8$ dm, $R=5$ dm.

OK topilsin (12.3.2-chizma).



12.3.2-chizma.

Yechilishi. O markazni PQ vatarning P va Q uchlari bilan tutashtirsak, ΔOPQ teng yonli bo'ladi: $OP=OQ=R$. O nuqtadan kesimgacha masofa PQ ga o'tkazilgan OK perpendikulyarning uzunligiga teng. OK kesma ΔOPQ ning medianasi ham bo'lganligidan, $PK=KQ$. Berilishiga ko'ra, PP_1QQ_1 — kvadrat va $PQ=PP_1=8$ dm va, demak, $PK=$ $=\frac{1}{2}PQ=4$ dm. Endi to'g'ri burchakli ΔOPK dan Pifagor teoremasiga asosan, $OK=\sqrt{OP^2-PK^2}=\sqrt{5^2-4^2}=3$ dm ekanligini olamiz.

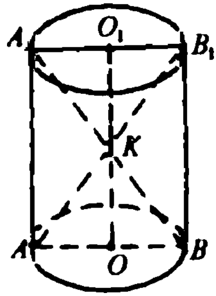
Javobi: A).

3. Berilgan. AB_1 — silindr, $S_{asos}:S_{kes}=\pi:4$

$\angle AKB$ topilsin (12.3.3-chizma).

Yechilishi. $\angle AKB=\alpha$, $OA=R$ belgilashlarni kiritamiz. ΔAKB teng yonli va $AK=BK$ bo'lganligidan, OK balandlik ham mediana, ham bissektrisa bo'ladi. Demak, $\angle AKO=\frac{\alpha}{2}$. To'g'ri burchakli ΔAKO dan

$\sin\frac{\alpha}{2} = \frac{OA}{AK} = \frac{R}{AK}$, $AK = \frac{R}{\sin\frac{\alpha}{2}}$ bo'lishi kelib chiqadi. Silindr o'q kesimining diagonali $AB_1 = 2AK = \frac{2R}{\sin\frac{\alpha}{2}}$ va uning yuzi $S_{kes} = \frac{1}{2}d^2 = = \frac{1}{2}(AB_1)^2 = \frac{1}{2}\left(\frac{2R}{\sin\frac{\alpha}{2}}\right)^2 = \frac{2R^2}{\sin^2\frac{\alpha}{2}}$. Silindr asosining yuzi $S_{asos} = \pi R^2$. Berilganiga ko'ra



12.3.3-chizma.

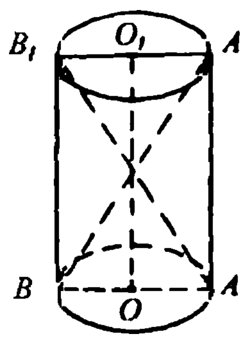
$S_{asos} : S_{kes} = \pi : 4$, shu sababli α ga nisbatan $\frac{S_{asos}}{S_{kes}} = \frac{\pi R^2}{2R^2} \times \sin^2\frac{\alpha}{2} = \frac{\pi}{4}$, $\sin^2\frac{\alpha}{2} = \frac{1}{2}$ tenglamani olamiz. Bu yerdan $\sin\frac{\alpha}{2} = \frac{1}{\sqrt{2}}$; $\frac{\alpha}{2} = \frac{\pi}{4}$; $\alpha = \frac{\pi}{2}$.

Javobi: B).

4. B e r i l g a n. AB_1 — silindr, $S_t = 62$ sm²; $S_{yon} = = 30$ sm².

H t o p i l s i n (12.3.4-chizma).

Y e c h i l i s h i. Silindr to'la sirtining yuzi va yon sirtining yuzi hisoblanadigan (12.2), (12.3) formulalardan foydalanib, quyidagi tenglamalar sistemasini tuzamiz:



12.3.4-chizma.

$$\begin{cases} 2\pi RH + 2\pi R^2 = 62, \\ 2\pi RH = 30. \end{cases}$$

Natijada $\begin{cases} 30 + 2\pi R^2 = 62 \\ 2\pi RH = 30 \end{cases} \Rightarrow \begin{cases} \pi R^2 = 16, \\ \pi RH = 15 \end{cases} \Rightarrow$

$$\Rightarrow \begin{cases} R = \dfrac{4}{\sqrt{\pi}}, \\ H = \dfrac{15}{4\sqrt{\pi}} \end{cases}$$

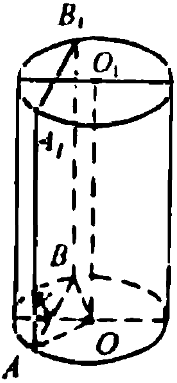
Javobi: D).

5. Berilgan. AB_1 — silindr, $OO_1 \| (AA_1B_1B)$. $S_{AA_1B_1B} = S$, $\cup ACB = \alpha$, $OK \perp (AA_1B_1B)$, $OK = a$,

V_s hisoblansin (12.3.5-chizma).

Yechilishi. $OA = OB = R$, $AA_1 = BB_1 = H$ belgilashlarni kiritamiz. ΔAOB — teng yonli va $\angle AOB = \alpha$ markaziy burchakdan iborat. ΔAOB ning asosidagi $\angle BAO = \angle ABO = \frac{180^\circ - \alpha}{2} = 90^\circ - \frac{\alpha}{2}$ bo'ladi. Sinuslar teoremasiga asosan, $\frac{AB}{\sin\alpha} = \frac{OB}{\sin\left(90^\circ - \frac{\alpha}{2}\right)}$, bu yerdan

$$AB = \frac{R\sin\alpha}{\cos\frac{\alpha}{2}} = \frac{2R\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = 2R\sin\frac{\alpha}{2}.$$



12.3.5-chizma.

Silindr asosining radiusini $OK = a$ orqali ifodalaymiz:

$$\sin\left(90^\circ - \frac{\alpha}{2}\right) = \frac{OK}{OB} = \frac{a}{R} \text{ va } R = \frac{a}{\cos\frac{\alpha}{2}}.$$

U holda $AB = \dfrac{2a\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = 2a\cdot\operatorname{tg}\frac{\alpha}{2}$.

Endi ABB_1A_1 to'g'ri to'rtburchakning balandligini hisoblaymiz:

$$H = \frac{S}{AB} = \frac{S}{2a\operatorname{tg}\frac{\alpha}{2}} = \frac{S}{2a}\operatorname{ctg}\frac{\alpha}{2}.$$

Demak, silindrning hajmi:

$$V = \pi R^2 \cdot H = \pi \cdot \frac{a^2}{\cos^2\frac{\alpha}{2}} \cdot \frac{S\cos\frac{\alpha}{2}}{2a\sin\frac{\alpha}{2}} = \frac{\pi a S}{\sin\alpha}.$$

Javobi: E).

12.4. Mustaqil yechish uchun masalalar

1. Silindr asosining radiusi r, o‘q kesimining diagonali d bo‘lsa, o‘q kesimning yuzi hisoblansin.

A) $h\sqrt{d^2-3r^2}$; B) $r\sqrt{d^2-2r^2}$; C) $2r\sqrt{d^2-4r^2}$;
D) $2d\sqrt{d^2-4r^2}$; E) $2r\sqrt{4r^2-3r^2}$.

2. Silindr o‘q kesimining diagonali d bo‘lib, asos tekisligiga α burchak ostida og‘ma bo‘lsa, silindr yon sirtining yuzi hisoblansin.

A) $\frac{1}{4}\pi d^2 \operatorname{ctg}\alpha$; B) $\frac{1}{6}\pi d^2\cos 2\alpha$; C) $\frac{1}{3}\pi d^2 \operatorname{tg} 2\alpha$;
D) $\frac{1}{2}\pi d^2 \sin 2\alpha$; E) $\pi d^2\cos\alpha$.

3. Silindrning balandligi 16 sm, asosining radiusi 10 sm. Silindrning o‘qiga parallel kesim o‘tkazilgan va u o‘qdan 60 mm uzoqlikda yotadi. Kesimning yuzi hisoblansin.

A) 216; B) 208; C) 256; D) 196; E) 160 sm^2.

4. Silindrning o‘qiga parallel tekislik o‘tkazilgan. Tekislikning silindr asosi bilan kesishish chizig‘i aylanani $m:n$ kabi nisbatda bo‘ladi. Agar kesimning yuzi S bo‘lsa, silindr yon sirtining yuzi hisoblansin.

A) $\frac{\pi S}{\sin\frac{\pi m}{m+n}}, (m \le n)$; B) $\frac{\pi S}{\cos\frac{\pi m}{m+n}}$; C) $\frac{\pi S n}{\cos\frac{\pi m}{n}}$;
D) $\frac{\pi S m}{\sin\frac{\pi m}{m}}$; E) $\frac{S(m+n)}{\sin\frac{\pi m}{\sqrt{mn}}}$.

5. Silindr asosining yuzi Q va o'q kesimining yuzi M bo'lsa, silindr to'la sirtining yuzi hisoblansin.

A) $2M+\pi Q$; B) $\pi M+2Q$; C) $\frac{M+\pi Q}{3}$; D) $\sqrt{M^2+4Q^2}$; E) $\sqrt{M^2+2Q^2}$.

6. Silindr yon sirtining yuzi uning to'la sirti yuzining yarmiga teng. Silindr o'q kesimining diagonali d bo'lsa, uning yon sirti yuzi hisoblansin.

A) πd^2+2; B) πd^2; C) $\frac{2\pi d^2}{7}$; D) $\frac{2\pi d^2}{3}$; E) $\frac{2\pi d^2}{5}$.

7. Silindrning asosida uzunligi a bo'lgan vatar α kattalikdagi yoyga tiralgan. Silindrning o'q kesimi kvadratdan iborat bo'lsa, uning hajmi hisoblansin.

A) $\frac{4\pi a^2}{\sin^2\alpha}$; B) $\frac{\pi a^2}{\cos^2\frac{\alpha}{2}}$; C) $\frac{\pi a^2}{\sin^2\frac{\alpha}{2}}$; D) $\frac{2\pi a^2}{\cos^2\alpha}$; E) $\frac{\pi a^2}{1+\sin\alpha}$.

8. Silindrning balandligi 15 sm, asosining radiusi 5 sm. Uzunligi 17 sm bo'lgan AB kesmaning uchlari silindr asoslarining aylanalarida yotadi. Shu kesmadan silindrning o'qigacha bo'lgan masofa topilsin.

A) 6; B) 4; C) 2; D) 3; E) 2,5 sm.

9. Silindr o'q kesimining diagonali asosining diametridan 25% uzun. Agar silindr asoslarining markazlari orasidagi masofa 18 sm bo'lsa, uning yon sirtining yuzi hisoblansin.

A) 432π; B) 360π; C) 448π; D) 396π; E) 460π.

10. Silindrning yon sirti 50π. Agar uning yon sirti asoslari yuzlarining yig'indisiga teng bo'lsa, silindrning hajmi hisoblansin.

A) 90π; B) 125π; C) 120π; D) 96π; E) 144π.

11. Silindr o'q kesimining yuzi Q. Asos radiusining o'rtasidan silindrning o'qiga parallel o'tuvchi kesimning yuzi hisoblansin.

A) $\frac{Q}{2}$; B) $\frac{Q\sqrt{2}}{3}$; C) $\frac{Q\sqrt{3}}{2}$; D) $\frac{Q\sqrt{5}}{2}$; E) $\frac{Q\sqrt{3}}{4}$.

12. Silindr o'q kesimining yuzi Q. Silindrning o'qiga parallel va undan asos radiusining $\frac{1}{4}$ qismiga teng uzoqlikda o'tuvchi kesimning yuzi hisoblansin.

A) $\frac{Q\sqrt{2}}{4}$; B) $\frac{Q\sqrt{7}}{9}$; C) $\frac{Q\sqrt{13}}{3}$; D) $\frac{Q\sqrt{11}}{4}$; E) $\frac{Q\sqrt{15}}{4}$.

13. Silindrning yasovchisi 4 dm, asosining radiusi 29 sm. Silindrning o'qiga parallel o'tuvchi kesim kvadrat shaklida bo'lsa, o'qdan shu kesimgacha bo'lgan masofa topilsin.

A) 21; B) 18; C) 24; D) 20; E) 12,5 sm.

14. Silindr o'q kesimining yuzi Q. Shu kesimning bitta yasovchisi orqali o'q kesim bilan 60° li burchak tashkil etuvchi kesimning yuzi hisoblansin.

A) 0,25 Q; B) 1,25 Q; C) 1,5 Q; D) 0,5 Q; E) 0,75 Q.

15. Silindr o'q kesimining yuzi Q. Shu kesim bilan 45° li burchak tashkil etuvchi kesimning yuzi hisoblansin.

A) $\frac{Q\sqrt{3}}{5}$; B) $\frac{Q\sqrt{2}}{2}$; C) $\frac{Q\sqrt{3}}{4}$; D) $\frac{Q}{3}$; E) $\frac{Q\sqrt{11}}{3}$.

16. Silindr o'q kesimining yuzi Q. Silindrning o'qiga parallel bo'lib, asosining aylanasidan 90° li yoyni ajratib o'tuvchi kesimning yuzi hisoblansin.

A) $\frac{Q\sqrt{11}}{4}$; B) $\frac{Q\sqrt{2}}{4}$; C) $\frac{Q\sqrt{2}}{2}$; D) $\frac{Q\sqrt{3}}{2}$; E) $\frac{Q\sqrt{5}}{3}$.

17. Silindr o‘q kesimining yuzi Q. Silindrning o‘qiga parallel bo‘lib, asosining aylanasidan 120° li yoyni ajratib o‘tuvchi kesimning yuzi hisoblansin.

A) $\frac{Q\sqrt{15}}{3}$; B) $\frac{Q}{2}$; C) $\frac{Q\sqrt{5}}{3}$; D) $\frac{Q\sqrt{3}}{4}$; E) $\frac{Q\sqrt{3}}{2}$.

18. Silindrning yasovchisi orqali o‘zaro perpendikulyar bo‘lgan ikkita kesim o‘tkazilgan. Bu kesimlarning yuzlari 45 dm^2 va 2 m^2 bo‘lsa, o‘q kesimning yuzi hisoblansin.

A) 205; B) 210; C) 216; D) 196; E) 180 dm^2.

19. Silindrda o‘tkazilgan ikkita kesim o‘zaro perpendikulyar va ularning kesishish chizig‘i — silindrning o‘qiga parallel. Shu chiziq bitta kesimni yuzlari 77 dm^2 va 27 dm^2 bo‘lgan qismlarga, ikkinchi kesimni esa yuzlarining nisbati 7:33 kabi bo‘lgan qismlarga ajratadi. Silindr o‘q kesimining yuzi hisoblansin.

A) 144; B) 96; C) 80; D) 130; E) 120 dm^2.

20. Silindrning yasovchisi orqali ikki AA_1B_1B va AA_1C_1C kesim o‘tkazilgan va ular orasidagi burchak 60°. Kesimlarning yuzlari mos ravishda 420 sm^2 va 1 dm^2 bo‘lsa, BCC_1B_1 kesimning yuzi hisoblansin.

A) 296; B) 380; C) 360; D) 344; E) 320 sm^2.

21. Silindrning balandligi 15 sm, asosidagi aylananing radiusi 5 sm bo‘lib, $AB=17$ sm kesmaning uchlari silindr asoslarining aylanalarida yotadi. AB kesma va silindrning o‘qi orasidagi masofa topilsin.

A) 4; B) 2; C) 3; D) 2,5; E) 1,5 sm.

22. Silindr asosining yuzi 36 π sm^2, uning o‘q kesimi kvadratdan iborat. Silindr yon sirtining yuzi hisoblansin.

A) 144π; B) 120π; C) 156π; D) 136π; E) 134π sm^2.

23. Silindr yon sirtining yuzi uning to'la sirti yuzining yarmiga teng. Silindr o'q kesimining diagonali d bo'lsa, silindr to'la sirtining yuzi hisoblansin.

A) $\frac{3}{5}\pi d^2$; B) $\frac{4}{3}\pi d^2$; C) $\frac{3}{4}\pi d^2$; D) $\frac{2}{3}\pi d^2$; E) $\frac{4}{5}\pi d^2$.

24. Silindr o'q kesimining diagonali asosining radiusidan 5,2 marta katta. Silindr yon sirtining yuzi 120π sm² bo'lsa, uning to'la sirti yuzi hisoblansin.

A) 180; B) 160; C) 144; D) 145; E) 120 sm².

25. Silindr asosining yuzi S, uning o'q kesimi diagonallarining kesishish nuqtasida silindrning yasovchisi 60° li burchak ostida ko'rinayapti. Silindr yon sirtining yuzi hisoblansin.

A) $\frac{4}{3}S\sqrt{2}$; B) $\frac{4}{3}S\sqrt{3}$; C) $\frac{5}{4}S\sqrt{3}$; D) $\frac{3}{4}S\sqrt{3}$;

E) $\frac{3}{4}S\sqrt{2}$.

26. Silindrning balandligi 6 dm, asosining radiusi 5 dm. $AB=10$ dm kesmaning uchlari har xil asoslarning aylanalarida yotadi. AB kesma va silindrning o'qi orasidagi eng qisqa masofa topilsin.

A) 3,5; B) 5; C) 3; D) 4; E) 2,5 dm.

27. Silindrning o'q kesimi — kvadrat va uning diagonali $3\sqrt{2}$ m bo'lsa, uning yon sirti yuzi hisoblansin.

A) 9π; B) 8π; C) 12π; D) 10π; E) 15π m².

28. Silindrning yon sirti tekislikka yoyilganda kvadrat hosil bo'ladi. Kvadratning tomoni a bo'lsa, silindrning hajmi hisoblansin.

A) $\frac{a^3}{3\pi}$; B) $\frac{a^3}{2\pi}$; C) $\frac{a^3}{27\pi}$; D) $\frac{a^3\pi}{15}$; E) $\frac{a^3}{4\pi}$.

29. Silindrning balandligi h bo'lib, silindr yoyilmasining diagonali yasovchi bilan 60° li burchak tashkil qiladi. Silindrning hajmi hisoblansin.

A) $\frac{8\pi}{3}h^3$; B) $\frac{4}{5\pi}h^3$; C) $\pi \cdot h^3$; D) $\frac{3}{4\pi}h^3$; E) $\frac{4}{5}h^3\pi$.

30. Silindrning o'qiga parallel kesim o'tkazilgan. Silindr asosining radiusi r, balandligi h, kesim bilan ajratilgan kichik yoyning kattaligi 120°. Silindr kichik qismining hajmi hisoblansin.

A) $\frac{\pi}{4}rh^2(4\pi - \sqrt{3})$; B) $\frac{\pi}{3}r^2h$; C) $\frac{\pi}{6}rh^2$; D) $\frac{\pi}{8}r^2h$;

E) $\frac{\pi}{4}r^2h$.

31. Silindr quyi asosining markazidan o'tkazilgan tekislik asosga α burchak ostida og'ma. U yuqori asosni uzunligi b bo'lgan vatar orqali kesib o'tadi va kattaligi β bo'lgan yoyni ajratadi. Silindrning hajmi hisoblansin.

A) $\frac{\pi b^3}{24}\sin 4\alpha \cdot \mathrm{tg}^2\beta$; B) $\frac{\pi b^3}{12}\sin 2\alpha \cdot \mathrm{tg}\beta$;

C) $\frac{\pi b^3}{8} \cdot \frac{\mathrm{ctg}\frac{\beta}{2} \cdot \mathrm{tg}\alpha}{\sin^2\frac{\beta}{2}}$; D) $\frac{\pi b^3}{6} \cdot \frac{\mathrm{tg}^2\alpha \cdot \sin\beta}{\cos^2\frac{\beta}{2}}$;

E) $\frac{\pi b^3}{6} \cdot \frac{\mathrm{ctg}\alpha \cdot \mathrm{tg}\frac{\alpha}{2}}{\cos\frac{\beta}{2}}$.

32. Tomoni a bo'lgan muntazam uchburchakning ikkita uchi — silindr pastki asosining aylanasida, uchinchi uchi esa silindr yuqori asosining aylanasida joylashgan. Uchburchak tekisligi silindrning yasovchisi bilan α burchak tashkil qiladi. Silindr yon sirtining yuzi hisoblansin.

A) $\frac{\pi a^2 \mathrm{ctg}\alpha(4-3\cos^2\alpha)}{4}$; B) $\frac{\pi a^2 \mathrm{tg}\alpha(3-4\cos^2\alpha)}{4}$;

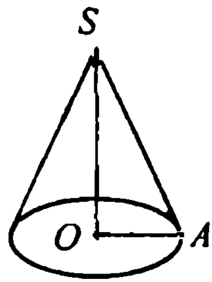
C) $\frac{\pi a^2 \sin 2\alpha(1-2\sin^2\alpha)}{12}$; D) $\frac{\pi a^2 \cos 2\alpha}{8}$;

E) $\frac{\pi a^2 \mathrm{tg}\alpha(2-4\sin^2\alpha)}{8}$.

13-§. KONUS VA KESIK KONUS

13.1. Asosiy tushunchalar va tasdiqlar

Konus to'g'ri burchakli uchburchakning kateti atrofida aylanishidan hosil bo'lgan jism. To'g'ri burchakli ΔSOA o'zining SO kateti atrofida (tevaragida) aylansa, uchburchakning SA gipotenuzasi konusning *yon sirtini*, OA kateti — konusning *asosi* bo'lgan doirani chizadi. S nuqta konusning *uchi* (13.1-chizma), SA *gipotenuza* — konusning *yasovchisi*, $OA=R$ — konus asosining *radiusi*, SO katet — konusning *balandligi* va konusning *simmetriya* o'qi bo'ladi. Konusning yasovchisi $SA=l$ bilan, balandligi $SO=H$ bilan belgilanadi. Konusning balandligidan o'tkazilgan tekislik kesimda teng yonli ΔASB hosil qiladi, u konusning *o'q kesimidan* iborat.



13.1-chizma.

Agar konusning yon sirtini bitta yasovchi bo'yicha kesib, tekislikka yoysak, konusning yoyilmasini hosil qilamiz. Yasovchisi l, asosining radiusi R bo'lgan konusning yoyilmasi radiusi l va yoy uzunligi $2\pi R$ bo'lgan doiraviy sektordir, uning yuzi konus yon sirtining yuziga teng.

1. Konus yon sirtining yuzi:

$$S_{yon}=\pi \cdot R \cdot l, \qquad (13.1)$$

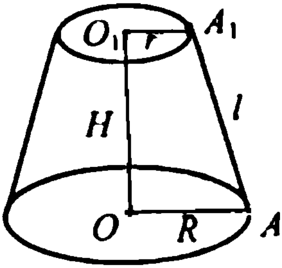
bu yerda l — konusning yasovchisi, R — konus asosining radiusi.

2. Konus to'la sirtining yuzi:

$$S_t=S_{yon}+S_{asos},$$

ya'ni

$$S_t=\pi Rl+\pi R^2=\pi R\,(l+R). \qquad (13.2)$$



13.2.-chizma.

3. Konusning hajmi:

$$V_k = \frac{1}{3}\pi R^2 \cdot H \qquad (13.3)$$

H — konusning balandligi.

Konusning asosiga parallel va u bilan kesishadigan tekislik o'tkazilganda tekislik konusni doira bo'ylab kesadi. Kesik konus — konusning asosi va unga parallel tekislik bilan kesilgan qismidir (13.2.-chizma). Chizmada: AA_1 — kesik konusning yasovchisi, $OO_1=H$ — kesik konusning balandligi, $OA=R$ va $O_1A_1=r$ kesik konus asoslarining radiuslari.

4. Kesik konus yon sirtining yuzi:

$$S_{yon}=\frac{P_1+P_2}{2}\cdot l, \qquad (13.4)$$

yoki

$$P_1=2\pi R, \; P_2=2\pi r \qquad (13.5)$$

bo'lishini hisobga olsak,

$$S_{yon}=\pi(R+r)l. \qquad (13.6)$$

5. Kesik konus to'la sirtining yuzi:

$$S_t=S_{yon}+S_{yu.as}+S_{q.as}$$

yoki

$$S_t=\pi(R+r)l+\pi R^2+\pi r^2. \qquad (13.7)$$

6. Kesik konusning hajmi:

$$V_{k.k.}=\frac{1}{3}\pi H(R^2+Rr+r^2) \qquad (13.8)$$

formula bo'yicha hisoblanadi.

13.2. Mavzuga doir masalalar

1. Konus asosining radiusi R bo'lib, uning o'q kesimi esa to'g'ri burchakli uchburchakdan iborat. Kesik konus o'q kesimining yuzi hisoblansin.

A) $\frac{R^2}{2}$; B) R^2; C) $\frac{3}{4}R^2$; D) $2R^2$; E) $1{,}5\ R^2$.

2. Konusning balandligi h ga teng. Agar kesimning yuzi konus asosining yuzidan to'rt marta kichik bo'lsa, kesim asosdan qanday uzoqlikda o'tishi kerak?

A) $\frac{2}{3}h$; B) $\frac{3}{4}h$; C) $\frac{5}{6}h$; D) $\frac{h}{2}$; E) $\frac{1}{4}h$.

3. Konus asosining radiusi R, konusning uchidan kesim o'tkazilgan bo'lib, kesim asos tekisligi bilan 60° li burchak tashkil qiladi va asosidagi 120° li yoyni ajratadi. O'tkazilgan kesimning yuzi hisoblansin.

A) $\frac{1}{2}R^2\sqrt{3}$; B) $R^2\sqrt{3}$; C) $R^2\sqrt{2}$; D) $\frac{3}{4}R^2$; E) $\frac{1}{5}R^2$.

4. Konusning balandligi 4, yasovchisi 5 bo'lsa, konus yoyilmasining burchagi kattaligi hisoblansin.

A) 150°; B) 186°; C) 204°; D) 196°; E) 216°.

5. Konusning to'la sirti πS kvadrat birlikka teng, konusning yoyilmasi esa burchagi 60° ga teng bo'lgan doiraviy sektordan iborat. Konusning hajmi hisoblansin.

A) $\frac{\pi S\sqrt{6S}}{14}$; B) $\frac{\pi S}{7}$; C) $\frac{\pi S\sqrt{5S}}{21}$; D) $\frac{\pi S\sqrt{S}}{21}$; E) $\frac{\pi S\sqrt{3S}}{14}$.

6. Konus asosining markazidan yasovchisigacha bo'lgan masofa d, yasovchi va konusning balandligi orasidagi burchak α bo'lsa, konus to'la sirtining yuzi hisoblansin.

A) $\frac{2\pi d^2 \operatorname{tg}\alpha}{1+\sin\alpha}$; B) $\frac{2\pi d^2}{1+\cos\alpha}$; C) $\frac{\pi d^2}{1+\cos\alpha}$;

D) $\frac{2\pi d^2 \operatorname{ctg}\alpha\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\sin 2\alpha}$; E) $\frac{3\pi d^2 \operatorname{tg}\alpha}{\sin\left(\frac{\pi}{4}+\alpha\right)}$.

7. Kesik konus asoslarining radiuslari R va r. Kesik konusning ikkita yasovchisi orqali uning asosidagi aylanadan 90° li yoy ajratuvchi kesim o'tkazilgan. Bu kesim asosning tekisligi bilan 60° li burchak tashkil qilsa, uning yuzi hisoblansin.

A) r^2+R^2; B) r^2-R^2; C) rR; D) $R\sqrt{R^2+r^2}$;
E) $r\sqrt{R^2-r^2}$.

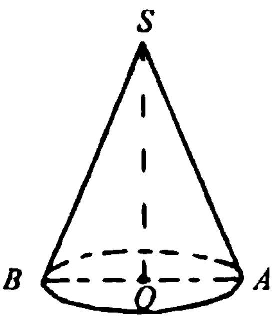
8. Kesik konus o'q kesimining diagonali d bo'lib, konusning pastki asosi bilan α burchak, yasovchisi bilan 90° li burchak tashkil etadi. Kesik konus yon sirtining yuzi hisoblansin.

A) $\pi d^2 \sin\alpha$; B) $\pi d^2 \cos\alpha$; C) $\pi d^2 \sin 2\alpha$; D) $\pi d^2 \mathrm{tg}\alpha$;
E) πd^2.

9. Kesik konus asoslarining radiuslari R va r, yasovchi esa asos tekisligi bilan 45° li burchak tashkil etadi. Kesik konusning hajmi hisoblansin.

A) $\frac{2\pi(R^3-r^3)}{3}$; B) $\frac{\pi(R^3+r^3)}{3}$; C) $\frac{\pi R r^2}{3}$;
D) $\frac{\pi R^2 r}{3}$; E) $\frac{\pi(R^3-r^3)}{3}$.

13.3. Mavzuga oid masalalarning yechimlari



13.3.1-chizma.

1. Berilgan. SAB — konus, ΔSAB — to'g'ri burchakli, $OA=R$.

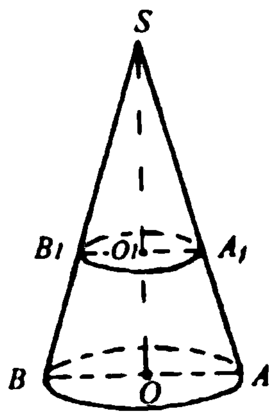
$S_{\Delta SAB}$ hisoblansin (13.3.1-chizma).

Yechilishi. Konusning o'q kesimi teng yonli to'g'ri burchakli ΔSAB dan iborat. Shuning uchun, $\angle SAB=\angle SBA=45°$. SO balandlikni o'tkazsak, teng yonli ΔSAO ni hosil qilamiz, chunki $\angle SAO=45^\circ$. Demak, $SO=OA=R$. U holda

$$S_{\Delta SAB}=\frac{1}{2}AB\cdot SO=\frac{1}{2}2R\cdot R=R^2.$$

2. B e r i l g a n. SAB — konus, $SO\perp AB$, $SO=h$, $A_1B_1 \parallel AB$, $S_{as}=2\cdot S_{kes}$.

OO_1 t o p i l s i n (13.3.2-chizma).



13.3.2-chizma.

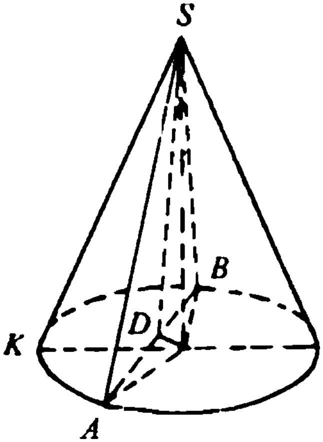
Y e c h i l i s h i. Berilishiga ko'ra, kesim asosga parallel bo'lganligidan, $\Delta SOA\sim\Delta SO_1A_1$ bo'ladi va o'xshash uchburchaklar uchun $\frac{OA}{O_1A_1}=\frac{SO}{SO_1}$ proporsiyani yozamiz. Ikkinchi tomondan, $S_{as}=2S_{kes}$ yoki $\pi\cdot OA^2=$ $=2\pi O_1A_1^2$. U holda $OA=2O_1A_1$ va $\frac{OA}{O_1A_1}=2$. Shartga ko'ra, $SO=h$, $SO_1=SO-OO_1=h-x$, $x=OO_1$. Bu ifodalarni proporsiyaga keltirib qo'ysak, $\frac{h}{h-x}=2$, $2h-2x=h$, $2x=2h-h=h$; $x=\frac{h}{2}$. Demak, $OO_1=\frac{h}{2}$.

Javobi: D).

3. B e r i l g a n. SAB — konus, ΔSAB — kesim, $\angle SDO=60^\circ$, $\cup AKB=120^\circ$, $OA=OB=R$.

S_{kes} h i s o b l a n s i n (13.3.3-chizma).

Y e c h i l i s h i. A nuqtani aylanada tanlab va A nuqtadan boshlab aylananing $\frac{1}{3}$ qismini olib, $\cup AKB=120^\circ$ yoyni ajratamiz. A va B nuqtalarni aylana markazi O bilan tu-



13.3.3-chizma.

tashtirib, $\angle AOB=120°$ va teng yonli ΔAOB ni hosil qilamiz. Berilishiga ko'ra, ΔSAB — teng yonli ($SA=SB$). Uning S uchidan $SD\perp AB$ o'tkazsak, u mediana ham bo'ladi, ya'ni $AD=DB$. Uch perpendikulyar haqidagi teoremaga asosan, $OD\perp AB$ va $\angle SDO=60°$. Kesimning yuzi $S_{kes}=\frac{1}{2}AB\cdot SD$ formuladan topiladi. ΔAOB teng yonli va $\angle AOB=120°$ bo'lganligidan, $\angle ABO=$ $=\angle BAO=\frac{180°-120°}{2}=30°$.

To'g'ri burchakli ΔOBD dan: $OD=OB\cdot\sin 30°=\frac{1}{2}R$, $BD=OB\cdot\cos 30°=\frac{1}{2}R\sqrt{3}$ va $AB=2BD=R\sqrt{3}$.

To'g'ri burchakli ΔSOD dan: $\frac{OD}{SD}=\cos 60°$, $SD=$ $=\frac{OD}{\cos 60°}=\frac{\frac{R}{2}}{\frac{1}{2}}=R$.

Demak, kesimning yuzi $S_{kes}=\frac{1}{2}AB\cdot SD=\frac{1}{2}R^2\sqrt{3}$ bo'ladi.

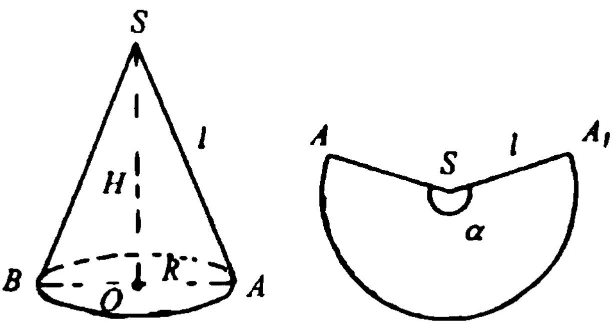
Javobi: A).

4. Berilgan. SAB — konus, $SO\perp AB$, $SO=4$, $SA=5$.

α topilsin (13.3.4-chizma).

Yechilishi. To'g'ri burchakli ΔSOA dan, Pifagor teoremasiga asosan, konus asosining radiusini topamiz:

$$R^2=l^2-H^2=5^2-4^2=9,\ R=3$$



13.3.4-chizma.

U holda konus asosi aylanasining uzunligi $C=2\pi\cdot 3=6\pi$ bo'ladi. Yoyilmada AA_1 yoyning uzunligi 6π ga teng va $SA=SA_1=5$.

Ma'lumki, 1° markaziy burchakka mos kelgan yoyning uzunligi $\frac{2\pi l}{360^\circ}=\frac{10\pi}{360^\circ}=\frac{\pi}{36^\circ}$, u holda α gradusga mos kelgan yoyning uzunligi ifodasini tenglashtiramiz: $\frac{\pi\alpha}{36^\circ}=6\pi$ va $\alpha=6\cdot 36^\circ=216^\circ$.

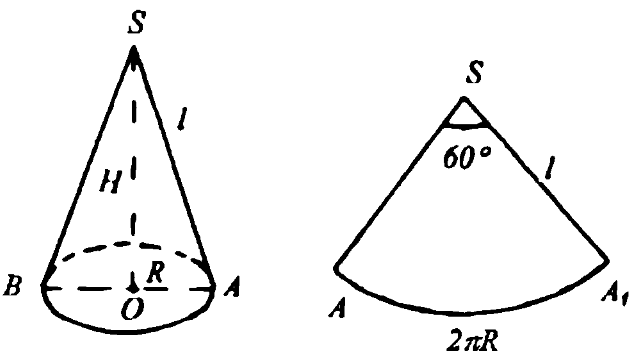
Javobi: E).

5. B e r i l g a n. SAB — konus, $S_t=\pi S$, ASA_1 — sektor, $\angle ASA_1=60^\circ$, $SA=l$.

V_k h i s o b l a n s i n (13.3.5-chizma).

Y e c h i l i s h i. Agar konus asosining radiusi R, balandligi H, yasovchisi l bo'lsa, uning to'la sirti $S_t=\pi Rl+\pi R^2$, hajmi esa $V_k=\frac{1}{3}\pi R^2\cdot H$ formula bo'yicha hisoblanadi.

Konus yoyilmasining burchagi 60° bo'lganligidan uning yuzi doira yuzining $\frac{1}{6}$ qismiga teng, ya'ni agar doiraning yuzi $S_d=\pi l^2$ bo'lsa, $S_{sek}=\frac{1}{6}\pi l^2$ bo'ladi. Ikkinchi tomondan,



13.3.5-chizma.

sektorning yuzi konus yon sirtining yuziga tengdir: $S_{yon} = S_{sek}$ yoki $\pi \cdot R \cdot l = \frac{1}{6}\pi l^2$ va $l=6R$.

U holda konusning to'la sirti uchun $\pi Rl + \pi R^2 = RS$ ifodani olamiz va $l=6R$ ni keltirib qo'ysak, $6R^2 + R^2 = S$, $7R^2 = S$, $R^2 = \frac{1}{7}S$ bo'ladi.

To'g'ri burchakli ΔSOA dan: $H^2 = l^2 - R^2$, $H^2 = (6R)^2 - R^2 = 35R^2$ va $H = R\sqrt{35}$.

Endi konusning hajmini hisoblaymiz:

$$V = \frac{1}{3}\pi \cdot \frac{S}{7} \cdot \sqrt{\frac{S}{7} \cdot 35} = \frac{\pi S\sqrt{5S}}{21}.$$

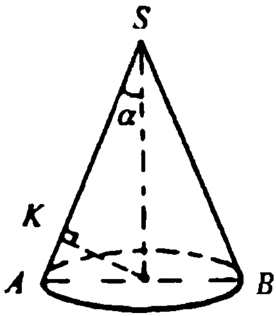
Javobi: C).

6. B e r i l g a n. SAB — konus, $\angle ASO = \alpha$, $OK = d$.

S_1 h i s o b l a n s i n (13.3.6-chizma).

Y e c h i l i s h i. Konus asosining markazidan yasovchisigacha bo'lgan masofa SA yasovchiga asosning markazidan o'tkazilgan OK perpendikulyarning uzunligiga teng:

$OK=d$. OK perpendikulyar yordamida to'g'ri burchakli ΔSKO ni hosil qilamiz: $OK=d$, $\Delta OSK=\alpha$. Bu uchburchakdan:



13.3.6-chizma.

$$\sin\alpha=\frac{OK}{SO} \text{ va } SO=\frac{d}{\sin\alpha}$$

To'g'ri burchakli ΔSAO dan:

$$\operatorname{tg}\alpha=\frac{AO}{SO},\ AO=R=SO\cdot\operatorname{tg}\alpha=$$

$$=\frac{d}{\sin\alpha}\cdot\frac{\sin\alpha}{\cos\alpha}=\frac{d}{\cos\alpha},$$

$$\sin\alpha=\frac{R}{l},\ l=\frac{R}{\sin\alpha}=\frac{d}{\sin\alpha\cdot\cos\alpha}.$$

Endi konusning to'la sirti yuzini hisoblaymiz:

$$S_t=\frac{\pi d}{\cos\alpha}\cdot\frac{d}{\sin\alpha\cdot\cos\alpha}+\frac{\pi d^2}{\cos^2\alpha}(1+\sin\alpha)$$

yoki $2\sin\alpha\cdot\cos\alpha=\sin 2\alpha$; $1+\sin\alpha=1+\cos\left(\frac{\pi}{2}-\alpha\right)$

bo'lganligidan, $S_t=\dfrac{4\pi dd^2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\sin 2\alpha\cdot\cos\alpha}$. Ikkinchi tomondan,

$$\cos\alpha=\sin\left(\frac{\pi}{2}-\alpha\right)=2\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cdot\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right).$$

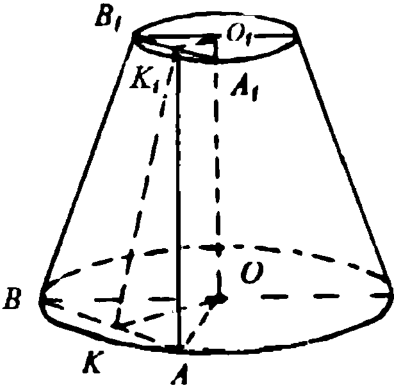
Shuning uchun,

$$S_t=\frac{2\pi d^2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{2\sin 2\alpha\,\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}=\frac{2\pi d^2\operatorname{ctg}\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\sin 2\alpha}.$$

Javobi: D).

7. B e r i l g a n. $OO_1AA_1BB_1$ — kesik konus, $\cup ACB=90^\circ$, $\angle K_1KO=60^\circ$, $OA=R$, $O_1A_1=r$.

$S_{AA_1B_1B}$ h i s o b l a n s i n (13.3.7-chizma).



13.3.7-chizma.

Yechilishi. AA_1B_1B teng yonli trapetsiyadir. Uning asoslarining o'rtalaridagi K va K_1 nuqtalarni tutashtirsak, $KK_1 \perp AB$ bo'ladi. Uch perpendikulyar haqidagi teoremaga asosan (8-§), $OK \perp AB$ bo'ladi. Shu sababli kesim va asos tekisligi orasidagi ikki yoqli burchakning chiziqli burchagi $\angle K_1KO = 60°$ bo'ladi.

Berilishiga ko'ra, AOB to'g'ri burchakli va teng yonlidir: $OA = OB$, $AOB = 90°$. U holda $AB = \sqrt{R^2 + R^2} = R\sqrt{2}$, $OK = BK = \frac{R\sqrt{2}}{2}$. Shunga o'xshash, $A_1B_1 = r\sqrt{2}$; $OK_1 = \frac{r\sqrt{2}}{2}$ bo'ladi. K nuqtadan kesik konusning pastki asosiga perpendikulyar o'tkazamiz: $K_1E \perp OK$, $KE = OK - OE = = OK - O_1K_1$;

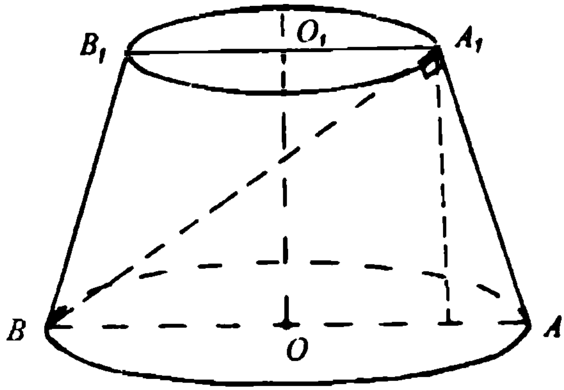
$KE = \frac{R\sqrt{2}}{2} - \frac{r\sqrt{2}}{2} = \frac{(R-r)\sqrt{2}}{2}$ bo'ladi. So'ngra to'g'ri burchakli ΔKEK_1 dan:

$$\cos 60° = \frac{KE}{KK_1}, \quad KK_1 = \frac{KE}{\cos 60°} = \frac{(R-r)\sqrt{2}}{2 \cdot \frac{1}{2}} = (R-r)\sqrt{2}.$$

U holda teng yonli AA_1B_1B trapetsiyaning yuzi:

$$S_{kes} = \frac{AB + A_1B_1}{2} KK_1 = \frac{(R+r)\sqrt{2}}{2} \cdot (R-r)\sqrt{2} = R^2 - r^2.$$

8. Berilgan. ABB_1A_1 — kesik konus, $A_1B = d$, $\angle A_1BA = a$, $\angle BA_1A = 90°$.



13.3.8-chizma.

S_{yon} hisoblansin (13.3.8-chizma).

Yechilishi. Ma'lumki, kesik konus yon sirtining yuzi

$$S_{yon.}=\pi l(R-r)$$

formula bo'yicha hisoblanadi. ΔA_1BA to'g'ri burchakli bo'lganligidan, $\frac{A_1B}{AB}=\cos\alpha$, $AB=\frac{A_1B}{\cos\alpha}=\frac{d}{\cos\alpha}$; $AA_1=d\cdot \text{tg}\alpha$; $AK=AA_1\cdot d\cdot \text{tg}\alpha\cdot \sin\alpha$.

Kesik konus yuqori asosining radiusini topamiz:

$$r=OK=O_1A_1=AO-AK=\frac{d}{2\cos\alpha}-\frac{d\sin^2\alpha}{\cos\alpha}=$$

$$=\frac{d}{2\cos\alpha}(1-2\sin^2\alpha)$$

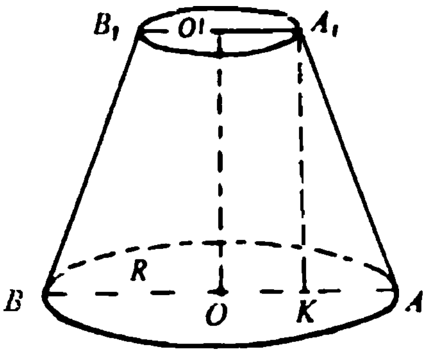
yoki $r=\frac{d\cos 2\alpha}{2\cos\alpha}$. U holda, kesik konusning yon sirti

$$S_{yon}=\pi d\text{tg}\alpha\left(\frac{d}{2\cos\alpha}+\frac{d\cos 2\alpha}{2\cos\alpha}\right)=\frac{\pi d^2\text{tg}\alpha}{2\cos\alpha}(1+\cos 2\alpha)=$$

$$=\frac{\pi d^2\text{tg}\alpha\cdot 2\cos^2\alpha}{2\cos\alpha}=\pi d^2\sin\alpha\,.$$

9. Berilgan. $OO_1ABB_1A_1$ — kesik konus, $OA=R$, $O_1A_1=r$, $\angle A_1AO=45°$.

V_{kk} hisoblansin (13.3.9-chizma).



13.3.9-chizma.

Yechilishi. Ma'lumki, kesik konusning hajmi

$$V = \frac{1}{3}\pi H(R^2 + r^2 + Rr)$$

formula bo'yicha hisoblanadi. A_1 nuqtadan $A_1K \perp OA$ o'tkazamiz. U holda, $OK=O_1A_1$, $AK=OA-OK=R-r$. ΔAA_1K to'g'ri burchakli va $\angle A_1AK=$ $=45°$ bo'lganligidan u teng yonli ham bo'ladi, $A_1K=AK=R-r$.

Endi kesik konusning hajmini hisoblaymiz:

$$V_{kk}=\frac{1}{3}\pi(R-r)(R^2+Rr+r^2) \text{ yoki } V_{kk}=\frac{1}{3}\pi(R^3-r^3).$$

Javobi: E).

13.4. Mustaqil yechish uchun masalalar

1. Konus asosining radiusi R ga teng. Konusning balandligini (uchidan asosiga qarab) $m:n$ nisbatda bo'luvchi parallel kesimning yuzi hisoblansin.

A) $\frac{4\pi R^2}{m^2+n^2}$; B) $\frac{2\pi R^2 m}{(m+n)^2}$; C) $\frac{\pi R^2 m^2}{(m+n)^2}$; D) $\frac{\pi R^2 n^2}{(m+n)^2}$;

E) $\frac{2\pi R^3 n}{(m-n)^2}$.

2. Konus balandligining o‘rtasidan uning l yasovchisiga parallel to‘g‘ri chiziq o‘tkazilgan. Konusning ichida yotuvchi to‘g‘ri chiziq kesmasining uzunligi topilsin.

A) 0,75 l; B) 2 l; C) 3 l; D) 0,5 l; E) 1,25 l.

3. Teng tomonli (o‘q kesimi muntazam uchburchakdan iborat) konus asosining radiusi R. Oralaridagi burchak 30° bo‘lgan ikki yasovchi orqali o‘tkazilgan kesimning yuzi hisoblansin.

A) 2,5 R^2; B) 4 R^2; C) $\frac{1}{2}R^2$; D) R^2; E) $2R^2$.

4. Agar konusning o‘q kesimi to‘g‘ri burchakli uchburchakdan iborat bo‘lsa, konus yoyilmasining burchagi topilsin.

A) 225°; B) 255°; C) 280°; D) 270°; E) 235°.

5. Konus asosining radiusi R. Konusning uchidan asos tekisligi bilan 60° li burchak tashkil qilib, asosidagi aylanadan 120° li yoy ajratuvchi kesim o‘tkazilgan. Shu kesimning yuzi hisoblansin.

A) $\frac{R^2\sqrt{2}}{8}$; B) $\frac{R^2\sqrt{2}}{4}$; C) $\frac{R^2\sqrt{3}}{4}$; D) $\frac{R^2\sqrt{2}}{2}$; E) $\frac{R^2\sqrt{3}}{2}$.

6. Yarim doiradan konus yasalgan bo‘lsa, konusning o‘q kesimi uchidagi burchak topilsin.

A) 75°; B) 90°; C) 60°; D) 45°; E) 30°.

7. Konusning o‘q kesimi uchidagi burchak 2α, o‘q kesimning yuzi Q. Konus to‘la sirtining yuzi hisoblansin.

A) $\frac{2\pi Q\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\cos\alpha}$; B) $\frac{\pi Q\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\sin\alpha}$; C) $\pi Q\cdot \mathrm{ctg}\alpha$;

D) $\frac{\pi Q\sin^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\cos\alpha}$; E) $\frac{\pi Q\cdot \mathrm{ctg}\alpha}{\sin^2\alpha}$.

8. Konusning balandligi asosining diametriga teng. Konusning asosi va yon sirti yuzlarining nisbati topilsin.

A) $\frac{5}{4}$; B) $\frac{3}{2}$; C) $\frac{\sqrt{7}}{7}$, D) $\frac{\sqrt{5}}{5}$; E) $\frac{\sqrt{3}}{3}$.

9. Konus asosining radiusi R, yoyilmasidagi markaziy burchak 90°. Konusning hajmi hisoblansin.

A) $\frac{2}{3}\pi R^2\sqrt{11}$; B) $\frac{1}{3}\pi R^3\sqrt{15}$; C) $\frac{1}{3}\pi R^3\sqrt{11}$;

D) $\frac{1}{3}\pi R^3\sqrt{13}$; E) $\frac{2}{3}\pi R^3\sqrt{5}$.

10. Konusning o‘q kesimi uchidagi burchak 2α, balandligi va yasovchisining yig‘indisi m ga teng. Konusning hajmi hisoblansin.

A) $\frac{1}{3}\pi m^3 \mathrm{tg}^2\frac{\alpha}{2}\sin\alpha$; B) $\frac{\pi m^3\cos^2\frac{\alpha}{2}}{6\,\mathrm{tg}\alpha}$; C) $\frac{\pi m^3\sin 2\alpha}{6\cos\alpha}$;

D) $\frac{1}{3}\pi m^3\mathrm{ctg}\alpha$; E) $\frac{\pi m^3\cos\alpha\cdot\sin^2\frac{\alpha}{2}}{6\cos^4\frac{\alpha}{2}}$

11. Kesik konus asoslarining radiuslari 11 sm va 27 sm, yasovchisi va balandligining nisbati 17:15 kabi bo‘lsa, o‘q kesimning yuzi hisoblansin.

A) 1020; B) 980; C) 1140; D) 1440; E) 1200 sm².

12. Kesik konusning balandligi H, yasovchisi l bo‘lib, yon sirti S ga teng. Kesik konus o‘q kesimining yuzi hisoblansin.

A) $\frac{SH}{4\pi}$; B) $\frac{SH}{2\pi}$; C) $\frac{SH}{\pi}$; D) $\frac{SH}{l\pi}$; E) $\frac{SH}{l}$

13. Kesik konusning yasovchisi 17 sm, o‘q kesimining yuzi 420 sm² va o‘rta kesimning yuzi 196π sm². Kesik konusning hajmi hisoblansin.

A) 3020π; B) 2860π; C) 3240π; D) 2980π;
E) 3080π sm³.

14. Konusning o‘q kesimi balandligi $5\sqrt{3}$ sm bo‘lgan teng tomonli uchburchak bo‘lsa, konus yon sirtining yuzi hisoblansin.

A) 60π; B) 50π; C) 48π; D) 44π; E) 64π sm^2.

15. Konusning yasovchisi 10 m va asos tekisligi bilan 30° li burchak tashkil qiladi. Konus o‘q kesimining yuzi hisoblansin.

A) $24\sqrt{2}$; B) $25\sqrt{2}$; C) 25; D) $25\sqrt{3}$; E) 28 sm^2.

16. Konusning balandligi 8 sm, asosining radiusi 6 sm. Konusning ikkita yasovchisi orasidagi burchak 90° bo‘lsa, ular orqali o‘tkazilgan kesimning yuzi hisoblansin.

A) 44; B) 56; C) 48; D) 60; E) 50 sm^2.

17. Konusda oralaridagi burchak 60° bo‘lgan yasovchilar orqali kesim o‘tkazilgan. Konusning asosi markazidan kesimgacha bo‘lgan masofa 3 sm, konusning balandligi va kesim orasidagi burchak 30° bo‘lsa, kesimning yuzi hisoblansin.

A) 24; B) $12\sqrt{3}$; C) $16\sqrt{3}$; D) $16\sqrt{2}$; E) $8\sqrt{3}$ sm^2.

18. Konusning balandligi h, balandlik va yasovchisi orasidagi burchak α bo‘lsa, konus yon sirtining yuzi hisoblansin.

A) $\frac{\pi h^2 \operatorname{tg}\alpha}{\cos\alpha}$; B) $\frac{\pi h^2 \sin\alpha}{1+\cos^2\alpha}$; C) $\frac{\pi h^2 \operatorname{ctg}\alpha}{1+\sin^2\alpha}$; D) $\frac{\pi h^2 \operatorname{ctg}\alpha}{\cos\alpha}$;
E) $\frac{\pi h^2}{\sin\alpha}$.

19. Konusning yasovchisi l, uning o‘q kesimi uchidagi burchak 2α bo‘lsa, o‘q kesimning perimetri topilsin.

A) $2l\sin2\alpha$; B) $2l(1+\sin\alpha)$; C) $l(2+\sin\alpha)$; D) $l(2+\cos\alpha)$;
E) $2l(1+\cos\alpha)$.

20. Konus asosining yuzi S, yasovchisi esa asos tekisligi bilan α burchak tashkil etadi. Konus yon sirtining yuzi hisoblansin.

A) $S(1+\cos\alpha)$; B) $S\cdot\cos\alpha$; C) $S\cdot\sin\alpha$; D) $\frac{S}{\cos\alpha}$;

E) $\frac{S}{\sin\alpha}$.

21. Konus asosining yuzi S, to'la sirtining yuzi $3S$ ga teng. Konusning yasovchisi asos tekisligi bilan qanday burchak tashkil etadi?

A) 15°; B) 45°; C) 30°; D) 75°; E) 60°.

22. Konusning yasovchisi l bo'lib, asosi tekisligi bilan α burchak tashkil qiladi. Konus o'q kesimining yuzi hisoblansin.

A) $\frac{1}{4}l^2\sin\alpha$; B) $2l^2\sin\alpha$; C) $\frac{1}{2}l^2\sin2\alpha$; D) $l^2\sin2\alpha$;

E) $\frac{1}{2}l^2\sin\alpha$.

23. Konusning balandligi h, asosining radiusi r. Konusning asosidagi vatar 60° li yoyning uchlarini tutashtiradi. Shu vatar va konusning uchi orqali tekislik o'tkazilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) $\frac{1}{4}r\sqrt{4h^2+3r^2}$; B) $r\sqrt{4h^2+3r^2}$; C) $\frac{1}{2}r\sqrt{4h^2+3r^2}$;

D) $r^2\sqrt{4h+3r}$; E) $\frac{1}{2}r^2\sqrt{4h^2+3r^2}$.

24. Kesik konus asoslarining radiuslari 3 va 6 dm, yasovchisi esa 5 dm. Kesik konus o'q kesimining yuzi hisoblansin.

A) 40; B) 36; C) 42; D) 32; E) 48 dm².

25. Konusning yoyilmasi yoyining kattaligi 270° bo'lgan doiraviy sektordan iborat. Konusning o'q kesimi uchidagi burchak topilsin.

A) $2\arccos\frac{3}{4}$; B) arctg2; C) $\arcsin\frac{3}{4}$; D) $2\arcsin\frac{3}{4}$;

E) $\arccos\frac{3}{4}$.

26. Konusning balandligi h bo'lib, uning asosiga parallel tekislik o'tkazilgan. Hosil qilingan kesimning yuzi konus asosining yuzidan ikki marta kichik. Konusning uchidan kesimgacha bo'lgan masofa topilsin.

A) $1{,}5h$; B) $2h$; C) $\sqrt{2}\,h$; D) $\frac{1}{2}h$; E) $\frac{\sqrt{2}}{2}h$.

27. Konus asosining yuzi S, yon sirtining yuzi Q bo'lsa, uning o'q kesimining yuzi hisoblansin.

A) $\sqrt{Q^2-S^2}$; B) $\frac{Q+S}{2}$; C) $\frac{\sqrt{Q^2-S^2}}{\pi}$; D) $\frac{\sqrt{QS}}{\pi}$;

E) $\frac{\sqrt{Q^2+S^2}}{\pi}$.

28. Konusning asosidagi $120°$ li yoyga tiralgan vatar va konusning uchi orqali tekislik o'tkazilgan bo'lib, u asos tekisligi bilan $45°$ li burchak tashkil etadi. Agar konus asosining radiusi 4 sm bo'lsa, hosil qilingan kesimning yuzi hisoblansin.

A) $6\sqrt{5}$; B) $4\sqrt{6}$; C) $5\sqrt{6}$; D) $4\sqrt{3}$; E) $5\sqrt{3}$ sm^2.

29. Konusning yon sirti va to'la sirti yuzlarining nisbati 7:8 kabi bo'lsa, konusning yasovchisi va asosi tekisligi orasidagi burchak topilsin.

A) $\arccos\frac{1}{7}$; B) $\arccos\frac{1}{8}$; C) $\arccos\frac{1}{15}$; D) $\arccos\frac{1}{56}$;

E) $\arccos\frac{7}{8}$.

30. Konus yon sirtining yoyilmasi kattaligi $270°$ bo'lgan doiraviy sektordan iborat. Konusning yasovchisi va balandligi orasidagi burchak topilsin.

A) $\arccos\frac{3}{4}$; B) $\arccos\frac{3}{5}$; C) $\arcsin\frac{3}{5}$; D) $\arcsin\frac{3}{4}$;

E) arctg2.

31. Konus o'q kesimining uchidagi burchagi α. Agar konusning yon sirti tekislikka yoyilgan bo'lsa, yoyilmaning markaziy burchagi topilsin.

A) $180°\sin\frac{\alpha}{2}$; B) $360°\cos\frac{\alpha}{2}$; C) $\frac{360°}{\cos\alpha}$; D) $180°\cos\frac{\alpha}{2}$;

E) $360°\sin\frac{\alpha}{7}$.

32. Konusning uchidan uning asosi tekisligiga φ burchak ostida og'ma tekislik o'tkazilgan. Bu tekislik asos aylanasidan kattaligi α bo'lgan yoy ajratadi. Hosil qilingan kesimning uchidagi burchak topilsin.

A) $\operatorname{arctg}\left(\sin\frac{\alpha}{2}\cos\varphi\right)$; B) $\operatorname{arcctg}\left(\cos\frac{\alpha}{2}\cos\varphi\right)$;

C) $2\operatorname{arctg}\left(\operatorname{tg}\frac{\alpha}{2}\cos\varphi\right)$; D) $2\arccos\left(\operatorname{tg}\frac{\alpha}{2}\cos\varphi\right)$;

E) $2\arcsin\left(\operatorname{tg}\frac{\alpha}{2}\right)$.

33. Kesik konus asoslarining radiuslari 3 va 6 dm, yasovchisi 5 dm bo'lsa, uning yasovchisi va asos tekisligi orasidagi burchak topilsin.

A) $\operatorname{arctg}\frac{1}{2}$; B) $\arcsin\frac{4}{5}$; C) $\arcsin\frac{3}{5}$; D) $\arcsin\frac{3}{4}$;

E) $2\operatorname{arctg}\frac{1}{2}$.

34. Konusning balandligi 2 dm, asosining radiusi 17 sm. Konusning uchidan o'tib, asosining markazidan 12 sm uzoqlikda yotgan kesimning yuzi hisoblansin.

A) 2; B) 3; C) 2,4; D) 2,8; E) 2,1 dm^2.

35. Konusning balandligi a. Konus balandligining o'rtasidan uning yasovchisiga parallel to'g'ri chiziq o'tkazilgan. Agar konus asosining radiusi $\frac{a}{2}$ bo'lsa, o'tkazilgan to'g'ri chiziq va konusning asosi orasidagi burchak topilsin.

A) $\arccos\frac{3}{5}$; B) $\arcsin\frac{3}{4}$; C) arctg4; D) arctg2;

E) arcctg2.

36. Kesik konus asoslarining radiuslari 11 va 27 sm, yasovchisi va balandligining nisbati 17:15 kabi. Kesik konus yon sirtining yuzi hisoblansin.

A) 1144π; B) 1200π; C) 960π; D) 1180π; E) 1292π sm^2.

37. Kesik konusning yasovchisi a bo'lib, asos tekisligi bilan α burchak tashkil qiladi. Kesik konus asoslari radiuslarining nisbati 2:3 kabi bo'lsa, uning asoslari yuzlari hisoblansin.

A) $4\pi a^2\cos2\alpha$, $9\pi a^2\cos2\alpha$; B) $3\pi a^2\sin^2\alpha$, $7\pi a^2\sin^2\alpha$;
C) $4\pi a^2\cos^2\alpha$, $9\pi a^2\cos^2\alpha$; D) $6\pi a^2$, $14\pi a^2$;
E) $6\pi a^2\cos^2\alpha$, $12\pi a^2\cos^2\alpha$.

38. Kesik konus o'q kesimining diagonali a ga teng bo'lib, asos tekisligi bilan α burchak tashkil qiladi. Kesik konus asoslari radiuslarining nisbati 1:3 kabi bo'lsa, kesik konusning yasovchisi topilsin.

A) $2a(1+\sin2\alpha)$; B) $\frac{a}{2}\sqrt{1+3\sin^2\alpha}$; C) $a\sqrt{1+2\sin^2\alpha}$;

D) $\frac{a}{4}\sqrt{2+3\sin^2\alpha}$; E) $\frac{a}{3}\sqrt{1+2\cos^2\alpha}$.

39. Konusning yasovchisi 25 sm, to'la sirtining yuzi 224π sm^2 bo'lsa, uning balandligi topilsin.

A) 2; B) 18; C) 26; D) 24; E) 21 sm.

40. Konusning yasovchisi 25 sm, balandligining o'rtasidan yasovchisigacha bo'lgan masofa 6 sm bo'lsa, konus to'la sirtining yuzi hisoblansin.

A) 6π yoki 9π; B) 4π yoki 8π; C) 8π yoki 9π;
D) $6{,}4\pi$ yoki $5{,}6\pi$; E) $7{,}2\pi$ yoki $8{,}0\pi$ dm^2.

41. To'g'ri burchakli uchburchakning katetlari 3 va 4 dm bo'lib, uchburchak o'z gipotenuzasi atrofida ay-

lanadi. Hosil bo'lgan aylanma jism sirtining yuzi hisoblansin.

A) $18{,}6\pi$; B) 18π; C) 14π; D) $15{,}4\pi$; E) $16{,}8\pi$ dm^2.

42. Teng yonli trapetsiyaning asoslari 14 sm va 5 dm, diagonali esa 4 dm. Trapetsiya o'zining katta asosi atrofida aylanadi va aylanishdan hosil bo'lgan jism sirtining yuzi topilsin.

A) 2056π; B) 2108π; C) 2112π; D) 2020π; E) 1986π sm^2.

43. Kesik konus asoslarining radiuslari 12 sm va 3 dm, yasovchisining balandligiga nisbati 41:40 kabi bo'lsa, kesik konus yon sirtining yuzi hisoblansin.

A) 3456π; B) 2196π; C) 3260π; D) 3444π;
E) 3244π sm^2.

44. Kesik konusning yasovchisi a va asosi tekisligi bilan 60° li burchak tashkil etadi. Kesik konus o'q kesimining diagonali shu burchakni teng ikkiga bo'ladi. Kesik konus yon sirtining yuzi hisoblansin.

A) $\frac{11}{4}\pi a^2$; B) $3\pi a^2$; C) $\frac{19}{4}\pi a^2$; D) $\frac{5}{2}\pi a^2$; E) $4\pi a^2$.

45. Konusning yasovchisi a va asosi tekisligi bilan α burchak tashkil qiladi. Konusning hajmi hisoblansin.

A) $\frac{1}{3}\pi a^3 \sin 2\alpha \cdot \mathrm{tg}\alpha$; B) $\frac{1}{3}\pi a^3 \sin\alpha \cdot \cos^2\alpha$; C) $\frac{1}{2}\pi a^3 \mathrm{tg}\alpha$;
D) $\frac{1}{2}\pi a^3 \cos 2\alpha$; E) $\frac{1}{4}\pi a^3 \sin 2\alpha$.

46. Ikkita konus umumiy balandlikka ega bo'lib, ularning asoslari o'zaro paralleldir. Agar ularning balandligi $H=15$ sm, asoslarining radiuslari 4 va 6 sm bo'lsa, konuslar umumiy qismining hajmi hisoblansin.

A) 27,4; B) 27,8; C) 26,4; D) 28,5; E) 28,8 sm^3.

47. Kesik konusning hajmi 129 π sm^3, balandligi 9 m, yasovchisining asosidagi proyeksiyasi 5 m. Kesik konus yuqori asosining radiusi topilsin.

A) 4; B) 2,5; C) 1; D) 3; E) 2 sm.

14-§. SHAR VA SFERA

14.1. Asosiy tushunchalar va tasdiqlar

Shar — fazoning berilgan nuqtadan berilgan masofadan katta bo'lmagan uzoqlikda yotgan hamma nuqtalaridan iborat jismdir. Berilgan nuqta sharning *markazi*, berilgan masofa esa sharning *radiusidir* (14.1-chizma).

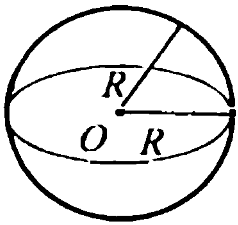
Shar sirti yoki sfera sharning chegarasidir, ya'ni sharning markazidan radiusga teng masofa qadar uzoqlashgan barcha nuqtalari sferaning nuqtalaridir.

Agar sfera markazining koordinatalari (a; b; c), radiusi R bo'lsa, uning tenglamasi

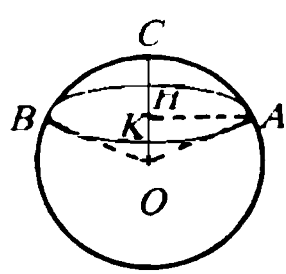
$$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \qquad (14.1)$$

ko'rinishda yoziladi.

Shar segmenti sharning sharni kesuvchi tekislik bilan chegaralangan qismidan iborat, 14.2-chizmada $AKBCA$ shar segmentidir. Kesuvchi tekislik sharni ikkita segmentga ajratadi.



14.1-chizma.



14.2-chizma.

Agar shar segmenti asosidagi nuqtalarni (agar u yarimshardan kichik bo'lsa) shar markazi bilan tutashtirsak, konus hosil bo'ladi va uning sirti shar segmenti bilan birgalikda *shar sektorini* tashkil qiladi. Agar shar segmenti yarimshardan katta bo'lsa, sharning shu konus chiqarib tashlangan qismi *shar sektoridir* (14.2-chizmada $AOBCA$ — shar sektori).

Sharning uni kesuvchi parallel tekisliklar bilan chegaralangan qismi *shar kamaridir*.

Shar segmentining *balandligi* segment asosining markazidan asosga o'tkazilgan perpendikulyarning shar sirti bilan kesishish nuqtasigacha bo'lgan masofadir.

14.1. Sferaning radiusi R bo'lsa, uning sirti

$$S=4\pi R^2 \qquad (14.2)$$

formula bo'yicha hisoblanadi.

14.2. Agar berilgan shardagi segmentning balandligi H, sferaning radiusi R ga teng bo'lsa, shar segmenti yon sirtining yuzi

$$S=2\pi RH \qquad (14.3)$$

formuladan topiladi.

14.3. Shar sektori sirtining yuzi

$$S_{sekt}=S_{segm}+S_{konus} \qquad (14.4)$$

formula bo'yicha hisoblanadi.

14.4. Sharning radiusi R bo'lsa, uning hajmi

$$V_t=\frac{4}{3}\pi R^3 \qquad (14.5)$$

formula bo'yicha hisoblanadi.

14.5. Sharning radiusi R, shar segmentining balandligi H bo'lsa, shar segmentining hajmi

$$V_{segm}=\pi H^2\left(R-\frac{1}{3}H\right), \qquad (14.6)$$

shar sektorining hajmi esa

$$V_{segm} = \frac{2}{3}\pi R^2 H \qquad (14.7)$$

formula bo'yicha hisoblanadi.

14.2. Mavzuga oid masalalar

1. Sharning radiusi 63 sm. Sharga urinma tekislikdagi bitta nuqta urinish nuqtasidan 16 sm uzoqlikda yotadi. Shu nuqtadan shar sirtigacha bo'lgan eng qisqa masofa topilsin.

A) 8 sm; B) 6 sm; C) 4 sm; D) 2 sm; E) 3 sm.

2. Ikkita sharning radiuslari 25 va 29 dm, markazlari orasidagi masofa 36 dm. Shu sharlarning sirtlari kesishgan chiziq uzunligi topilsin.

A) 8π; B) 4π; C) 3π; D) 6π; E) 12π dm.

3. Sharning radiusi a. Radiusning uchidan o'tkazilgan tekislik shu radius bilan 60° li burchak tashkil qiladi. Hosil qilingan kesimning yuzi hisoblansin.

A) $\frac{\pi a^2}{4}$; B) $\frac{\pi a^2}{3}$; C) $\frac{\pi a^2}{8}$; D) $\frac{\pi a^2}{2}$; C) $2\pi a^2$.

4. Shar kamari asoslarining radiuslari 3 va 4 m, uning sferasining radiusi 5 m. Agar kamarning asoslari shar markazining har xil tomonida yotsa, kamarning hajmi hisoblansin.

A) 96π; B) 86π; C) 72π; D) 66π; E) 64π m³.

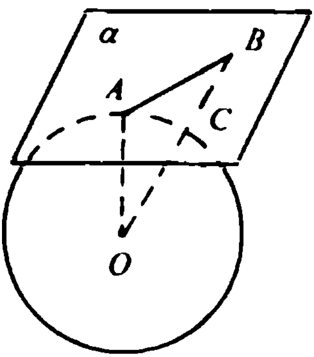
5. Shar sektorining hajmi 512π sm³, mos segment sirtining yuzi esa 96π sm². Shar sektorining balandligi topilsin.

A) 4,5; B) 5; C) 2,5; D) 4; E) 3 sm.

14.3. Mavzuga oid masalalarning yechimlari

1. Berilgan. (O, R) shar, $B \in \alpha$, $OA=R=63$ sm, α — urinma tekislik, $AB=16$ sm.

BC topilsin (14.3.1-chizma).



14.3.1-chizma.

Yechilishi. α urinma tekislikdagi B nuqtani sharning markazi O nuqta bilan tutashtiramiz. OB kesmaning shar sirti bilan kesishish nuqtasini S deb belgilasak, izlangan masofa BC kesmaning uzunligiga teng bo'ladi. Agar A urinish nuqtasi bo'lsa, $OA \perp \alpha$ va, demak, $OA \perp AB$ bo'ladi. Natijada to'g'ri burchakli ΔAOB hosil bo'ladi va unda:

$$OB^2=OA^2+AB^2=63^2+16^2=3969+256=4225,$$
$$OB=OC+CB=65 \text{ sm}.$$

U holda $BC=OB-OC=65-63=2$ sm.

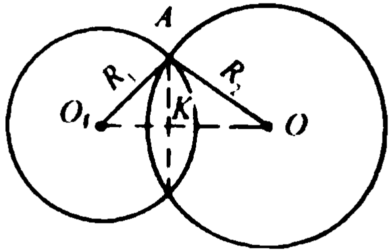
Javobi: D).

2. Berilgan. (O, R); (O_1, R_1) — sharlar, $R=29$ dm, $R_1=25$ dm, $OO_1=36$ dm.

l — kesishish chizig'i uzunligi topilsin (14.3.2-chizma).

Yechilishi. Uchburchak bir tomonining uzunligi qolgan ikki tomoni uzunliklari yig'indisidan kichik bo'lganligidan, ΔOO_1A da $36<25+29$ $(OO_1<OA+O_1A)$, demak,

sharlar kesishadi va kesishish aylanasining radiusini $AK=r$ deb belgilaymiz. Bundan tashqari, $OK=x$ deb belgilaymiz, $O_1K=$ $=36-x$ bo'ladi. Ikkita to'g'ri burchakli ΔO_1AK va ΔOAK ni qaraymiz. Ulardan Pifagor teoremasiga asosan:



14.3.2- chizma.

$$\begin{cases} r^2 = R^2 - x^2, \\ r^2 = R_1^2 - (36-x)^2 \end{cases} \Rightarrow \begin{cases} r^2 = 29^2 - x^2, \\ 29^2 - x^2 = 25^2 - (36-x)^2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} r^2 = 29^2 - x^2, \\ 29^2 - 25^2 + 36^2 = x^2 + 72x + x^2 \end{cases} \Rightarrow \begin{cases} r^2 = 29^2 - x^2, \\ 4 \cdot 54 + 36^2 = 72x \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} r^2 = 29^2 - x^2, \\ 4 \cdot 3 \cdot 9 \cdot 2 + 4^2 9^2 = 72x \end{cases} \Rightarrow \begin{cases} r^2 = 29^2 - x^2, \\ x = \frac{36 \cdot 42}{72} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} r^2 = 29^2 - x^2, \\ x = 21 \end{cases} \Rightarrow \begin{cases} r^2 = 29^2 - 21^2, \\ x = 21 \end{cases} \Rightarrow \begin{cases} r = 20, \\ x = 21 \end{cases} \text{dm.}$$

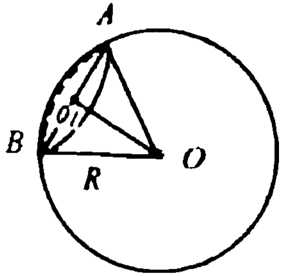
Demak, kesishish chizig'i radiusi 20 dm bo'lgan aylanadan iborat, uning uzunligi $l = 2\pi \cdot 20$ dm$=4\pi \cdot$ m.

Javobi: B).

3. Berilgan. (O, a) shar, $\angle OAB=60°$, $OA=a$.

$S_{kes.}$ hisoblansin (14.3.3-chizma).

Yechilishi. Kesimdagi doiraning radiusi $O_1A=R$ bo'lsa, uning yuzi $S_{kes}=\pi R^2$ bo'ladi. Berilishiga ko'ra, ΔAOB —



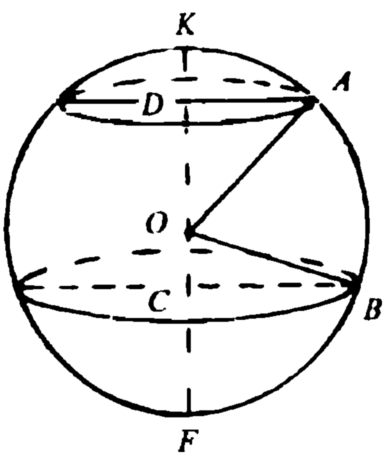
14.3.3-chizma.

teng yonli $OA = OB = a$ va asosidagi burchaklar 60° dan bo'lsa, $\angle AOB = 60°$ bo'ladi, ya'ni ΔAOB — teng tomonli va $AB = a$, ya'ni kesim doirasining diametri a bo'ladi. Demak, $R = \frac{1}{2}$, $AB = \frac{a}{2}$ va kesimning yuzi $S_{kes} = \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$ bo'ladi.

Javobi: A).

4. Berilgan. (O, R) — shar, $AD = r_1 = 3$ m, $BC = r_2 = 4$ m, $R = 5$ m.

V_{kamar} hisoblansin (14.3.4-chizma).



14.3.4-chizma.

Yechilishi. Shar kamarining hajmini topish uchun undagi ikkita shar segmentining hajmlarini topish yetarli bo'ladi, so'ngra sharning hajmidan shu hajmlarni ayirib tashlaymiz. Ma'lumki, shar segmentining hajmi $V_{segm} = \pi H^2\left(R - \frac{1}{3}H\right)$ formula bo'yicha hisoblanadi. Shakldа $CF = H_2$, $DK = H_1$ bo'lsin. To'g'ri burchakli

ΔAOD dan: $OD = \sqrt{R^2 - r_1^2} = \sqrt{5^2 - 3^2} = 4$ m;

ΔBOC dan: $OC = \sqrt{R^2 - r_2^2} = \sqrt{5^2 - 4^2} = 3$ m.

U holda $N_1=R-OD=5-4=1$ m; $H_2=R-OC=5-3=2$ m. Shar segmentlarining hajmlari, mos ravishda,

$$V_1=\pi\cdot 4^2\left(5-\frac{1}{3}\cdot 4\right)=16\pi\ \frac{11}{3}=\frac{1}{3}16\cdot 11\pi\ \text{m}^3;$$

$$V_2=\pi\cdot 3^2\left(5-\frac{1}{3}\cdot 3\right)36\pi\ \text{m}^3.$$

Sharning hajmi esa

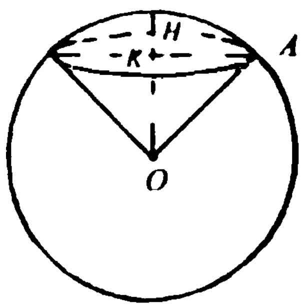
$$V_{sh.}=\frac{4}{3}\pi R^3=\frac{4}{3}\pi\cdot 5^3=\frac{5000}{3}\pi\ \text{m}^3.$$

U holda shar kamarining hajmi

$$V_{kamar.}=V_{sh}-V_1-V_2=\frac{500\pi}{3}-\frac{176}{3}\pi-36\pi=108\pi-36\pi=$$
$$=72\pi\ \text{m}^3.$$

Javobi: C).

5. Berilgan. (O, R) — shar, $V_{sh.sekt.}=512\pi$, $S_{sh.segm.}=96\pi$. H topilsin (14.3.5-chizma).



14.3.5-chizma.

Yechilishi. Ma'lumki, shar sektorining hajmi $V_{sh\,sekt}=\frac{2}{3}\pi R^2H$ formuladan, shar segmentining yuzi

$$S_{sh.segm.}=2\pi R\cdot H$$

formuladan topiladi. $V_{sh\,sekt}$: $S_{sh.segm.}$ nisbatni tuzamiz:

$$\frac{V_{sh.sekt.}}{S_{sh\,sekt.}}=\frac{\frac{2}{3}\pi R^2H}{2\pi RH}\Rightarrow\frac{512}{96\pi}=\frac{R}{3}\Rightarrow\frac{16}{3}=\frac{R}{3}\Rightarrow R=16\ \text{sm}$$

va $H=\frac{96\pi}{2\pi R}=\frac{96\pi}{2\pi\cdot 16}=3$ sm.

Javobi: E).

14.4. Mustaqil yechish uchun masalalar

1. Radiusi 41 sm bo'lgan shar markazidan 9 dm uzoqlikda yotuvchi tekislik bilan kesilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) 12π; B) 22π; C) 24π; D) 16π; E) 18π dm^2.

2. Sharning sirtida uchta nuqta berilgan bo'lib, ularning oralaridagi to'g'ri chiziq kesmalari 6, 8 va 10 sm. Sharning radiusi 13 sm bo'lsa, shar markazidan shu uchta nuqta orqali o'tkazilgan tekislikkacha bo'lgan masofa topilsin.

A) 12; B) 10; C) 13; D) 14; E) 16 sm.

3. Shar kamari asoslarining radiuslari 20 va 24 m, sharning radiusi 25 m. Kesimlar shar markazining bir tomonida yotishi ma'lum bo'lsa, shar kamari sirtining yuzi hisoblansin.

A) 289π; B) 440π; C) 400π; D) 360π; E) 424π m^2.

4. Shar segmentining balandligi h, o'q kesimidagi yoyning uzunligi 120°. Shar segmenti to'la sirtining yuzi hisoblansin.

A) $12\pi h^2$; B) $7\pi h^2$; C) $8\pi h^2$; D) $9\pi h^2$; E) $6\pi h^2$.

5. Sharning diametriga perpendikulyar bo'lgan tekislik o'tkazilgan va shu tekislik bilan sharning diametri uzunliklari 3 va 9 sm bo'lgan qismlarga ajraladi. Hosil bo'lgan shar qismlarining hajmlari hisoblansin.

A) 48π, 240π; B) 42π, 246π; C) 54π, 234π;
D) 336π, 252π; E) 45π, 243π sm^3.

6. Agar shar sektori asosining radiusi 60 sm, sharning radiusi 75 sm bo'lsa, shar sektorining hajmi hisoblansin.

A) 1160π; B) 1180π; C) 1200π; D) 1125π; E) 1196π m^2.

7. Sharning radiusi 37 sm. Sharning markazidan 23 sm uzoqlikda kesim o'tkazilgan. Shu kesimning yuzi hisoblansin.

A) 840π; B) 720π; C) 780π; D) 820π; E) 800π sm^2.

8. Sharning radiusi a bo'lib, radiusning uchidan 30° li burchak tashkil qiluvchi tekislik o'tkazilgan. Hosil qilingan kesimning yuzi hisoblansin.

A) $\frac{4}{5}\pi a^2$; B) $\frac{4}{3}\pi a^2$; C) $\frac{3}{4}\pi a^2$; D) $\frac{1}{2}\pi a^2$; E) $\frac{2}{3}\pi a^2$.

9. A va C nuqtalar sharning OK radiusini uchta teng qismga ajratadi. A va C nuqtalardan radiusga perpendikulyar bo'lgan kesimlar o'tkazilgan. Shu kesimlar yuzlarining nisbati topilsin.

A) 4:9; B) 5:8; C) 4:3; D) 3:4; E) 7:12.

10. M nuqtadan sharga MK urinma o'tkazilgan va $MK=12$. Agar sharning radiusi 5 bo'lsa, M nuqtadan shargacha bo'lgan masofa topilsin.

A) 12; B) 7; C) 9; D) 6; E) 8.

11. Sharning K nuqtasidan o'zaro perpendikulyar bo'lgan uchta KA, KB, KC vatar o'tkazilgan bo'lib, $KA=6$ sm, $KB=13$ sm, $KC=18$ sm. Sharning radiusi uzunligi topilsin.

A) 10,5; B) 13; C) 15; D) 11,5; E) 13,5 sm.

12. Sharning radiusi 5 dm. Shar sirtidagi nuqtadan o'zaro perpendikulyar va uzunliklarining nisbati: 12:15:16 kabi bo'lgan uchta vatar o'tkazilgan. Har bir vatarning uzunligi topilsin.

A) 48, 60, 64; B) 36, 48, 56; C) 24, 46, 60;
D) 48, 56, 72; E) 42, 48, 56 sm.

13. Sharda ikkita o'zaro perpendikulyar va yuzlari 185π sm^2 va 320π sm^2 bo'lgan kesimlar o'tkazilgan. Bu kesimlar

o'zaro kesishadigan vatarning uzunligi 16 sm bo'lsa, shar radiusining uzunligi topilsin.

A) 18; B) 21; C) 20; D) 28; E) 25 sm.

14. Radiusi 18 sm ga teng bo'lgan sharda ikkita o'zaro perpendikulyar kesim o'tkazilgan. Agar kesimlar radiuslarining nisbati 2:3 kabi hamda kesimlar o'zaro kesishadigan vatarning uzunligi 2 sm bo'lsa, kesimlar radiuslarining uzunliklari topilsin.

A) 16, 12; B) 12, 17; C) 10, 13; D) 12, 14;
E) 10, 15 sm.

15. Ikkita shar berilgan bo'lib, ularning radiuslari 41 sm va 5 dm, markazlari orasidagi masofa 21 sm bo'lsa, sharlar kesishish chizig'ining uzunligi topilsin.

A) 12π; B) 6π; C) 8π; D) 9π; E) 7π dm.

16. Ikkita shar berilgan bo'lib, ularning radiuslari 25 sm va 3 dm, sharlar kesishish chizig'ining uzunligi 48π sm bo'lsa, sharlarning markazlari orasidagi masofa topilsin.

A) 20 yoki 16; B) 24 yoki 18; C) 23 yoki 12;
D) 25 yoki 11; E) 20 sm yoki 12 sm.

17. To'g'ri burchakli uchburchakning katetlari 3 dm va 4 dm. Radiusi 65 sm bo'lgan shar uchburchakning uchlaridan o'tadi. Shardan uchburchak tekisligigacha bo'lgan masofa topilsin.

A) 6; B) 5; C) 8; D) 10; E) 4,5 dm.

18. Uchburchakning tomonlari 13, 14 va 15 sm. Uchburchakning uchlaridan o'tuvchi sharning markazi uchburchak tekisligidan 9 sm uzoqlikda joylashgan bo'lsa, shar radiusining uzunligi topilsin.

A) $11\frac{3}{4}$; B) $12\frac{1}{8}$; C) $12\frac{1}{4}$; D) $10\frac{3}{4}$; E) $11\frac{1}{2}$ sm.

19. Trapetsiyaning asoslari 4 dm va 48 sm, balandligi 8 sm. Shu teng yonli trapetsiyaning uchlaridan o'tuvchi sharning markazi trapetsiya tekisligidan 6 dm uzoqlikda bo'lsa, shar radiusining uzunligi topilsin.

A) 54; B) 62; C) 56; D) 60; E) 65 sm.

20. Radiusi 1 dm bo'lgan shar rombning barcha tomonlariga urinadi. Agar romb diagonallarining uzunliklari 15 sm va 2 dm bo'lsa, sharning markazidan romb tekisligigacha bo'lgan masofa topilsin.

A) 6; B) 12; C) 9; D) 8; E) 7 sm.

21. Radiusi 15 sm bo'lgan shar teng yonli trapetsiyaning barcha tomonlariga urinadi. Trapetsiyaning asoslari 16 sm va 36 sm bo'lsa, sharning markazidan trapetsiya tekisligigacha bo'lgan masofa topilsin.

A) 12; B) 8; C) 9; D) 10; E) 4 sm.

22. Markazi O nuqtada bo'lgan shar a tekislikka B nuqtada urinadi va A nuqta shu tekislikda yotadi hamda $OA=26$ sm, $AB=24$ sm. Shar sirtining yuzi hisoblansin.

A) 6π; B) 4π; C) 5π; D) $6{,}25\pi$; E) $5{,}7\pi$ dm^2.

23. Sharning markazidan 8 sm uzoqlikda tekislik o'tkazilgan bo'lib, hosil qilingan kesimdagi doiraning radiusi 6 sm. Sharning hajmi hisoblansin.

A) $\frac{4}{3}\pi$; B) $\frac{3}{4}\pi$; C) $\frac{4}{5}\pi$; D) 4π; E) $\frac{4}{7}\pi$ dm^3.

24. Birinchi shar sirtining yuzi 396π m^2. Ikkinchi sharning radiusi birinchi sharning radiusidan 3 marta kichik bo'lsa, ikkinchi shar sirtining yuzi hisoblansin.

A) 48π; B) 46π; C) 42π; D) 56π; E) 44π m^2.

25. Birinchi shar sirtining yuzi 43π ga teng. Ikkinchi sharning hajmi birinchi sharning hajmidan 27 marta katta bo'lsa, uning sirti yuzi hisoblansin.

A) 368π; B) 356π; C) 422π; D) 387π; E) 400π.

26. Shar segmentining balandligi H, o'q kesimidagi yoyning kattaligi α ga teng. Shar segmentining sferik qismi yuzi hisoblansin.

A) $\dfrac{\pi(H^2+4^2)}{\cos^2\frac{\alpha}{2}}$; B) $\dfrac{\pi H}{\sin^2\frac{\alpha}{4}}$; C) $\dfrac{\pi H^2}{\sin^2\frac{\alpha}{4}}$;

D) $\dfrac{\pi H}{\sin^4\frac{\alpha}{4}}$; E) $\dfrac{\pi H}{\mathrm{tg}^2\frac{\alpha}{2}}$.

27. Shar segmenti asosining radiusi R, o'q kesimidagi yoyning kattaligi 60°. Shu segment to'la sirtining yuzi hisoblansin.

A) $\pi R^2(1+2\sqrt{3})$; B) $\pi R^2(9-4\sqrt{3})$; C) $\pi R^2(8-4\sqrt{2})$; D) $\pi R^2(2+\sqrt{3})$; E) $\pi R^2(3+2\sqrt{3})$.

28. Sferik kamarning balandligi 7 sm, asoslarining radiuslari 16 sm va 33 sm. Kamar to'la sirtining yuzi hisoblansin.

A) 910π; B) 1080π; C) 920π; D) 1108π; E) 966π sm^2.

29. Sferik kamar asoslarining radiuslari 20 va 24 m, sharning radiusi esa 25 m. Kamar sferik qismining yuzi hisoblansin.

A) 850π yoki 650π; B) 360π yoki 1200π; C) 280π yoki 1360π; D) 440π yoki 960π; E) 400π yoki 1100π m^2.

30. Shar segmentning balandligi h, o'q kesimidagi yoyning kattaligi 120° bo'lsa, segment to'la sirtining yuzi hisoblansin.

A) $3\pi h^2$; B) $9\pi h^2$; C) $6\pi h^2$; D) $7\pi h^2$; E) $5\pi h^2$.

31. Shar segmenti asosining radiusi r, o'q kesimidagi yoyning kattaligi 90° bo'lsa, segment to'la sirtining yuzi hisoblansin.

A) $\frac{\pi r^2\sqrt{5}}{6}$; B) $\pi r^2(3-2\sqrt{2})$; C) $\pi r^2(5-2\sqrt{2})$;
D) $\frac{\pi r^2(4-\sqrt{3})}{3}$; E) $\pi r^2(6-2\sqrt{3})$.

32. Sharning diametriga perpendikulyar tekislik uni uzunliklari 3 va 9 sm bo'lgan qismlarga ajratadi. Hosil qilingan shar qismlarining hajmlari hisoblansin.

A) 60π, 180π; B) 45π, 243π; C) 54π, 224π;
D) 62π, 218π; E) 56π, 200π sm³.

33. Radiusi 13 sm bo'lgan shar markazining har xil tomonlarida o'zaro parallel va teng kesimlar o'tkazilgan. Kesimlarning har birining radiusi 5 sm bo'lsa, parallel tekisliklar orasidagi shar qismining hajmi hisoblansin.

A) 2904π; B) 2800π; C) 2860π; D) 2780π;
E) 3024π sm³.

34. Shar sektorining radiusi R, o'q kesimidagi burchagi 120° bo'lsa, shar sektorining hajmi hisoblansin.

A) $\frac{4}{5}\pi R^3$; B) $\frac{2\pi R^3}{3}$; C) πR^3; D) $\frac{1}{2}\pi R^3$; E) $\frac{1}{3}\pi R^3$.

35. Shar sektori asosining radiusi 60 sm, sharning radiusi esa 75 sm bo'lsa, shar sektorining hajmi hisoblansin.

A) 960π; B) 1260π; C) 1160π; D) 1125π; E) 1180π dm³.

36. Sharning radiusi R, shar sektorining o'q kesimidagi yoyning kattaligi α bo'lsa, shar sektorining hajmi hisoblansin.

A) $\frac{2}{3}\pi R^3\cos^2\frac{\alpha}{2}$; B) $\frac{1}{3}\pi R^3\sin^2\frac{\alpha}{2}$; C) $\frac{4}{3}\pi R^3\sin^2\frac{\alpha}{4}$;

D) $\frac{2}{5}\pi R^3\cos^2\alpha$; E) $\frac{3}{5}\pi R^3\sin2\alpha$.

37. Radiusi $R=4$ bo'lgan sharning markazi $A(2;\ -4;\ 7)$ nuqtada bo'lsa, sferaning tenglamasi topilsin.

A) $(x+2)^2+(y-4)^2+z^2=15$;
B) $(x-2)^2+(y+4)^2+(z-7)^2=16$;
C) $(x-2)^2+(y+4)^2+(z-7)^2=1$;
D) $(x+2)^2+(y-4)^2+(z+7)^2=16$;
E) $(x-2)^2+(y+4)^2+(z-7)^2=25$.

38. Markazi A$(-2,\ 2,\ 0)$ nuqtada, radiusi $R=2$ bo'lgan sferaning tenglamasi topilsin.

A) $(x+2)^2+(y-4)^2+z^2=4$; B) $(x-2)^2+(y+4)^2=4$;
C) $(x+2)^2+(y+2)^2+z^2=9$; D) $(x-2)^2+(y+2)^2+z^2=4$;
E) $x^2-4x+y^2-4y+z^2=0$

39. Markazi $A(-2,\ 2,\ 0)$ nuqtada bo'lib, $R\ (5,\ 0,\ -1)$ nuqtadan o'tuvchi sferaning tenglamasi topilsin.

A) $(x+2)^2+(y-2)^2+z^2=36$; B) $(x-2)^2+(y+2)^2+z^2=64$;
C) $(x+2)^2+(y-2)^2+z^2=49$; D) $(x+2)^2+(y+2)^2+z^2=48$;
E) $(x+2)^2+(y-2)^2+z^2=54$.

40. Tenglamasi $(x-3)^2+(y-2)^2+(z-1)^2=2$ ko'rinishda bo'lgan sferaning yuzi hisoblansin.

A) 10π; B) 12π; C) 4π; D) 8π; E) 6π.

41. Tenglamasi $x^2+y^2+z^2+6x-2y-2z-5=0$ ko'rinishda bo'lgan sferaning markazi va radiusi topilsin.

A) $C(-3,\ 1,\ 1)$, $R=3$; B) $C(3,\ -1,\ -1)$, $R=4$;
C) $A\ (-3,\ 1,\ 1)$, $R=4$; D) $C(-3,\ 1,\ -1)$, $R=5$;
E) $C(-3,\ 1,\ -1)$, $R=4$.

42. Sferaning radiusi 112 sm. Sferaning A nuqtasidan urinma tekislik o'tkazilgan va shu tekislikda B nuqta olingan. Agar $AB=15$ sm bo'lsa, B nuqtadan sferagacha bo'lgan masofa topilsin.

A) 2; B) 1; C) 3; D) 4,5; E) 1,5 sm.

43. Sharda ikkita parallel kesim o'tkazilgan. Agar kesimlarning radiuslari 9 va 12 sm, kesimlar orasidagi masofa 3 sm bo'lsa, sferaning yuzi hisoblansin.

A) 900π; B) 960π; C) 880π; D) 848π; E) 942π sm^2.

44. Shar sirtidagi nuqtadan uchta o'zaro teng bo'lgan vatar o'tkazilgan. Vatarlar o'zaro α kattalikdagi burchaklar tashkil qilsa va sharning radiusi R bo'lsa, vatarning uzunligi topilsin.

A) $2R\sqrt{3\cos(60 +\alpha)\sin(60^\circ-\alpha)}$;

B) $\frac{2R}{\sqrt{3}}\sqrt{\sin\left(60^\circ+\frac{\alpha}{2}\right)\sin\left(60^\circ-\frac{\alpha}{2}\right)}$;

C) $\frac{2}{3}R\sqrt{\cos(60^\circ+\alpha)\sin(60^\circ-\alpha)}$;

D) $\frac{4R}{\sqrt{3}}\sqrt{\sin\left(60^\circ+\frac{\alpha}{2}\right)\sin\left(60^\circ-\frac{\alpha}{2}\right)}$;

E) $4R\sqrt{3\sin\left(60^\circ+\frac{\alpha}{2}\right)\sin\left(60^\circ-\frac{\alpha}{2}\right)}$.

45. Sharning hajmi V. Shar sektori o'q kesimining markaziy burchagi α bo'lsa, sektorning hajmi hisoblansin.

A) $2V\cdot\cos\frac{\alpha}{4}$; B) $V\cdot\operatorname{tg}\frac{\alpha}{2}$; C) $V\cdot\sin^2\frac{\alpha}{4}$; D) $V\cdot\cos^2\frac{\alpha}{4}$;

E) $V\cdot\operatorname{tg}^2\frac{\alpha}{2}$.

46. Sharning radiusi R bo'lib, shar sektorining markaziy burchagi α ga teng bo'lsa, shar sektori to'la sirtining yuzi hisoblansin.

A) $2\pi R^2\sin\alpha$; B) $\pi R^2\cos\frac{\alpha}{2}$; C) $\pi R^2\operatorname{tg}\frac{\alpha}{2}$;

D) $\pi R^2\sin\frac{\alpha}{2}\operatorname{tg}^2\varphi$; E) $\frac{\pi R^2\sin\frac{\alpha}{2}}{\cos^2\varphi}$ va $\operatorname{tg}\varphi=\sqrt{2\operatorname{tg}\frac{\alpha}{4}}$.

47. Agar sfera AB diametrining $A(2, -3, 5)$, $B(4, 1, -3)$ uchlari berilgan bo'lsa, sferaning tenglamasi topilsin.

A) $(x+3)^2+(y-1)^2+z^2=25$;
B) $(x-3)^2+(y+1)^2+(z-1)^2=21$;
C) $(x-3)^2+(y+1)^2+(z-1)^2=36$;
D) $(x-2)^2+(y+3)^2+(z-5)^2=25$;
E) $(x-4)^2+(y-1)^2+(z+3)^2=36$.

48. Sferasining tenglamasi $x^2+y^2+z^2-4x-6y+2z+5=0$ ko'rinishda bo'lgan sharning hajmi hisoblansin.

A) 56π; B) 24π; C) 32π; D) 36π; E) 48π.

15-§. SHARGA ICHKI VA TASHQI CHIZILGAN KO'PYOQLAR VA JISMLAR

15.1. Asosiy tushunchalar va tasdiqlar

Agar ko'pyoqning hamma uchlari sferaga tegishli bo'lsa, ko'pyoq sferaga *ichki chizilgan* bo'ladi (sferaning o'zi ko'pyoqqa *tashqi chizilgan* bo'ladi).

Agar ko'pyoqning barcha yoqlari sferaga urinsa, ko'pyoq sferaga *tashqi chizilgan* bo'ladi (bunda sferaning o'zi ko'pyoqqa *ichki chizilgan* bo'ladi).

Quyidagi tasdiqlar o'rinli:

1. Agar piramida asosiga aylanani tashqi chizish mumkin bo'lsa, piramidaga tashqi sfera chizish mumkin.

2. Agar prizma to'g'ri bo'lib, uning asosiga aylanani tashqi chizish mumkin bo'lsa, prizmaga sferani tashqi chizish mumkin.

Bulardan: muntazam prizma va muntazam piramidaga sferani tashqi chizish mumkinligi kelib chiqadi.

3. Ixtiyoriy tetraedrga sferani ichki chizish mumkin. Uning markazi tetraedr ikkiyoqli burchaklari bissektor (teng ikkiga bo'luvchi) tekisliklarining kesishish nuqtasidan iborat bo'ladi.

4. Agar piramidaning asosiga aylanani ichki chizish mumkin bo'lsa va piramidaning balandligi o'sha aylana markazidan o'tsa, piramidaga sferani ichki chizish mumkin.

Demak, muntazam piramidaga sferani doimo ichki chizish mumkin.

5. Agar: 1) prizmaning perpendikulyar kesimiga aylanani ichki chizish mumkin bo'lsa;

2) prizmaning balandligi aylana diametriga teng bo'lsa, prizmaga sferani ichki chizish mumkin.

6. Ixtiyoriy silindr va konusga sferani tashqi chizish mumkin. Tashqi chizilgan sferaning markazi silindr yoki konusning o'q kesimiga tashqi chizilgan aylananing markazidir.

7. Agar silindrning balandligi uning asosi diametriga teng bo'lsa, silindrga sferani ichki chizish mumkin.

8. Har qanday konusga sferani ichki chizish mumkin.

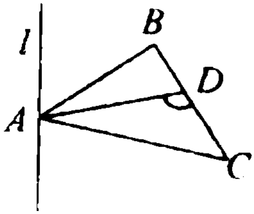
9. P. Gyulden teoremasi: tekis shaklning shu shakl tekisligida yotib, uni kesib o'tmaydigan o'q atrofida aylanishidan hosil bo'lgan jismning hajmi shakl yuzining shakl og'irlik markazi chizgan aylana uzunligiga ko'paytmasiga teng:

$$V_{o} = 2\pi \cdot S \cdot d_s,$$

bunda S — aylanayotgan shaklning yuzi, d_s — shakl og'irlik markazidan aylanish o'qigacha bo'lgan masofa.

Quyidagi tasdiq ham o'rinli:

10. Lemma. Agar ΔABC uchburchak tekisligida yotib, uning A uchidan BC tomonni kesmasdan o'tuvchi l o'q



atrofida aylansa — bu aylanish natijasida hosil qilingan jismning hajmi — qarshida yotgan BC tomon hosil qilgan sirtning S_{BC} yuzi bilan uchburchakning shu tomonga tushirilgan balandligi uchdan birining ko'paytmasiga teng:

$$V_{a.j.} = \frac{AD}{3} \cdot S_{BC}.$$

15.2. Mavzuga oid masalalar

1. Teng yonli silindr o'q kesimining diagonali d. Silindrga ichki chizilgan oltiburchakli muntazam prizma eng kichik diagonal kesimining yuzi hisoblansin.

A) $\frac{d^2\sqrt{2}}{4}$; B) $\frac{d^2}{8}$; C) $\frac{d^2\sqrt{3}}{4}$; D) $\frac{d^2\sqrt{2}}{3}$; E) $\frac{d^2\sqrt{6}}{4}$.

2. Konus asosining radiusi r, balandligi h. Konusga barcha qirralari o'zaro teng bo'lgan uchburchakli muntazam prizma ichki chizilgan. Prizmaning qirrasi topilsin.

A) $\frac{rh\sqrt{2}}{h+r\sqrt{2}}$; B) $\frac{h^2\sqrt{3}}{h+r}$; C) $\frac{r^2\sqrt{3}}{h+r}$; D) $\frac{r^2\sqrt{2}}{h+r}$; E) $\frac{rh\sqrt{3}}{h+r\sqrt{3}}$.

3. Radiusi R bo'lgan sharga uchburchakli muntazam prizma tashqi chizilgan. Prizma to'la sirtining yuzi hisoblansin.

A) 24 $R^2\sqrt{3}$; B) 18 $R^2\sqrt{3}$; C) 16 $R^2\sqrt{3}$; D) 20 R^2; E) 24 $\sqrt{2}\ R^2$.

4. Konusga shar ichki chizilgan. Shar va konusning urinish nuqtalaridan hosil bo'lgan aylananing radiusi r, konusning balandligi va yasovchisi orasidagi burchak α. Konusning hajmi hisoblansin.

A) $\frac{\pi r^3(1+\text{tg}\alpha)^3}{8\text{tg}^2\alpha}$; B) $\frac{\pi r^3 \cos^2\alpha}{2\sin^2\alpha\cos^3\alpha}$; C) $\frac{\pi r^3 \sin^2\alpha}{4\sin\alpha\cos^2\alpha}$;

D) $\frac{\pi r^3(1+\sin\alpha)^3}{3\sin\alpha\cdot\cos^5\alpha}$; E) $\frac{\pi r^3(1+\sin\alpha)^3}{3\cos\alpha\cdot\sin^3\alpha}$.

5. Muntazam tetraedrning qirrasi 1 sm. Shu tetraedrga tashqi chizilgan sharning radiusi topilsin.

A) $\frac{\sqrt{3}}{2}$; B) $\frac{\sqrt{6}}{4}$; C) $\frac{\sqrt{5}}{3}$; D) $\frac{\sqrt{6}}{2}$; E) $\frac{1}{2}$.

6. To'la sirtining yuzi S bo'lgan piramidaga R radiusli sfera ichki chizilgan. Piramidaning hajmi hisoblansin.

A) $\frac{1}{2}SR$; B) $\frac{1}{8}S^2R$; C) $\frac{R}{4}(S^2+R^2)$; D) $\frac{S}{4}(S^2+R^2)$;

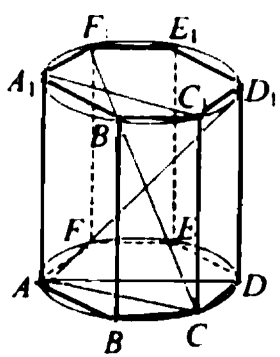
E) $\frac{1}{3}SR$.

15.3. Mavzuga oid masalalarning yechimlari

1. Berilgan. $AB...E_1F_1$ — mintazam prizma, AD_1 — tashqi chizilgan silindr, $AD_1=d$; $AA_1=AD$.

$S_{AA_1C_1C}$ hisoblansin (15.3.1-chizma).

Yechilishi. Teng yonli silindrning o'q kesimi kvadrat bo'ladi. Agar silindr asosining radiusi R ga teng bo'lsa, aylanaga ichki chizilgan muntazam oltiburchakning tomoni shu radiusga teng bo'ladi (5-§): $AB=$ $=BC=...FA=R$. To'g'ri burchakli ΔADD_1 dan Pifagor teoremasi (2-§) yordamida $AD^2+DD_1^2=AD_1^2$, $(2R)^2+(2R^2)=d^2$, $8R=d^2$, $R^2=\frac{d^2}{8}$, $R=\frac{d\sqrt{2}}{4}$ bo'ladi.



15.3.1-chizma.

U holda silindrning balandligi: $H=2R=\frac{d\sqrt{2}}{4}$. Ma'lumki, muntazam oltiburchakning ichki burchagi (5-§): $\angle ABC=$ $=\frac{180°(6-2)}{6}=30°\cdot4=120°$. ΔABC dan, kosinuslar teoremasi yordamida, oltiburchakning AC kichik diagonalini topamiz:

$$AC^2=2\cdot AB^2-2AB^2\cdot\cos120°=2AB^2+2AB^2\cdot\frac{1}{2}=3AB^2,$$

$$AC=AB\sqrt{3}=R\sqrt{3}=\frac{d\sqrt{2}}{4}\sqrt{3}=\frac{d\sqrt{6}}{4}.$$

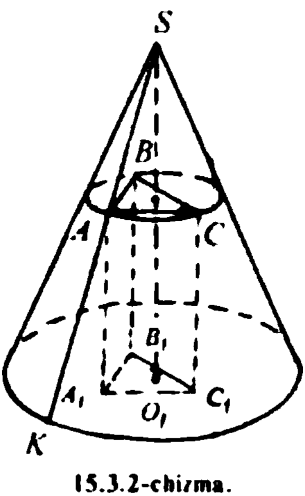
Demak, endi kichik diagonal kesimning yuzi:

$$S_{AA_1C_1C}=AC\cdot H=\frac{d\sqrt{6}}{4}\cdot\frac{d\sqrt{2}}{2}=\frac{d^2\sqrt{3}}{4}.$$

Javobi: C).

2. B e r i l g a n. SK — konus, $ABCA_1B_1C_1$ — ichki chizilgan muntazam prizma, $O_1K=r$; $SO=h$; $AB=AA_1$.

AA_1 t o p i l s i n (15.3.2-chizma).



15.3.2-chizma.

Y e c h i l i s h i. Konusning SA yasovchisini o'tkazamiz va uning asos tekisligi bilan kesishish nuqtasini K deb belgilaymiz. Prizma muntazam bo'lganligidan, konusning balandligi prizma asoslarining O va O_1 markazlaridan o'tadi, OA kesma ΔABC ga tashqi chizilgan aylananing radiusi $OA=R$ bo'ladi va $O_1A_1=R$. Lekin $O_1K=r$. To'g'ri burchakli $\Delta SOA\sim\Delta SKO_1$ va ulardan $\frac{O_1K}{OA}=\frac{SO_1}{SO}$ bo'ladi.

$AB=a$ bo'lsin, u holda $OA=\frac{a}{2\sin 60^\circ}=\frac{a}{\sqrt{3}}$ (sinuslar teoremasiga asosan). Berilganiga binoan, $OO_1=AA_1==a$ va $SO=h-a$. Yuqorida yozilgan proporsiyaga keltirib qo'ysak, $\frac{r}{a\sqrt{3}}=\frac{h}{h-a}$ va uni a ga nisbatan yechamiz:

$$\sqrt{3}rh-\sqrt{3}ra=ah,\ a(h+r\sqrt{3})=rh\sqrt{3},\ a=\frac{rh\sqrt{3}}{h+r\sqrt{3}}.$$

Javobi: E).

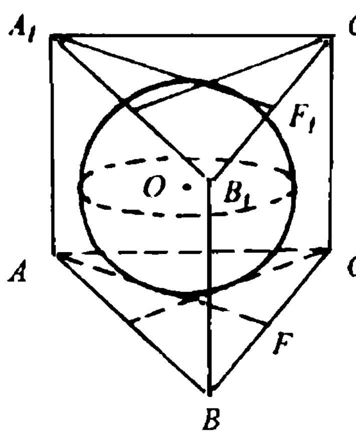
3. Berilgan. (O, R) — shar, $ABCA_1B_1C_1$ — tashqi chizilgan prizma.

$S_{\text{t pr}}$ hisoblansin (15.3.3-chizma).

Yechilishi. Shar ichki chizilgan bo'lganligidan, uning diametri prizmaning balandligiga teng: $H=2R$. Agar prizma asosining tomoni $AB=a$, balandligi H bo'lsa, to'la sirti

$$S_{\text{t.pr.}}=3a\cdot H+2\cdot S_{\text{ж}},$$
$$S_{\text{ж}}=\frac{a^2\sqrt{3}}{4}$$

formula bo'yicha hisoblanadi.



15.3.3-chizma.

Sharning markazidan prizmaning asosiga parallel tekislik o'tkazamiz va kesimda prizmaning asosi ΔABC ga teng bo'lgan uchburchakni hamda unga ichki chizilgan R radiusli aylanani hosil qilamiz. Muntazam ΔABC ga ichki chizilgan aylananing O markazi uchburchak bissektrisalarning kesishish nuqtasidir (5-§). Shuning uchun, ΔAOK — to'g'ri burchakli ($OK \perp AB$), $\angle AOK = 30°$; $AK = \frac{a}{2}$ va $AK = OK \cdot \text{ctg}30°$ yoki $\frac{a}{2} = R \cdot \text{ctg}30°$, $a = 2R\sqrt{3}$.

U holda $S_{yon} = 3 \cdot 2R\sqrt{3} \cdot 2R = 12R^2\sqrt{3}$;

$$S_s = \frac{1}{4}(2R\sqrt{3})^2 \cdot \sqrt{3} = 2R^2\sqrt{3}$$

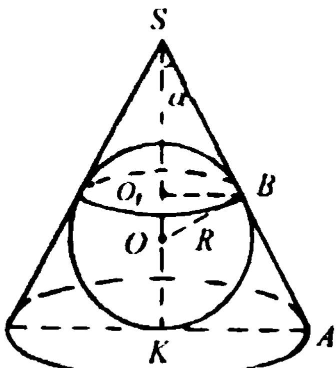
va

$$S_{t.pr} = S_{yon} + 2S_{as} = 12R^2\sqrt{3} + 6R^2\sqrt{3} = 18R^2\sqrt{3}.$$

Javobi: B).

4. Berilgan. (SAK) — konus, (O, R) — ichki chizilgan shar, (O_1, r) — kesim, $\angle ASK = \alpha$.

V_k hisoblansin (15.3.4-chizma).



15.3.4-chizma.

Yechilishi. Ma'lumki, konusning hajmi $V_k = \frac{1}{3} S_{.c} \times$ $\times H$ formula bo'yicha hisoblanadi. Agar konus asosining radiusi $AK = R_1$, konusning balandligi $SK = H$ bo'lsa, $V_k = \frac{1}{3}\pi R_1^2 H$ bo'ladi. Kesimdagi doiraning $O_1B = r$ radiusi urinmaga perpendikulyardir. Shuning uchun, ΔABC va ΔO_1BS to'g'ri burchakli bo'ladi va $O_1B \perp SK$, $OB \perp SA$ bo'lgani uchun $\angle O_1BO = \angle O_1SB_1$, chunki ularning mos tomonlari o'zaro perpendikulyar. ΔOBO_1 dan ichki chizilgan sharning radiusini topamiz:

$$\cos\alpha = \frac{O_1B}{OB}; \quad R = OB = \frac{O_1B}{\cos\alpha} = \frac{r}{\cos\alpha}.$$

To‘g‘ri burchakli ΔOBS dan:

$$\sin\alpha = \frac{OB}{SO};\ SO = \frac{R}{\sin\alpha} = \frac{r}{\sin\alpha\cdot\cos\alpha}.$$

U holda

$$H = SO + OK = \frac{r}{\sin\alpha\cdot\cos\alpha} + \frac{r}{\cos\alpha} = \frac{r(1+\sin\alpha)}{\sin\alpha\cdot\cos\alpha}.$$

Konus asosining radiusi

$$R_1 = H\cdot \mathrm{tg}\alpha = \frac{r(1+\sin\alpha)}{\sin\alpha\cdot\cos^2\alpha};\ R_1 = \frac{r(1+\sin\alpha)}{\cos^2\alpha}$$

bo‘ladi, demak, konusning hajmi

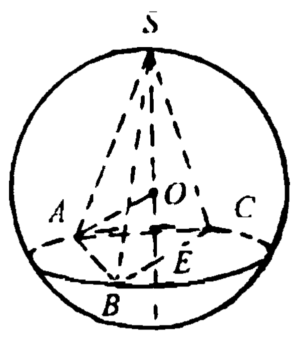
$$V_k = \frac{1}{3}\pi\frac{r^2(1+\sin\alpha)^2}{\cos^4\alpha}\cdot\frac{r(1+\sin\alpha)}{\sin\alpha\cdot\cos\alpha} = \frac{\pi r^3(1+\sin\alpha)^3}{3\sin\alpha\cdot\cos^5\alpha}.$$

Javobi: D).

5. Berilgan $SABC$ — muntazam tetraedr, $AS=AB=1$, $OA=OS$, (O, R) — tashqi chizilgan shar.

R topilsin (15.3.5-chizma).

Yechilishi. Piramida muntazam bo‘lganligidan, uning SE balandligi piramidaning asosi — ΔABC medianalarining kesishish nuqtasi E dan o‘tadi. U holda $AE=r$ shu ΔABC ga ichki chizilgan aylananing radiusidan iborat va $2r = \frac{AB}{\sin 60^\circ}$, $r = \frac{1}{2\cdot\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$.



15.3.5-chizma.

To‘g‘ri burchakli ΔAES dan

$$H = SE = \sqrt{AS^2 - AE^2},\quad H = \sqrt{1-\frac{1}{3}} = \sqrt{\frac{2\cdot 3}{3\cdot 3}} = \frac{\sqrt{6}}{3}.$$

Endi SE balandlikni shar bilan K nuqtada kesishguncha davom ettiramiz va $SK=2R$ sharning diametri bo‘ladi hamda ΔSAK — to‘g‘ri burchakli va AE uning SK gipotenuzasiga o‘tkazilgan balandlikdan iborat.

Shu balandlikning xossasiga ko‘ra: $AE^2=SE\cdot EK$.

Bizning holda $AE=r$, $EK=2R-H$. Shuning uchun,

$$\left(\frac{1}{\sqrt3}\right)^2=\frac{\sqrt6}{3}\left(2R-\frac{\sqrt6}{3}\right),\ \frac13=\frac{\sqrt6}{3}\left(2R-\frac{\sqrt6}{3}\right),\ 1=2\sqrt6\,R=\frac63,$$

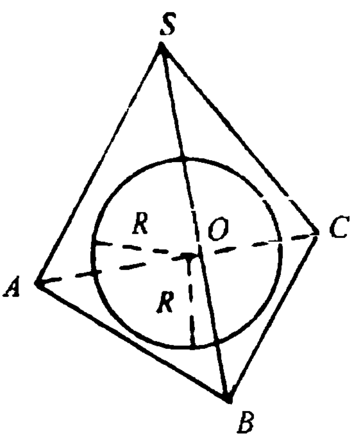
$$3=2\sqrt6\,R,\ R=\frac{3\cdot\sqrt6}{2\sqrt6\cdot\sqrt6},\ R=\frac{3\sqrt6}{2\cdot6}=\frac{\sqrt6}{4}.$$

Javobi: B).

6. B e r i l g a n. $SABC$ — piramida, $S=S_1(O,R)$ — ichki chizilgan shar.

V_{pir} h i s o b l a n s i n (15.3.6-chizma).

Y e c h i l i s h i. Masalani uch burchakli piramida uchun yechamiz. Faraz qilaylik, piramida yoqlarining yuzlari, mos ravishda, $S_{ABS}==S_1$, $S_{BSC}=S_2$, $S_{ASC}=S_3$, $S_{ABC}=S_4$ bo‘lsin.



15.3.6-chizma.

Ichki chizilgan sharning O markazidan piramidaning yoqlariga radiuslar o‘tkazamiz va bu radiuslar mos yoqlarga perpendikulyar bo‘ladi. O markazni piramidaning uchlari bilan tutashtirsak, u to‘rtta $OSAB$, $OSBC$, $OSAC$, $OABC$ piramidaga ajraladi. Natijada berilgan piramidaning hajmi shu to‘rtta piramida hajmlarining yig‘indisiga teng bo‘ladi:

$$V = V_1 + V_2 + V_3 + V_4,$$

ya'ni

$$V = \frac{1}{3} S_1 R + \frac{1}{3} S_2 R + \frac{1}{3} S_3 R + \frac{1}{3} S_4 R;$$

$$V = \frac{1}{3} R(S_1 + S_2 + S_3 + S_4).$$

Lekin, $S_1 + S_2 + S_3 + S_4 = S_t = S$. Shuning uchun piramidaning hajmi

$$V = \frac{1}{3} S \cdot R$$

bo'ladi. Bu formula ixtiyoriy piramida uchun ham isbot qilinishi mumkin.

Javobi: E).

15.4. Mustaqil yechish uchun masalalar

1. Radiusi 9 dm bo'lgan sharga to'rt burchakli muntazam prizma ichki chizilgan. Agar prizmaning balandligi 14 dm bo'lsa, prizma asosi tomonining uzunligi topilsin.

A) 6; B) 8; C) 10; D) 12; E) 9 dm.

2. Oltiburchakli muntazam prizmaning balandligi 8 m, yon yog'ining diagonali 13 m. Unga tashqi chizilgan sharning radiusi topilsin.

A) 8; B) 14; C) 12; D) 11; E) 10 m.

3. Tomonlari 6, 8 va 10 sm bo'lgan uchburchak to'g'ri prizmaning asosidan iborat. Prizmaning balandligi 24 sm bo'lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) 15; B) 12; C) 13; D) 11; E) 16 sm.

4. To'rt burchakli muntazam piramidaning balandligi h, yon qirrasi b bo'lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) $3b-2h$; B) $\frac{b^2-h^2}{h}$; C) $\frac{b^2+h^2}{2bh}$; D) $\frac{h^2}{2b}$; E) $\frac{b^2}{2h}$.

5. Muntazam tetraedrning qirrasi a berilgan bo'lsa, unga ichki chizilgan sharning radiusi topilsin.

A) $\frac{a\sqrt{6}}{12}$, B) $\frac{a\sqrt{2}}{3}$; C) $\frac{a\sqrt{3}}{12}$; D) $\frac{a\sqrt{3}}{14}$; E) $\frac{a\sqrt{2}}{2}$.

6. Berilgan piramida yon qirralarining har biri 9 sm dan, balandligi esa 5 sm bo'lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) 6,8; B) 8,1; C) 7,2; D) 9; E) 7 sm.

7. Balandligi h, asosidagi ikkiyoqli burchagi 60° bo'lgan muntazam piramidaga ichki chizilgan sharning radiusi topilsin.

A) $\frac{h}{2}$; B) $\frac{3}{5}h$; C) $\frac{2}{5}h$; D) $\frac{1}{3}h$; E) $\frac{2}{3}h$.

8. Uch burchakli muntazam piramidaning balandligi h, yon qirralari esa o'zaro perpendikulyar bo'lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) $1{,}8h$; B) $2h$; C) $1{,}5h$; D) $0{,}75h$; E) $1{,}2h$.

9. Piramidaning asosi tomoni 3 dm bo'lgan muntazam uchburchakdan iborat, yon qirralaridan biri 2 dm va asosiga perpendikulyardir. Unga tashqi chizilgan sharning radiusi topilsin.

A) 2,5; B) 1,5; C) 1; D) 3; E) 2 dm.

10. To'rt burchakli muntazam prizma asosining tomoni 6 sm, yon qirrasi 17 sm bo'lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) 9,5; B) 8; C) 10; D) 8,5; E) 12 sm.

11. To'g'ri burchakli parallelepipedning o'lchamlari nisbati 2:3:6 kabi, to'la sirtining yuzi 1152 sm^2 bo'lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) 16; B) 14; C) 12; D) 15; E) 10 sm.

12. To‘g‘ri burchakli parallelepiped yoqlarining diagonallari, mos ravishda, a, b, c bo‘lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) $\frac{1}{8}\sqrt{a^2+2(b^2+c^2)}$; B) $\frac{1}{4}\sqrt{2(a^2+c^2)-b^2}$;

C) $\frac{1}{4}\sqrt{2(b^2+c^2)-a^2}$; D) $\frac{1}{4}\sqrt{2(a^2+b^2+c^2)}$;

E) $\frac{1}{4}\sqrt{2(a^2+b^2)-c^2}$.

13. Radiusi 21 sm bo‘lgan sharga balandligi 14 sm bo‘lgan to‘rt burchakli muntazam prizma ichki chizilgan. Prizma to‘la sirtining yuzi hisoblansin.

A) 2780; B) 3242; C) 3136; D) 2960; E) 3164 sm^2.

14. Uch burchakli muntazam prizma asosining tomoni 12 sm, balandligi 2 sm bo‘lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) 8; B) 10; C) 5; D) 6; E) 7 sm.

15. Radiusi 14 sm bo‘lgan sharga uchburchakli muntazam prizma ichki chizilgan. Prizmaning balandligi asosining tomonidan 17 sm katta bo‘lsa, prizma yon sirtining yuzi hisoblansin.

A) 702; B) 696; C) 760; D) 792; E) 640 sm^2.

16. To‘g‘ri prizmaning asosi teng yonli uchburchakdir. Uchburchakning asosi 6 sm, balandligi 1 sm hamda prizmaning balandligi 24 sm bo‘lsa, prizmaga tashqi chizilgan sharning radiusi topilsin.

A) 12; B) 13; C) 14; D) 16; E) 10 sm.

17. Olti burchakli muntazam prizma asosining tomoni 4 dm, prizmaning balandligi 15 dm bo‘lsa, unga tashqi chizilgan sharning radiusi topilsin.

A) 10; B) 9,5; C) 9; D) 8,5; E) 8 dm.

18. To'rt burchakli piramida qirralarining har biri a. Piramidaga tashqi chizilgan sharning radiusi topilsin.

A) $a\sqrt{6}$; B) $\frac{a\sqrt{3}}{4}$; C) $\frac{a\sqrt{2}}{2}$; D) $\frac{a\sqrt{2}}{4}$; E) $\frac{a\sqrt{3}}{2}$.

19. To'rt burchakli muntazam piramida asosining tomoni 8 dm, yon qirrasi 9 dm. Piramidaga tashqi chizilgan sharning radiusi topilsin.

A) $4\frac{13}{15}$; B) 4; C) 5; D) 6; E) $5\frac{11}{14}$ dm.

20. Uch burchakli muntazam piramida asosining tomoni a, piramidaning uchidagi yassi burchaklari 90° bo'lsa, piramidaga tashqi chizilgan sharning radiusi topilsin.

A) $a\sqrt{6}$; B) $\frac{a\sqrt{6}}{4}$; C) $\frac{a\sqrt{6}}{2}$; D) $\frac{a\sqrt{3}}{4}$; E) $\frac{a\sqrt{2}}{3}$.

21. Uch burchakli muntazam piramidaning balandligi 5 dm, yon qirrasi va asosi tomonining nisbati 2:3 kabidir. Piramidaga tashqi chizilgan sharning radiusi topilsin.

A) 1; B) 0,5; C) 2; D) 2,5; E) 0,75 dm.

22. Olti burchakli muntazam piramidaga tashqi chizilgan sharning markazi piramida balandligini uzunliklari 1 va 7 sm bo'lgan kesmalarga ajratadi. Piramidaning hajmi hisoblansin.

A) $196\sqrt{3}$; B) $184\sqrt{3}$; C) 202; D) $192\sqrt{3}$; E) 196 sm^3.

23. Prizmaga shar ichki chizilgan. Prizmaning asosi teng yonli trapetsiyadan iborat bo'lib, uning asoslari 8 sm va 5 sm. Prizma to'la sirtining yuzi hisoblansin.

A) 3740; B) 3080; C) 3480; D) 3250; E) 3560 sm^2.

24. Piramidaning asosi tomonlari 25, 29 va 36 sm bo'lgan uchburchakdan iborat. Agar piramidaning uchi asosining tomonlaridan bir xil uzoqlikda bo'lsa, piramidaga ichki chizilgan sharning radiusi topilsin.

A) 1,5; B) 3; C) $2\frac{4}{7}$; D) 2; E) $2\frac{2}{3}$ sm.

25. Konusning yasovchisi 17 sm, balandligi 15 sm. Konusga shar ichki chizilgan bo'lsa, shar va konusning urinish nuqtalari hosil qilgan aylananing uzunligi topilsin.

A) $\frac{121\pi}{42}$; B) $\frac{144}{17}\pi$; C) $\frac{169\pi}{24}$; D) $\frac{156\pi}{37}$; E) $\frac{144\pi}{13}$.

26. Konus asosining radiusi 6 dm. Konusga shar ichki chizilgan va ular uringan aylananing uzunligi 4π dm. Konusning hajmini va yon sirtining yuzi hisoblansin.

A) $36\sqrt{5}\pi$, 90π; B) $48\sqrt{3}\pi$, 72π; C) $36\sqrt{2}\pi$, 72π;
D) $36\sqrt{2}\pi$, 84π; E) $36\sqrt{3}\pi$ dm^3, 90π dm^2.

27. Kesik konusning yasovchisi 13 sm, asoslaridan birining radiusi 4 sm. Agar shu kesik konusga sharni ichki chizish mumkin bo'lsa, uning yon sirti yuzi va hajmi hisoblansin.

A) 272π, 542π; B) 216π, 532π; C) 266π, 532π;
D) 266π, 486π; E) 272π sm^2, 486π sm^3.

28. Radiusi 12 sm bo'lgan sharga kesik konus tashqi chizilgan. Kesik konus asoslari radiuslarining nisbati: 4:9 kabi bo'lsa, uning hajmi hisoblansin.

A) 5216π; B) 3576π; C) 4526π; D) 4256π; E) 3976π sm^3.

29. Radiusi 6 sm bo'lgan sharga yasovchisi 15 sm bo'lgan kesik konus tashqi chizilgan. Shar va kesik konus urinish chizig'ining uzunligi topilsin.

A) $6,8\pi$; B) $10,2\pi$; C) $8,4\pi$; D) $7,2\pi$; E) $9,6\pi$ sm.

30. Asosining radiusi 9 sm bo'lgan konusga shar ichki chizilgan. Ularning urinish chizig'idan tekislik o'tkazilgan va bu tekislik konusning hajmini, uchidan hisoblaganda, 8:117 nisbatda bo'ladi. Sharning radiusi topilsin.

A) 3; B) 4,5; C) 5; D) 4; E) 6,5 sm.

31. Piramidaning asosi — romb va uning diagonallari 6 va 8 m. Piramidaning balandligi 1 m va asosining markazidan o'tadi. Piramidaga ichki chizilgan sharning radiusi topilsin.

A) 0,36; B) 0,72; C) 0,48; D) 1,2; E) 0,52 m.

32. Uch burchakli muntazam kesik piramida asoslarining tomonlari a va b. Shu piramidaga shar ichki chizilgan. Kesik piramida yon sirtining yuzi hisoblansin.

A) $\frac{\sqrt{3}}{4}(a+b)^2$; B) $\frac{\sqrt{3}}{2}(a+b)^2$; C) $\frac{1}{4}(a+b)^2$;

D) $\frac{3}{5}(a+b)^2$; E) $\frac{\sqrt{3}}{2}(a-b)^2$.

33. Olti burchakli muntazam kesik piramida asoslarining tomonlari a va b. Unga shar ichki chizilgan bo'lsa, kesik piramida yon sirtining yuzi hisoblansin.

A) $\frac{\sqrt{3}}{4}(a+b)^2$; B) $\frac{3\sqrt{3}}{2}(a-b)^2$; C) $\frac{3}{2}(a-b)^2$;

D) $\frac{3\sqrt{3}}{2}(a+b)^2$; E) $\frac{\sqrt{3}}{2}(a+b)^2$.

34. Sharga to'rt burchakli muntazam kesik piramida tashqi chizilgan. Uning asoslari tomonlari a va b bo'lsa, kesik piramida yon sirtining yuzi hisoblansin.

A) $\frac{1}{2}(a-b)^2$; B) $(a-b)^2$; C) $\frac{a^2+b^2}{2}$; D) $\frac{1}{2}(a+b)^2$;

E) $(a+b)^2$.

35. Radiusi $2r$ bo'lgan sharga asosining radiusi r bo'lgan konus ichki chizilgan. Konusning hajmi hisoblansin.

A) $\frac{r\pi}{3}$ yoki $\frac{r\pi}{2}$; B) $\frac{r\pi}{3}(2+\sqrt{3})$ yoki $\frac{r\pi}{3}(2-\sqrt{3})$;

S) $\frac{r\pi}{3}(1+\sqrt{3})$ yoki $\frac{r\pi}{3}(1-\sqrt{3})$;

D) $\frac{r\pi}{3}(1+\sqrt{2})$ yoki $\frac{r\pi}{3}(1-\sqrt{2})$;

E) $\frac{r\pi}{2}(1+\sqrt{3})$ yoki $\frac{r\pi}{2}(1-\sqrt{3})$.

36. Konusning yasovchisi l va asosi tekisligi bilan α burchak tashkil qiladi. Konusga tashqi chizilgan sharning hajmi hisoblansin.

A) $\frac{\pi l^3 \sin\alpha}{\cos^3\alpha}$; B) $\frac{\pi l^3 \cos\alpha}{4\sin^2\alpha}$; C) $\frac{\pi l^3}{6\sin^3\alpha}$; D) $\frac{\pi l^3 \cos\alpha}{6\sin^2\alpha}$;

E) $\frac{\pi l^3 \sin\alpha}{4\cos^2\alpha}$.

37. Kubga ichki va tashqi chizilgan sharlar hajmlarining nisbati topilsin.

A) $\sqrt{3}$:9; B) 3:7; C) $\sqrt{2}$:5; D) 5:9; E) 7:9.

38. Konusning o'q kesimi muntazam uchburchakdan iborat. Konusga tashqi va ichki chizilgan sferalar yuzlarining nisbati topilsin.

A) 6:5; B) 3:1; C) 4:5; D) 2:3; E) 4:1.

39. Muntazam piramidaning apofemasi m va asosi tekisligi bilan α burchak tashkil qiladi. Piramidaga ichki chizilgan sfera sirtining yuzi hisoblansin.

A) $9\pi m^2 \cos\alpha \cdot \mathrm{tg}\frac{\alpha}{2}$; B) $\pi(2m\cos\alpha \cdot \mathrm{tg}\frac{\alpha}{2})^2$;

C) $4\pi m^2 \cdot \mathrm{tg}^2\frac{\alpha}{2}$; D) $2\pi m^2\cos^2\alpha \cdot \mathrm{tg}\frac{\alpha}{2}$;

E) $4(m\cos\alpha \cdot \mathrm{tg}\frac{\alpha}{2})^2$.

40. Silindrning balandligi h va asosining radiusi r. Silindrga tashqi chizilgan sfera sirtining yuzi hisoblansin.

A) $\pi(2r^2-h^2)$; B) $\pi(4r^2-2h^2)$; C) $\pi(4r^2+h^2)$;
D) $\pi(r^2+2h^2)$; E) $\pi(2r^2+4h^2)$.

41. To'rt burchakli muntazam piramidaning yon qirrasi b va asosi tekisligi bilan α burchak tashkil qiladi. Piramidaga tashqi chizilgan sfera sirtining yuzi hisoblansin.

A) $\frac{\pi b^2}{\cos^2\alpha}$; B) $\pi b^2\sin^2\alpha$; C) $\frac{\pi b^2}{\sin^2\alpha}$; D) $\frac{\pi b^2}{\mathrm{tg}\alpha}$;

E) $\pi b^2\cos^2\alpha$;

42. Uch burchakli muntazam piramida asosining tomoni a, asosi qirrasidagi ikki yoqli burchak α. Piramidaga ichki chizilgan sfera sirtining yuzi hisoblansin.

A) $\frac{\pi}{3}a^2\cos 2\alpha$; B) $\frac{\pi}{2}a^2\sin^2\alpha$; C) $\frac{\pi}{4}a^2\text{ctg}^2\frac{\alpha}{2}$;

D) $\frac{\pi}{3}a^2\text{tg}^2\frac{\alpha}{2}$; E) $\frac{1}{3}a^2\text{ctg}\alpha$.

43. To'rt burchakli muntazam piramida asosining tomoni a, piramidaning ichidagi yassi burchak α. Piramidaga ichki chizilgan sfera sirtining yuzi hisoblansin.

A) $\pi a^2\text{ctg}^2 2\alpha$; B) $\frac{\pi a^2}{4}\cos\left(45^\circ-\frac{\alpha}{2}\right)$; C) $\pi a^2\text{ctg}\left(45^\circ+\frac{\alpha}{2}\right)$;

D) $\pi a^2\sin(60^\circ-\alpha)$; E) $\pi a^2\text{tg}\left(45^\circ-\frac{\alpha}{2}\right)$.

44. Sharga kesik konus ichki chizilgan. Kesik konusning asoslari sfera sirtini yuzlari 10π, 70π, 20π ga teng uchta qismlarga ajratadi. Kesik konusning hajmi hisoblansin.

A) 64π; B) $\frac{259\pi}{3}$; C) $\frac{264}{3}\pi$; D) $\frac{289\pi}{3}$; E) 32π.

45. Shar segmentining o'q kesimidagi yoyning kattaligi a bo'lib, shu segmentga hajmi V ga teng bo'lgan shar ichki chizilgan. Shar segmenti va ichki chizilgan shar hajmlarining ayirmasi topilsin.

A) $3V\cdot\text{ctg}^2\frac{\alpha}{4}$; B) $3V\cdot\text{tg}^2\frac{\alpha}{2}$; C) $2V\cdot\cos^2\frac{\alpha}{4}$;

D) $4V\cdot\cos^2\frac{\alpha}{4}$; E) $3V\cdot\sin 2\alpha$.

Mustaqil yechish uchun berilgan masalalarning javoblari

Paragraflar / Topshiriq raqamlari	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	D	A	B	E	C	D	A	D	B	BEC	B	C	C	D	B
2	E	E	D	C	D	B	C	A	A	D	D	D	A	A	D
3	A	D	A	B	E	A	B	D	C	A	A	C	D	C	C
4	C	B	E	A	D	C	D	E	E	E	C	A	B	B	E
5	B	C	C	D	B	D	E	C	B	C	E	B	E	E	A
6	C	A	B	E	D	A	B	A	D	B	D	E	C	D	B
7	A	B	C	B	E	E	D	C	A	D	B	C	A	A	D
8	E	C	D	A	B	CDB A	C	E	D	E	A	D	D	C	C
9	D	E	A	C	D	BCA DEA	A	D	E	C	E	A	B	B	E
10	B	D	E	D	C	A	E	C	B	A	C	B	E	E	A
11	E	B	C	A	A	DDA	B	B	B	B	A	C	C	D	B
12	D	C	A	B	E	ECB	D	A	C	E	E	E	D	A	D
13	A	E	B	C	A	A	C	E	A	B	B	A	A	B	C
14	C	A	D	E	C	EAB	E	C	D	D	D	D	B	E	E
15	B	D	E	A	E	D	A	D	C	E	C	B	D	C	A
16		B	A	D	D	E	C	B	B	A	A	C	E	D	B
17		E	C	B	B	C	D	D	A	C	E	E	C	A	D
18		A	B	E	A	A	E	B	E	D	B	A	A	B	C
19		C	D	B	D	B	A	C	C	A	D	D	B	E	E
20		B	E	C	E	D	D	E	B	B	A	B	D	D	B
21		D	A	B	C	C	B	A	C	D	C	C	E	C	A
22		A	C	E	B	B	E	D	E	C	E	A	C	B	D
23		E	B	D	A	E	D	B	A	B	B	E	A	A	C
24		E	D	A	E	B	C	C	B	D	D	D	B	E	E

Davomi

Paragraflar / Topshiriq raqamlari	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
25		B	E	C	D	A	E	E	D	C	A	B	D	D	B
26		D	C	B	C	C	A	A	C	A	C	C	E	C	A
27		E	A	A	B	E	B	D	B	E	D	A	C	B	C
28		A	B	C	D	D	C	E	D	C	B	E	B	A	D
29		C	D	E	A	B	D	B	C	B	E	D	A	E	E
30		B	C	D	E	A	E	C	A	D	A	B	D	D	B
31		A	E	B	C	B	A	A	B	A	C	C	E	C	C
32		D	B	A	B	D	D	D	E	E	D	A	C	B	A
33		E	A	C	D	B	E	B	C	C	B		B	A	D
34		C	B	D	E	C	C	E	B	D	E		A	E	E
35		B	C	E	A	C	B	C	D	B	A		D	D	B
36		E	D	A	B	D	A	A	E	A	C		E	C	C
37		A	E	B	C	D	D	A	A		D		C	B	A
38		D	A	C	E	A	E	B	B		B		B	A	E
39		C	D	E	D	C	B	C	C		E		D	E	B
40		E	B	D	A	E	C	D	E		D		A	D	C
41		A	C	E	B	B	A	E	D		C		E	C	A
42		C	E	C	E	E	D	A	A		B		C	B	D
43		D	A	B	C	C	E	B	B		A		D	A	E
44		B	D	A	D	D	B	C	C		D		A	D	B
45		E	C	D	E	E	A	D	D		E		B	C	A
46		C	E	B	A	B	C	E	E		B		E	E	
47		D	B	A	B	B	D	A	A		C		C	B	
48		A	D	C		E	E	B	B		D			D	
49		E	B	E		E	A	C	C		A				
50		B				C	B	D	D						
51								E	E						

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Bichimi 84×108$^1/_{32}$. Kegli 11 shponli. Tayms garn. Ofset bosma usulida bosildi. Bosma t. 19,0. Shartli b.t. 15,96. Shartli kr.-ott. 16,38. Nashr. t. 9,31. 40000 nusxada bosildi. Buyurtma № 15

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